Coding efficiency of CIE color spaces

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Abstract

Estimates were made of the efficiency with which color spaces code color information from images of natural scenes. Six spaces were tested, namely, CIE XYZ tristimulus space, and the spaces CIELUV, CIELAB, CIELAB and S-CIELAB after chromatic adaptation with CMCCAT2000, and the space CIECAM02. For each space, the information available and the information retrieved in color matching were calculated for images of 50 natural scenes under different daylight illuminants. The information available was decomposed into components associated with the individual variables of the space and the interactions between them, including redundancy and illuminant-dependence. It was found that the information retrieved was much less than the information available, and that its decomposition depended on the space. The differing efficiencies of the spaces were interpreted in relation to the effectiveness of opponent-color and chromatic-adaptation transformations, and the statistics of images of natural scenes.

Introduction

Color spaces are used routinely for specifying or describing the color of individual samples. But they may also be incorporated within more general image descriptions, for example, of natural scenes, which often contain a complex combination of spatial and chromatic detail. Each point in the image might have its color coded by its tristimulus values (X,Y,Z), or perhaps by the coordinates (L^*,a^*,b^*) of CIELAB space [1]. Although the choice of color space will depend on several factors, there is an advantage in choosing one that provides an efficient color code, in the sense of maximizing the amount of information about the scene provided by the description and minimizing any redundancy within it [2, 3, 4, 5].

The aim of this work was to analyze how efficiently information from a scene is coded by each of the main CIE color spaces. The images were of natural scenes under different illuminants, and information was expressed in terms of Shannon's mutual information [6].

Information was calculated by two methods. The first was based on the information theoretically available from images of a scene. It depends only on the statistical distribution of the color-code values in each image of the scene and how they vary with changes in illuminant. The second method was based on the information actually retrieved with a particular matching task, by which points in an image of the scene under one illuminant are matched, by color, to points in an image of the same scene under another illuminant. Estimates from the first method set an upper limit on estimates from the second.

As a precursor to the analysis, the theoretical section which follows gives the definition of mutual information and an explanation of its decomposition into components. These components are associated with the individual variables of a color space and the interactions between them, including redundancy and illuminant dependence. The methods section contains details of the two kinds of information estimator, along with a brief description of the scenes and illuminants. The results for six color spaces are then summarized. Differences in the information retrieved across spaces and in redundancy and the illuminant-dependent component are considered in the discussion section. Opponent-color and chromatic-adaptation transformations were both critical in determining the efficiency of coding and the retrieval of information.

Some partial results on the decomposition of the information available for different color spaces have been reported previously [7].

Theory

To fix ideas, suppose that the color at each point in an image of a scene under some illuminant E is coded by its tristimulus values (X,Y,Z), and consider, in particular, the luminance variable Y. Suppose that y is the value of Y at a particular point in an image of the scene under illuminant E and y' is the corresponding value in the image of the scene under illuminant E'. If the point is chosen randomly, then the values y and y' can be thought of as samples from random variables Y and Y', respectively. Suppose that the probability density functions of Y, Y', and of the pair (Y,Y') are f_E, $f_{E'}$, and $f_{EE'}$, respectively. Then the mutual information $I(Y;Y')$ between Y and Y' is given [6] by

$$I(Y;Y') = \iint f_{EE'}(y,y') \log \frac{f_{EE'}(y,y')}{f_E(y)\, f_{E'}(y')}\, \mathrm{d}y\mathrm{d}y' , \qquad (1)$$

where the integrations are taken over the spaces spanned by Y and Y'. The logarithm is to the base 2, and mutual information expressed is in bits.

Mutual information can be expressed as a combination of differential entropies [8], which are also based [6] on the probability density functions f_E, $f_{E'}$, and $f_{EE'}$, thus

$$\begin{aligned} h(Y) &= -\int f_E(y) \log f_E(y)\, \mathrm{d}y , \\ h(Y') &= -\int f_{E'}(y') \log f_{E'}(y')\, \mathrm{d}y' , \\ h(Y,Y') &= -\int f_{EE'}(y,y') \log f_{EE'}(y,y')\, \mathrm{d}y\mathrm{d}y' . \end{aligned} \qquad (2)$$

The mutual information (1) is then given by

$$I(Y;Y') = h(Y) + h(Y') - h(Y,Y') . \qquad (3)$$

The definition of mutual information can be straightforwardly extended from the single variables Y and Y' to the tristimulus values (X,Y,Z) and (X',Y',Z'), thus

$$\begin{aligned} I(X,Y,Z;X',Y',Z') &= h(X,Y,Z) + h(X',Y',Z') \\ &\quad - h(X,Y,Z,X',Y',Z') , \end{aligned} \qquad (4)$$

where all differential entropies are calculated as in (2) with the corresponding multivariate probability density functions. In an exactly analogous way, mutual information can be defined for the same two images of a scene for any other color space.

Redundancy and illuminant dependence

Again, suppose that coding is by tristimulus coordinates (X,Y,Z). To simplify notation, let I stand for the mutual information $I(X,Y,Z;\ X',Y',Z')$ as in (4) and let I_1, I_2, and I_3 stand for the individual mutual-information components associated with the first, second, and third variables; that is,

$$\begin{aligned} I_1 &= I(X;X'), \\ I_2 &= I(Y;Y'), \\ I_3 &= I(Z;Z'). \end{aligned} \tag{5}$$

The difference between I and the sum $I_1+I_2+I_3$ represents the contribution from the interactions between the variables of the color space. These interactions can be measured by the multi-information [9, 10], which is a form of generalization of mutual information [4], and which here divides into two components.

The first component is quantified by the multi-information between (X,Y,Z) and the multi-information between (X',Y',Z'); that is,

$$M(X;Y;Z) = h(X)+h(Y)+h(Z)-h(X,Y,Z)\,,$$

and

$$M(X';Y';Z') = h(X')+h(Y')+h(Z')-h(X',Y',Z')\,.$$

The sum of these two quantities defines an intrinsic redundancy R; that is,

$$R = M(X;Y;Z)+M(X';Y';Z')\,. \tag{6}$$

The redundancy R arises from the dependence between the variables of the space [4], [11], and should not be confused with the notion of redundancy considered by Barlow and others [2].

The second component is quantified by the multi-information between (X,X'), (Y,Y'), and (Z,Z'). This quantity defines an extrinsic component D; that is,

$$\begin{aligned} D &= M(X,X';Y,Y';Z,Z'), \\ &= h(X,X')+h(Y,Y')+h(Z,Z'), \\ &\quad -h(X,X',Y,Y',Z,Z'). \end{aligned} \tag{7}$$

This component is strongly illuminant dependent.

It may then be shown that the mutual information I in (4) has the following decomposition

$$I = I_1+I_2+I_3-R+D\,. \tag{8}$$

Both R and D are necessarily positive. An efficient representation is one that maximizes $I_1+I_2+I_3$ and minimizes R and D [4, 11].

Methods

The information available and the information retrieved were estimated separately for the chosen set of images of natural scenes under different daylight illuminants.

Information available estimator

For each scene, daylight change, and color space, the individual information components I_1, I_2, and I_3 (5), the redundancy R (6), and illuminant-dependent component D (7) were obtained from the corresponding differential entropies (2). The information available I was then calculated from (8).

A modification of the Kozachenko-Leonenko estimator of differential entropy [8] was used to obtain the individual entropies. This modification involved estimating each entropy after previously whitening the data (so that the variance-covariance matrix coincided with the identity), which gave better estimates. If $\mathrm{Var}(X,Y,Z)$ is the variance-covariance matrix of the variables (X,Y,Z), then the differential entropy $h(X,Y,Z)$ is given [6] by

$$h(X,Y,Z) = h(X^*,Y^*,Z^*)+\frac{1}{2}\log|\mathrm{Var}(X,Y,Z)|, \tag{9}$$

where $|\cdot|$ denotes the determinant of a matrix and

$$\begin{pmatrix} X^* \\ Y^* \\ Z^* \end{pmatrix} = [\mathrm{Var}(X,Y,Z)]^{-1/2} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}.$$

Information retrieved estimator

With a complex scene, matching points in an image of the scene under one illuminant, solely on the basis of their color, to points in an image of the same scene under another illuminant is an uncertain process. To take a trivial example, the tristimulus values (X,Y,Z) of a point under illuminant E will not, in general, coincide with its tristimulus values (X',Y',Z') under illuminant E', so that a nearest-neighbor match will generally be wrong. By introducing a chromatic-adaptation transformation in the coding, such as CMCCAT2000 [12], errors lessen, but they do not disappear.

A relevant way to quantify the error in matching is to measure how far the incorrect match i is from the correct match j in terms of the number k_j of potential matches that are closer to j than i [13]. To accommodate the variance-covariance structure of the data, color matching was based not on the simple Euclidean distance between variables but on the Mahalanobis distance. The relative frequencies of these numbers k_j may then be interpreted as estimates of a probability mass function $\{p_0,p_1,\ldots,p_k,\ldots,p_{N-1}\}$ of a random variable K, so that $P\{K=k\}=p_k$. The discrete entropy of K [6] is

$$H(K) = -\sum_{k=0}^{N-1} p_k \log p_k\,,$$

where $\log 0 = 0$, as in [6]. From this discrete entropy, a mutual information I_{match} can be obtained in a way analogous to that with differential entropy (3), but taking the form

$$I_{\mathrm{match}} = \log N - H(K). \tag{10}$$

This mutual information quantifies the information retrieved. Unlike the information available, I_{match} depends strongly on the structure of the color space.

Color spaces and image data

The color spaces considered were as follows.

1. CIE XYZ [1],
2. CIELUV [1],
3. CIELAB [1],
4. CIELAB [1] with CMCCAT2000 [12, 1],
5. S-CIELAB [14] with CMCCAT2000 [12, 1],
6. CIECAM02 [15].

All were used with their default values for a 2-degree observer, and adaptation was assumed to be complete.

Image data were generated from 50 hyperspectral images ($\leq$ 1344×1024 pixels) of natural rural and urban scenes [16, 17] under three representative changes in daylight illuminant from a correlated color temperature of 4000 K to 6500 K, from 25,000 K to 6500 K, and from 25,000 K to 4000 K.The particular daylight illuminants were chosen because of their special role in the CIE specification [1], and were approximated by the CIE method for reconstructing illuminants [18]. Tristimulus values (X,Y,Z) at each point in the image of a scene were obtained from the CIE 1931 color-matching functions [1]. To reduce computation time, images were spatially subsampled, with just alternate pixels being used.

Results and Comment

Table 1 shows the mean information available and the mean information retrieved by matching across images of scenes for each of the six color spaces. The change in illuminant was from a daylight of correlated color temperature 4000 K to one of 6500 K. Means were taken over the 50 scenes. Standard deviations are shown in parentheses.

Table 1. Mean (SD) of information available and information retrieved with color images of natural scenes[a]

Color space	Available bits[b]	Retrieved bits[c]
CIE XYZ	20.4 (1.1)	7.4 (0.8)
CIELUV	19.8 (1.4)	8.2 (0.8)
CIELAB	20.1 (1.1)	8.5 (1.0)
CIELAB, CMCCAT2000	20.3 (1.1)	11.6 (1.3)
S-CIELAB, CMCCAT2000	19.8 (1.7)	12.2 (1.4)
CIECAM02	20.0 (1.0)	11.9 (1.2)

[a] Information was estimated from images of a scene under a change in illuminant from a daylight of correlated color temperature 4000 K to one of 6500 K. Means were taken over 50 scenes. Standard deviations are shown in parentheses.
[b] Estimate I from (8) and (9).
[c] Estimate I_{match} from (10).

The mean information available was closely similar over all the spaces at approximately 20 bits. For a change in correlated color temperature from 25,000 K to 6500 K, the estimates were very similar, but for a larger change, from 25,000 K to 4000 K (not shown here), the mean information available was lower, by approximately 3 bits.

The mean information retrieved by color matching was much less than the mean information available, particularly so with CIE XYZ, CIELUV, and CIELAB spaces. Some reasons for these differences are considered in the discussion section.

Figure 1 shows the decomposition of the mean information available for each of the six color spaces. The values shown are the mutual-information components I_1, I_2, and I_3 associated with the individual variables of the space (5), the redundancy R (6), and the illuminant-dependent component D (7), each averaged over the 50 scenes with a change in illuminant from a daylight of correlated color temperature 4000 K to one of 6500 K. Error bars indicate ±1 SD of the sample (the relatively large SDs of R and D in Fig. 1 are not necessarily inconsistent with the small SDs of the information available I in Table 1).

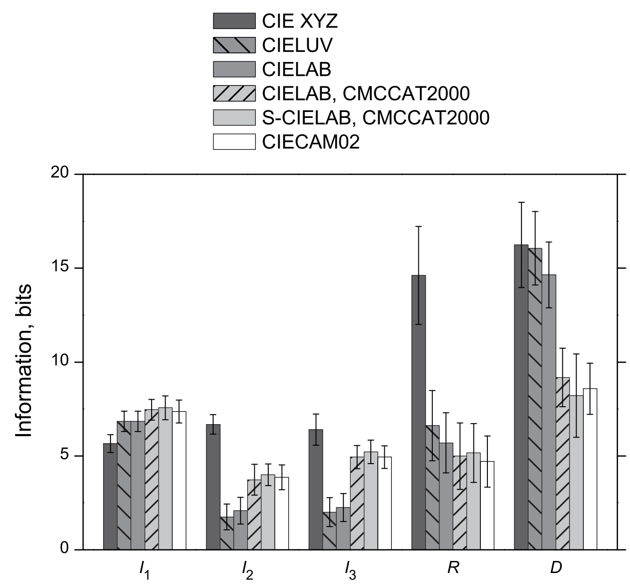

Figure 1. *Decomposition of the mean information available for six color spaces. The mutual information associated with the individual variables (3, 5) is indicated by I_1, I_2, and I_3, and the redundancy (6) and illuminant-dependent component (7) are indicated by R and D, respectively. Information was estimated from images of a scene under a change in illuminant from a daylight of correlated color temperature 4000 K to one of 6500 K. Results were averaged over 50 scenes. Error bars indicate ±1 SD of the sample.*

A similar pattern of performance was found with the other two illuminant changes (also not shown here).

Discussion

The color spaces considered here were smoothly invertible functions of each other. As a consequence, the information available with each should, in principle [19], have been the same for each set of images and illuminant changes. Despite the differences in the complexity of the spaces, with CIE XYZ tristimulus space and the color-appearance model CIECAM02 being the most extreme, the information available for each was almost constant. The small differences in the information available listed in Table 1 reflect different statistical errors in the estimates.

Although an inverse of the CIECAM02 model has been described [15], technical anomalies have been reported [20], the most important of which is the "brightness problem" [21]: the calculated brightness may fail to be a real number. A solution has been proposed [21], which was adopted here, but it leads to the non-invertibility of the model. Fortunately, the brightness problem occurs rarely in practice, and was not found here with daylights with correlated color temperatures 4000 K and 6500 K and only infrequently with 25,000 K.

The information available was, however, coded with varying efficiency (Fig. 1). The spaces CIELAB and S-CIELAB, each with the chromatic-adaptation transformation CMCCAT2000, and the space CIECAM02 had illuminant-dependent components D about half the size of those for CIE XYZ, CIELUV, and CIELAB, presumably because of the absence or inadequacies of their chromatic-adaptation transformations. By contrast, the levels of redundancy R were broadly similar for all the spaces, except CIE XYZ. This was the only space not to have an opponent-color transformation, which would have reduced the dependence between variables [22], especially between tristimulus values X and Y.

As noted earlier, the information available sets a theoretical limit to the maximum information that can be retrieved with a given set of scenes and illuminants. The information retrieved in color matching was, on average, about half of that available (Table 1), but some spaces were much poorer than the best, most notably, CIE XYZ, and slightly less so, CIELUV and CIELAB.

Two factors potentially underlying the poor performance in color matching have already been identified in considering coding efficiency: the effectiveness of opponent-color and chromatic-adaptation transformations. A third, more general factor affecting color-matching performance can be traced to the relationship between the metric used to make nearest-neighbor matches and the statistics of the images, specifically, the probability density distributions of color-code values and the differences in these values. For color matching to retrieve the maximum information possible, the distributions need to be Gaussian [6], [23], but for none of the spaces is this true. Examples of the non-Gaussian distributions of color differences under CIEDE2000 for images of natural scenes have been presented elsewhere [24].

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