Reflectance Prediction in Multi-Angle Measurement by Wiener Estimation Method

Pesal Koirala, Markku Hauta-Kasari, Jouni Hiltunen and Jussi Parkkinen
Department of Computer Science and Statistics, University of Joensuu , P.O. Box 111, FIN-80101 Joensuu, Finland
E-mail: pkoirala@cs.joensuu.fi

Abstract

Goodness of the reflectance prediction in multi-angle measurement of metallic and pearlescent samples mainly depends on the viewing and illumination directions. This study focuses on predicting the reflectance for all viewing directions from the reflectance for few number of selected best primary angles. The experiment was done on metallic and pearlescent samples. The principle component analysis and wiener estimation method were used to find best primary angles and predict the reflectance for all viewing angles. It has been found that wiener estimation method with higher polynomial order of Principle Components (PC) reduces prediction error significantly. Reflectance for only three best primary angles are sufficient to predict reflectance for all viewing angles between $125°$ *to* $-35°$ *in aspecular directions, if the estimation function was calculated from the training set of similar types. Similarly this experiment shows that reflectance for only five best primary angles are sufficient to predict reflectance for all viewing angles independent of the sample types.*

Keywords: *Reflectance, Pearlescent, Metallic, PCA, Wiener Estimation, CMC(l:c).*

Introduction

The aim of this study is to estimate the reflectance for large number of viewing directions from the reflectance for few number of primary angles using wiener estimation [8] on the top of Principle component analysis (PCA)[2]. The experiment was conducted on metallic samples coated with PVDF with flakes, Metallic samples coated with PVDF and Polyster and Pearlescent samples. Integrating sphere and $45°/0°$ geometry have been used traditionally for most of paints that absorb incident light and the rest is diffusely scattered as a result perceived color is independent of measuring geometry including the illumination and viewing angles. In contrary to this, the brightness of the metallic coating depends on the viewing angles but is independent of the illumination angles and in pearl interference pigments or pearlescent coating, perceived chroma, hue and brightness depend on both illumination and viewing angles [4]. In multi-angle measurement, measured color is simply a function of the angle away from specular [6]. So, multi-angle measurement is essential for metallic and pearlescent samples to assess the reflectance accurately in different viewing angles. The understanding of reflectance characteristics in multi-angle of these type of specular and glossy sample is useful for the quality assessment, sample characterization in terms of color and glossiness and is also useful in the complex color rendering process in computer graphics application. The measurement of reflectance for all viewing angles and for different illumination angles is not the feasible solution from the practical point of view since it costs more time and larger memory space. To avoid these situation, few number of primary angles should be selected in the way less color difference during prediction could be achieved. The primary angles and viewing angles are defined as aspecular angles. Aspecular angle is $0°$ in the specular direction and increases towards the normal of the surface from specular direction and decreases in the opposite direction. The measurement geometry has been shown in Figure 1.

There are different proposals for primary angles depending on the type of materials, estimation process, and color difference and spectral reflectance difference. Three primary angles, one near specular at $15°$, one far from specular at $110°$ and the third in between at $45°$ for the measurement of metallic surface were proposed [6]. Similarly $20°$, $45°$ and $70°$ were used to find more better result than previous geometry [5]. It was tested with both metallic and pearlescent samples. Both methods used polynomial modeling approach as the estimation method. Similarly combination of primary angles $25°$, $45°$ and $110°$ gave the highest correlation with visual assessment for the metallic paint films [11].

The ASTM recommends the aspecular angles $15°$, $45°$ and $110°$ and DIN recommends the aspecular angles of $25°$, $45°$, and $75°$ for the metallic samples [1]. Aspecular angles of $15°$, $35°$, $45°$, $70°$ and $85°$ viewing angles for illumination direction of $15°$, $45°$ and $65°$ have been proposed for pearlescent pigment [4]. Similarly Aspecular angles of $10°$, $18°$, $28°$, $40°$ and $90°$ viewing angles for illumination of $60°$ have been proposed for all type of painted surface [3]. Similarly the best angles can selected from the local minima and maxima points produced by the reconstruction error of principle component analysis [10]. These methods give quite good solution but not the optimal one.

In this study we selected best primary angles according to average minimum color difference of CMC(2:1) as prediction error function. We applied the wiener estimation method to the linear relationship between principle components of reflectance for primary angles and reflectance for all viewing angles. As the order of polynomial of principle components were increased, the better prediction results were achieved. Our result showed that up to fifth order of polynomial of principle components improves the prediction result significantly. In addition to that our result showed that reflectance for the five best primary angles are sufficient to predict the reflectance in all viewing angles between 125° to −35° in aspecular direction using the wiener estimation function from the training set containing metallic and pearlescent samples. The reflectance for three best primary angles are sufficient to predict reflectance for mentioned viewing angles for each type of samples, in the case if the estimation function was calculated from the training set containing similar type of samples. The three primary angles found with best combination are $[115^\circ \quad 25^\circ \quad -30^\circ]$, $[75^\circ \quad 20^\circ \quad -10^\circ]$ and $[70^\circ \quad 20^\circ \quad -15^\circ]$ for metallic coated with PVDF with flake, metallic coated with PVDF and polyster and pearlescent respectively. The five primary angles found with best combination independent of sample types are $[120^\circ \quad 70^\circ \quad 35^\circ \quad 10^\circ \quad -25^\circ]$.

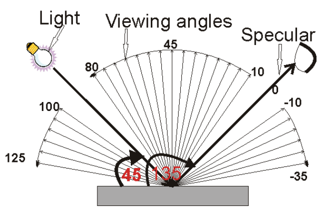

Figure 1. Measurement geometry, Angles shown across the viewing directions are aspecular angles. Aspecular angle is 0° in specular direction

Measurement

The intensity signal (S) of the samples at different angles were calculated using Hamamatsu Photenic Multichannel Analyzer within the visible range of 380 nm to 780 nm with 5 nm step under the light source halogen lamp with D65 filter. The position of the light source was set at 45° from the surface for all viewing directions. The reflectance of the samples was calculated using Eq. (1).

$$R_x(\lambda,\theta) = \frac{S_x(\lambda,\theta) - S_k(\lambda,\theta)}{S_w(\lambda,\theta) - S_k(\lambda,\theta)} R_w(\lambda,\theta) \tag{1}$$

Where λ and θ are the wavelengths and viewing angles respectively. S_x, S_w and S_k are the measured signals from samples, standard white and dark respectively. R_x and R_w are the reflectance for the sample and calibrated value for a white standard. The reflectance of the samples may exceed value one in specular direction since the sample is more glossy and specular than that of used white reference. Altogether fortyfive different samples were used in training set. Out of fortyfive samples, twelve samples were pearlescent, twenty one samples were metallic coated with Polyvinylidene Fluoride (PVDF) with flakes and six samples were metallic coated with polyester and remaining samples were metallic coated with PVDF. All these samples were divided in three classes a) Pearlescent b) Metallic with PVDF coating with mica c) Metalic coated with Polyster and PVDF. The classification was made according to the distribution of reflectance across viewing directions. The reflectance characteristics of these three types of samples are shown in Figure. 2. $X-axis$ shows the wavelength, $Y-axis$ shows the aspecular viewing direction and $Z-axis$ is the reflectance value.

The reflectance measured for each sample in 123 different viewing angles [125° to 100° 80° to 10° −10° to −35°] in aspecular direction with the sampling of one degree were used as training set as shown in Figure 1. The reflectance measured between aspecualr angles 135° to 124° and −36° to −45° produces the noise due to low reflectance value near to zero, sometimes goes to negative value so those angles were not considered for the measurement . Similarly reflectance between aspeucular angles 101° to 79° could not be measured since camera obscures the light source and produces the shadow in the surrounding angles. In the proximity of specular directions between 9° to −9° the reflectance gets saturated for large number of samples so the reflectance for these angles were not considered for the measurement.

Principle Component of Primary Reflectance

The idea of PCA is to reduce the dimensionality of a data set, consisting of large number of interrelated variables, while retaining as much as possible variations present in the data set. This is achieved by transforming to a new set of uncorrelated variables called the principle components (PC). The PC are ordered so that first few dimension retains most of the variations present in all of the original variables [2]. Here PCA gives the linear relation of reflectance for all viewing directions to the principle components of reflectance for best primary angles. This linear relation has been utilized to find Wiener estimation function for the higher polynomial order of PC.

The first step of PCA is to have original data preferably mean subtracted data. In this experiment, the data or training set is collection of reflectance of different samples in all viewing angles. The reflectance of training set for all different viewing angles of all different samples is represented in 2D matrix form in Eq. (2).

$$R = \begin{bmatrix} R_1(\theta_1) & .. & .. & R_m(\theta_1) \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ R_1(\theta_n) & .. & .. & R_m(\theta_n) \end{bmatrix} \tag{2}$$

If the matrix R is defined for each wavelength separately, the size of each R is $r \times c$, where r is number of viewing angles n and c is number of samples m. Similarly if the single matrix R is defined for all wavelength at a time, $R_i(\theta_j)$ is the reflectance value in all wavelengths of i^{th} sample at j^{th} viewing angle and is represented as $R_i(\theta_j) = \left[R_i^{380}(\theta_j) - - - R_i^{780}(\theta_j)\right]$. Then the size of matrix R is $r \times c$, where r is number of viewing angles and $c = m \times k$, m is number of samples and k is number of wavelengths. The correlation matrix K of the data set R is calculated as
$K = \frac{1}{c-1} R R^T$.
Here $[]^T$ denotes the transpose of the matrix. If R is mean sub-

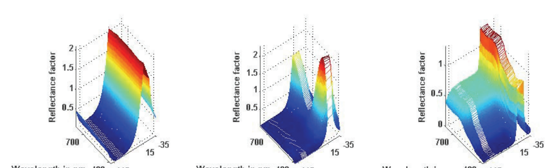

Figure 2. Reflectance characteristics of metallic with PVDF coating and mica (gray color), pearl sample and metallic with polyester coating (left to right)

tracted data, the correlation matrix K will be exactly covariance matrix. For the correlation matrix K, the eigen equation $K\nu = \sigma\nu$ gets satisfied. ν and σ are eigen vectors and eigen values respectively of size $n \times n$. The eigen value σ is a diagonal matrix. The eigen vectors are the orthogonal matrix. The eigen vectors corresponding to p largest eigen values are orthogonal basis function B. The size of basis matrix B is $n \times p$. The number of eigen vectors as basis functions were chosen according to the information content termed as fidelity ratio. The fidelity ratio f for the first p eigen vectors are calculated as the ratio of the sum of first p eigen values to the sum of total eigen values as shown in Eq. (3).

$$f = \frac{\sum_{i=1}^{p} \sigma_i}{\sum_{i=1}^{n} \sigma_i} 100 \tag{3}$$

During the data reduction process principle component P is calculated as in Eq. (4).

$$P = B^T R \tag{4}$$

For an orthogonal basis function B , the elements of P are optimally mutually uncorrelated. The basis function represented in matrix form is:

$$B = \begin{bmatrix} \nu_1(\theta_1) & .. & .. & \nu_p(\theta_1) \\ : & .. & .. & : \\ : & .. & .. & : \\ \nu_1(\theta_n) & .. & .. & \nu_p(\theta_n) \end{bmatrix}$$

The reconstruction of reflectance is calculated from Eq. (5).

$$R \approx \tilde{R} = B\ P \tag{5}$$

Reflectance for all viewing angles have been estimated by the linear combination of first p principle components as shown in Eq. (5). But the goal is to predict the reflectance for all viewing angles from the reflectance for p number of primary angles of the test set from the known basis function B derived from training set. Here the main problem is , we do not know the principle component P_t of the test reflectance for primary angles. The principle component P_t is solved as shown in Eq. (6) using test reflectance for primary angles and basis function corresponding to p angles. Here the primary angles are defined as $\alpha = (\alpha_1, ..., \alpha_p)$ where set of primary angles α should be the element of set of total viewing angles θ.

$$P_t = \begin{bmatrix} \nu_1(\alpha_1) & . & \nu_p(\alpha_1) \\ : & . & : \\ : & . & : \\ \nu_1(\alpha_p) & . & \nu_p(\alpha_p) \end{bmatrix}^T \begin{bmatrix} R_1(\alpha_1) \\ : \\ : \\ R_p(\alpha_p) \end{bmatrix} \tag{6}$$

Here $R_i(\alpha_j)$ is $\left[R_i^{380}(\alpha_j) - - - R_i^{780}(\alpha_j)\right]$, if the prediction is done by using the basis function calculated for training set matrix R using all wavelength at a time. Otherwise $R_i(\alpha_j)$ is reflectance value for each wavelength and P_t should be calculated for each wavelength. i and j indicate the index of principle component and primary angles respectively. The prediction of reflectance of test sample for all viewing angle is achieved as shown in Eq. (7)

$$R \approx \tilde{R} = B\ P_t \tag{7}$$

Wiener Estimation

The wiener estimation method is traditionally used to estimate the data sets in larger dimensional space from lower dimensional space. In this study, this method has been employed to estimate reflectance for all viewing angles from the reflectance for few number of selected primary viewing angles. The wiener estimation method is quite simple and provides accurate estimation [3].

The Wiener estimation rule to estimate the reflectance R for all viewing angles from reflectance r for few primary angles with mapping function G is shown in (8).

$$R = GP \tag{8}$$

Here P is principle component calculated for reflectance r as shown in Eq. (6). In Eq. (8), the size of R is $n \times m$, size of G is $n \times p$ and size of P is $p \times m$. Here n is number of total viewing angles, m is number of samples, and p is number of primary angles. If single calculation was done for all wavelengths, then new m is product of number of samples m and number of wavelengths k, $m = m * k$. For the first order of polynomial Eq. (8) is exactly the same as Eq. (5). The number of primary angles were selected according to fidelity value calculated from PCA. The fidelity value chosen was ≥ 99.9 percent. The purpose of the estimation matrix G is to minimize the square error between original R and estimated $\tilde{R}$ [8].
$e = \mid R - \tilde{R} \mid \longrightarrow min$
The estimation matrix G is explicitly represented in Eq. (9) [8].

$$G = C_{RP} C_{PP}^{-1} \tag{9}$$

The notation $[]^{-1}$ indicates the inverse of the matrix. Matrix C_{RP} is cross correlation between matrices R and P. Matrix C_{PP} is auto correlation of matrix P. The C_{RP} and C_{PP} are calculated as shown in Eq. (10).

$$C_{RP} = \frac{RP^T}{m-1} \quad , \quad C_{PP} = \frac{PP^T}{m-1} \tag{10}$$

Here notation $[]^T$ denotes the transpose of matrix. After calculating C_{RP} and C_{PP}, the estimation matrix G is calculated from Eq. (9). After having estimation function G calculated from training set, the estimation of reflectance $\tilde{R}$ for all viewing angles from principle components P of reflectance for particular primary angles are achieved from Eq. (8).

In this experiment, we tested our result using first order polynomial to fifth order polynomial of principle components. The predicted results were improved considerably as the order of polynomial increases. For the first order polynomial of principle component P of reflectance r for three primary angles $\alpha 1$, $\alpha 2$ and $\alpha 3$ is arranged as $P_1 = [P_{\alpha 1}\ \ P_{\alpha 2}\ \ P_{\alpha 3}]$. In the second order polynomial for three primary angles we appended the term $P_2 = [P_{\alpha 1} * P_{\alpha 1}\quad P_{\alpha 1} * P_{\alpha 2}\quad P_{\alpha 1} * P_{\alpha 3}\quad P_{\alpha 2} * P_{\alpha 2}\quad P_{\alpha 2} * P_{\alpha 3}\quad P_{\alpha 3} * P_{\alpha 3}]$. Similarly the third order polynomial consists the term $P_3 = [P_{\alpha 1} * P_{\alpha 1} * P_{\alpha 1}\quad P_{\alpha 1} * P_{\alpha 1} * P_{\alpha 2}\quad P_{\alpha 1} * P_{\alpha 1} * P_{\alpha 3}\quad P_{\alpha 1} * P_{\alpha 2} * P_{\alpha 2}\quad P_{\alpha 1} * P_{\alpha 1} * P_{\alpha 3}\quad P_{\alpha 1} * P_{\alpha 2} * P_{\alpha 2}\quad P_{\alpha 1} * P_{\alpha 2} * P_{\alpha 3}\quad P_{\alpha 1} * P_{\alpha 3} * P_{\alpha 3}\quad P_{\alpha 2} * P_{\alpha 2} * P_{\alpha 2}\quad P_{\alpha 2} * P_{\alpha 2} * P_{\alpha 3}\quad P_{\alpha 2} * P_{\alpha 3} * P_{\alpha 3}\quad P_{\alpha 3} * P_{\alpha 3} * P_{\alpha 3}]$. So for the second order polynomial $P = [P_1\ \ P_2]$ and for the third order polynomial $P = [P_1\ P_2\ P_3]$ are used to calculate estimation function. Similarly P can be arranged to higher order of polynomials for more number of primary angles. From Eq. (9) the estimation function G is derived. As the order of polynomial increases, the size of estimation function increases accordingly. The prediction results up to fifth order polynomial improves significantly.

Error Measures

In this study, we used root-mean-square error (RMSE) to check the goodness of predicted reflectance. The color difference formula ΔE from the society of Dyers and Colourists Color Measurement Committee CMC(l:c) was used to measure the color difference of the predicted result from the measured one. The RMSE was calculated as shown in Eq. (11).

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(R_m(i) - R_p(i))^2}{n}} \tag{11}$$

In Eq. (11) R_m and R_p are the measured and predicted reflectance factors at the same angle and n is the number of wavelengths.

To calculate ΔE for CMC(l:c), the luminance L^* and chrominance a^* and b^* were calculated, then chroma C^*_{ab} and hue h^*_{ab} were derived using International Commission on Illumination CIE 1976 $L^*a^*b^*$ system with standard day light source CIE $D65$ and 1931 standard observer CIE 2°. Then the color difference was calculated using Eq. (12).

$$\Delta E_{CMC}(l:c) = \sqrt{\left(\frac{L^*_m - L^*_p}{lS_L}\right)^2 + \left(\frac{C^*_m - C^*_p}{cS_c}\right)^2 + \left(\frac{h^*_m - h^*_p}{S_H}\right)^2} \tag{12}$$

In Eq. (12) the subscript m and p are the notations for the measured and predicted ones. l and c are the tolerances applied respectively in difference in lightness and chroma relative to hue difference. The c is usually smaller than l because human perceives smaller shifts in chroma than in lightness. In our experiment we have choosed ratio l:c equal to 2:1. The $\Delta E_{CMC}(l:c)$ is based on the optimization method to convert non uniform ellipsoidal in LAB space to more uniform spherical in LCH space. In Eq. (12) S_L, S_C and S_H are the relative attribute difference for luminance , chroma and hue calculated from the standard observer [9].

Results

The samples in the training sets were categorized in three groups a) PVDF coated metallic with flakes (metallic 1) b) PVDF and Polyester coated metallic (metallic 2) c) Pearlescent. The fidelity ratios calculated for each group and altogether have been listed in Table 1. The fidelity ratio suggests that only three principle components are enough to retain ≥ 99.9 percent of information, if the reconstruction was done from the training set containing the same group of samples. Similarly five principle components are enough to retain ≥ 99.9 percent of information, if the reconstruction was done from the training set containing all groups of samples. On that basis three best primary angles are enough to predict reflectance in all viewing angles for each type of samples using the estimation function calculated from the training set containing same group of samples. Similarly five best primary angles are enough to predict reflectance independent of the type of samples. Figure 4 shows the prediction error for different types of samples calculated from the basis function independent of sample types. There are methods to find the primary angles from the minimum and maximum points of principle component vector [3] and from the local maximum and minimum points of the reconstruction error. However these methods do not provide the optimal solution. The sequential backward selection method [12] can be used to find the primary angles and it requires the $(Nn - \frac{n(n-1)}{2})$ number of operations to choose n best primary angles from N set of angles as a sub optimal technique. The full search requires the $\frac{N!}{(N-n)!n!}$ number of operations. Here []! is factorial. As the number of viewing angles and number of required primary angles increase, the full search method becomes more inconvenient due to time complexity. In this study , as the first step we reduced the viewing angles from total 123 angles to 10 best viewing angles using sequential backward selection method and applied the full search method to find best primary angles among 10 best angles. The best three primary angles, and prediction error for metallic 1, metallic 2 and pearlescent have been shown in Table 4. The results in Table 4 were obtained using the training set of the same group of samples and wiener estimation of order five. Table 2 shows the 1^{st} to 5^{th} best angles and prediction error. The results in Table 2 were calculated using the training set containing all types (Metallic 1, Metallic 2 and Pearlescent) of samples using wiener estimation with polynomial order 5. Figure 5 shows the luminance, chroma and hue of the measured and predicted reflectance using five best primary angles and wiener estimation with order 5.

The prediction of the reflectance in all viewing angles was done by using wiener estimation method with polynomial order five of the principle components of reflectance for primary viewing angles. It was found that higher order of polynomial of principle components decreased the prediction error significantly. There was no significant decrement of prediction error after polynomial order five as shown in Figure 3. As a result polynomial order 5 is suitable for wiener estimation in multi-angle reflectance prediction. Table 3 shows the gradual improvement of prediction error due to increasing order of polynomial. When the order of polynomial is one, it is almost the same result predicted by linear PCA. The two-way analysis of variance (ANOVA) with factors viewing angles and sample type (Metallic 1, Metallic 2 and pearlescent) were computed. The criterion variables are mean values of color differences between measured reflectance and predicted reflectance using five best primary angles. The results of ANOVA in Table 5 show that the interaction between viewing angle and sample lies below one percent significance probability, that means reflectance for the five best primary angles can be used to predict reflectance in all viewing angles independent of the sample types.

Conclusions

The wiener estimation based reflectance prediction in multi-angle measurement for metallic and pearlescent samples has been proposed. The wiener estimation was applied to the linear relationship provided by principle component analysis. The wiener estimation method up to the fifth order of polynomial of principle components has improved prediction results significantly. The experiment shows that five best primary angles are sufficient to predict reflectance in all viewing angles independent of the types of samples. Additionally it has been found that only three best primary angles are sufficient to predict reflectance for all viewing angles using the estimation function calculated from the similar type of samples. As a future work, the number of samples will be increased in training set containing wide range of material types, colors and gloss value so that more stable viewing angles could be achieved.

References

[1] Roy S. Berns, Billmeyer and Saltzman's Principles of Color Technology, John Wiley & Sons, VA, 2000, pg.75-105.

[2] Jolliffe I.T., Principle Component Analysis, Springer series of statistics, Springer, 2002.

[3] Atsushi Takagi, Akihiro Watanabe and Grorw Baba, Prediction of Spectral Reflectance Factor Distribution of Automotive Paint Finishes, Color Res. and Appl., 30, 4 (2005).

[4] Maria E. Nadal and Edward A. Early, Color Measurements for Pearlescent Coatings, Color Res. and Appl., 29, 1 (2004).

[5] Allan B.j.Rodrigues and Troy, Measurement of Metallic and Pearlescent Colors, DIE FARBE 37 (1990), pg. 65-78

[6] Alman D. H., Directional color measurement of metallic flake finishes. proc. ISCC Williamsburg Conference on Appearence 1987, pg. 53

[7] H. Haneishi, T. Hasegawa and A. Hosoi, System Design for Accurately Estimating the Spectral Reflectance of Art paintings, Applied Optics 39 (2000), pg. 6621-6632.

[8] P. Stiggel, K. Miyata and M. Hauta-Kasari, Wiener Estimation Method in Estimating of Spectral Reflectance from RGB images, Pattern Recognition and Image Analysis 17 (2007), pg. 233-242.

[9] K J Smith, Colour-order systems, colour spaces, colour difference anc colour scales, Colour Physics for Industry, edition 2, edited by Roderick McDonald, 1997, pg.151-155.

[10] Koirala, P., Hauta-Kasari, M., Hiltunen, J. and Parkkinen, J., Accurate reflectance prediction in multi-angle measurement. CGIV 2008/MCS'08, 4th European Conference on Colour in Graphics, Imaging, and Vision, Terrassa, Spain, June 9-13, 2008, pp.489-493.

[11] Saris H.J.A., R.J.B. Gottenbos and H.van Houewelinegen, Correlation between visual and instrumental color differences of metallic paint films. Color Res. Appl. 15 (1990), pg. 200-205.

[12] Sergios Theodoridis and Konstantinos Koutroumbas,Pattern Recognition, Third Edition, Academic Press, Inc. , 2006.

Author Biography

Pesal Koirala is a PhD student in the Department of Computer Science and Statistics at the University of Joensuu, Finland. He received his MSc in Computer Science from the University of Joensuu in 2006 and BE in Computer Engineering from the Kathmandu University, Nepal in 2001. His research interest includes spectral imaging and pattern recognition.

Table 1. Fidelity ratio **calculated from training set containing only Metallic with PVDF with mica (Metallic 1), Metallic with PVDF and Polyster (Metallic 2), Pearlescent and all types of samples (All).**

PC	Metallic 1	Metallic 2	Pearlescent	All
1^{st}	98.82	96.14	84.56	89.43
$1^{st}-2^{nd}$	99.77	99.76	99.85	98.48
$1^{st}-3^{nd}$	99.90	99.97	99.93	99.45
$1^{st}-4^{nd}$	99.95	99.98	99.98	99.80
$1^{st}-5^{nd}$	99.97	99.99	99.99	99.89

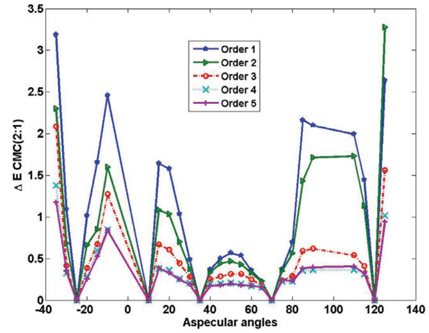

Figure 3. *Prediction error by wiener estimation method using first to fifth order of polynomial using five primary angles.*

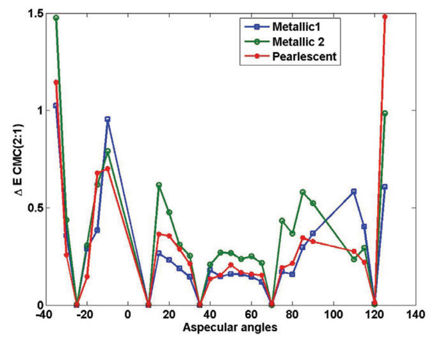

Figure 4. *Prediction error for different samples by wiener estimation method using fifth order of polynomial using five primary angles.*

Table 2. **Prediction result Mean values (μ) and Maximum values (m) of color difference (($\Delta E\ CMC(2:1)$) and the Root mean square errors (RMSE) according to best angles for all type of samples.**

Best angles	$\Delta E\ CMC(2:1)$		RMSE	
	μ	m	μ	m
$40°$	6.863	12.287	0.133	0.442
$70°20°$	3.089	10.688	0.040	0.212
$70°25°-20°$	1.425	5.262	0.020	0.116
$70°30°15°-20°$	0.762	3.587	0.009	0.045
$120°70°35°10°-25°$	0.309	1.177	0.004	0.022

Table 3. **Prediction from five best primary angles using wiener estimation with first to fifth order of polynomials of PC, mean values (μ) and maximum values (m) of color difference ($\Delta E\ CMC(2:1)$), and Root mean square errors (RMSE).**

Order	$\Delta E\ CMC(2:1)$		RMSE	
	μ	m	μ	m
1	1.029	4.522	0.617	0.099
2	0.695	3.579	0.008	0.047
3	0.389	1.670	0.005	0.028
4	0.310	1.520	0.004	0.020
5	0.277	1.427	0.003	0.018

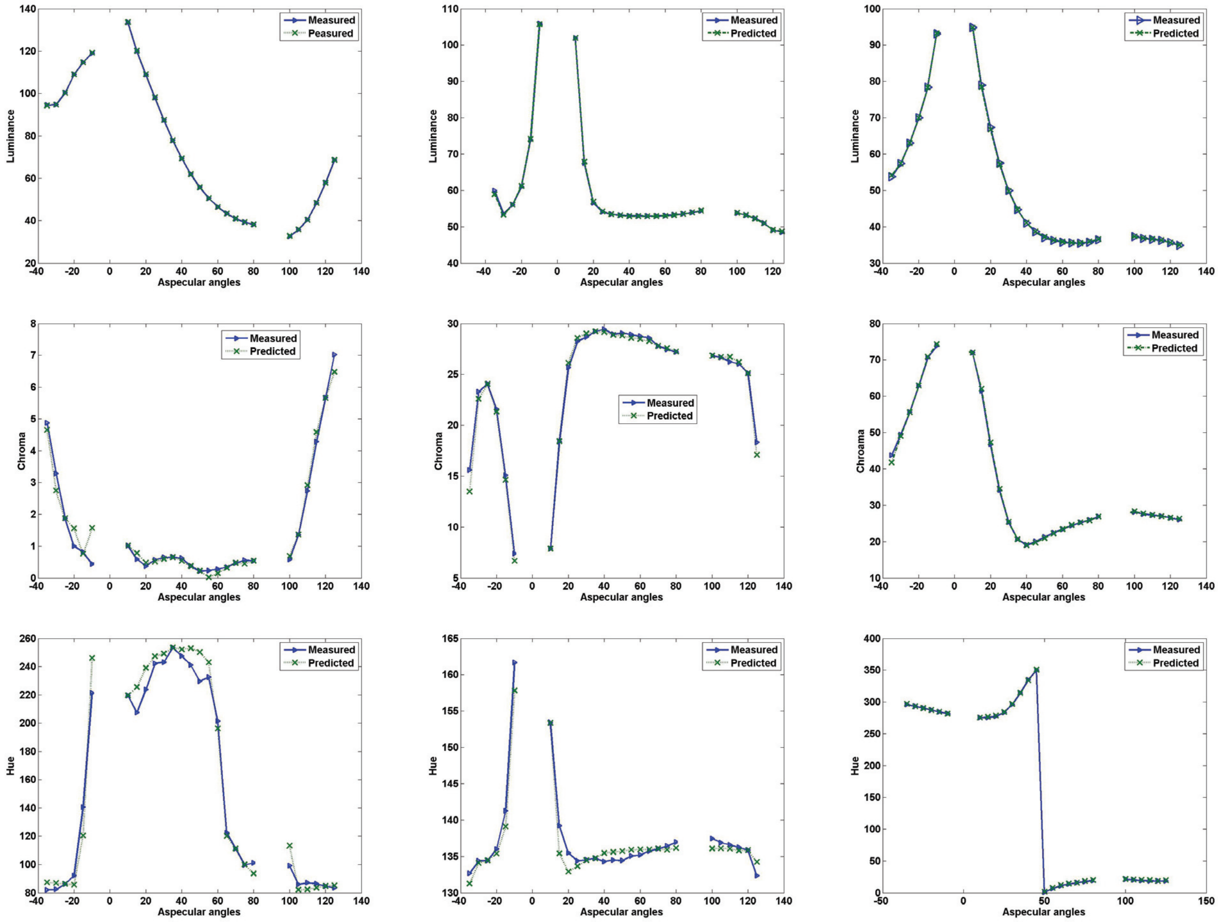

Figure 5. *Predicted and measured Luminance, Chroma and Hue (top to bottom) of Metallic 1, Metallic 2 and Pearlescent samples (left to right).*

Table 4. **Prediction from three best primary angles using wiener estimation with fifth order of polynomials of PC, mean values (μ) and maximum values (m) of color difference ($\Delta E\ CMC(2:1)$), and Root mean square errors (RMSE).**

Sample type	Primary angles	$\Delta E\ CMC(2:1)$		RMSE	
		μ	m	μ	m
Metallic 1	$115°\quad 25°\quad -30°$	0.458	1.515	0.008	0.038
Metallic 2	$75°\quad 20°\quad -10°$	0.384	1.263	0.005	0.012
Pearlescent	$70°\quad 20°\quad -15°$	0.498	2.902	0.007	0.023

Table 5. Anova result **Comparison verification between all and classified.X_1 represents the sample type and X_2 represents the viewing angles.**

Source	Sum of Squares	Degree of freedom	Mean Squares	F-value	P-value
X_1	73.697	26	2.835	25.09	0
X_2	1.359	2	0.680	6.02	0.003
$X_1 * X_2$	13.29	52	0.256	2.26	0
Error	103.73	918	0.113		
Total	206.932	998			