PCA-based Reflectance Analysis/Synthesis of Cosmetic Foundation

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Abstract

This paper analyzes the spectral reflection properties of skin surface with make-up foundation. Foundations with different material compositions are painted on a bio-skin. First we show the limitations of the previous models used for describing the complicated reflectance curves by a small number of parameters. A new approach based on PCA is then proposed for describing the detailed shape of surface-spectral reflectance. All skin surfaces exhibit the property of the standard dichromatic reflection, so that the observed reflectances are represented by only two spectral components of a constant reflectance and a diffuse reflectance. Moreover, the weighting coefficients are decomposed into two basis functions with a single parameter. Thus, the spectral reflectance at arbitrary angular conditions can be estimated by synthesizing the diffuse spectral reflectance and several onedimensional basis functions. Finally, the feasibility of the proposed method is examined using many skin samples with different material compositions of make-up foundation.

Introduction

The estimation of human skin color has been one of the most popular topics in many fields including color reproduction, image processing, computer graphics, medical imaging, and cosmetic development. There are recent research reports on the spectral analysis of skin color [1]-[3]. Foundation is a cosmetic preparation which is applied to the skin. It has various purposes. The essential role is to improve the appearance of skin surfaces. Therefore it is important to evaluate the change of skin color by foundation. However, there is no scientific discussion on the spectral analysis of foundation material and skin with make-up in the visible range, except for the IR spectrum [4].

The present paper analyzes the spectral reflection properties of skin surface with make-up foundation. Foundations with different material compositions are painted on a bio-skin. Light reflected from the skin surface is measured gonio-photometrically. Appearances of the surface, including specularity, gloss, and matte appearance, depend on the observation conditions of light incidence and viewing, and also the material compositions. First, we investigate a relationship between the surface appearances and the various conditions on observation and material composition. Next, we show the limitations of models used for describing the complicated reflectance curves by a small number of parameters.

We consider a new approach to creating the detailed shape of surface-spectral reflectance of the skin with make-up foundation. First, we investigate the basic property of the skin surface based on the principal-component analysis (PCA) of the whole set of spectral reflectances measured in the visible range under a variety of conditions of light incidence and viewing. We show that all skin surfaces satisfy the property of the standard dichromatic reflection. Then the observed reflectances can be represented by only two spectral components of a constant reflectance and a diffuse reflectance. Moreover, we analyze the weighting coefficients for the spectral components, depending on the incidence and viewing directions. It is found that the weighting coefficients can be decomposed into two basis functions with a single parameter. Thus, the spectral reflectance at arbitrary angular conditions can be estimated by synthesizing the diffuse spectral reflectance and several one-dimensional basis functions. Finally, the feasibility of the proposed method for reflectance estimation is examined using many skin samples with different material compositions of make-up foundation.

Foundation Samples

The make-up foundation is composed of different materials such as mica, talc, nylon, titanium, and oil. Two materials of mica and talc are the important components which affect the appearance of skin surface painted with the foundation. So many foundations were made by changing the quantity and the ratio of two materials. For instance, the combination ratio of mica (M) and talc (T) was changes as (M=0, T=60), (M=10, T=50), ..., (M=60, T=0), and the ratio of mica was changed with a constant T as (M=0, T=40), (M=10, T=40). Thus we have many combinations to produce a variety of foundations.

The bio-skin used in this study is made of urethane which looks like human skin. Powder foundations with the above compositions were painted on a flat skin surface with the fingers. Figure 1 shows a board sample of bio-skin with foundation. The foundation layer is very thin as 5-10 microns in thickness on the skin.



Figure 1 Sample of bio-skin with foundation.

Measurement

Gonio-spectrophotometric measurement is used for investigating 3D reflection properties of the skin surface with different foundations. In the gonio-meter system, the sensor position is fixed, and the light source can rotate around the sample as shown in Figure 2. The ratio of the radiance from the sample to the one from the reference white diffuser, called the radiance factor, is output as reflectance.



Figure 2 Measuring system of surface reflectance.



Figure 3 Reflectance measurements from sample IKD-20. (Upper: 3D view of spectral reflectances at $\theta = 10$, Lower: average reflectance as a function of viewing angle)

Figure 3 shows an example of the spectral radiance factors measured from the sample IKD-20 of (M=20, T=40). The upper figure in Figure 3 is a 3D perspective view at the incidence angle of 10 degrees, where the spectral curves are depicted as a function of viewing angle. The spectral reflectance depends not only on the viewing angle, but also on the incidence angle. In order to make

this point clear, we average the radiance factors on wavelength in the visible range. The lower figure in Figure 3 depicts a set of the average curves at different incidence angles as a function of viewing angle.

In comparison, Figure 4 shows the spectral radiance factors measured from the bio-skin itself. A comparison between Figure 3 and Figure 4 suggests that the typical features of foundation reflectance are (1) back-scattering at around -70 degrees, (2) reflectance hump at around the vertical viewing angle, and (3) rising reflectance with increasing viewing angle, rather than the absolute values of reflectance.



Figure 4 Reflectance measurements from bio-skin surface.

Model Fitting

The Phong model [5] and the Cook-Torrance model [6] are known as a 3D reflection model used for describing light reflection of an object surface. The former model is convenient for inhomogeneous dielectric object like plastics. The mathematical expression of the model is relatively simple, and the number of model parameters is small. The latter model is a physically precise model which is available for both dielectrics and metals. Tominaga et al. [7] examined the two models for describing 3D light reflection of oil painting surfaces. It was shown that the Cook-Torrance model could describe the reflection properties of oil paints more precisely than the Phong model.

The Cook-Torrance model can be written in terms of the spectral radiance factor as

$$Y(\lambda) = S(\lambda) + \beta \frac{D(\varphi, \gamma)G(\mathbf{N}, \mathbf{V}, \mathbf{L})F(\theta_{Q}, n)}{\cos \theta_{i} \cos \theta_{r}}, \qquad (1)$$

where the first and second terms represent, respectively, the diffuse and specular reflection components. β is the specular

reflection coefficient. A specular surface is assumed to be an isotropic collection of planar microscopic facets by Torrance and Sparrow [8]. The area of each microfacet is much smaller than the pixel size of an image. Note that the surface normal vector **N** represents the normal vector of a macroscopic surface. Let **Q** be the vector bisector of an **L** and **V** vector pair, that is, the normal vector of a microfacet. The symbol θ_i is the incidence angle, θ_r is the viewing angle, φ is the angle between **N** and **Q**, and θ_Q is the angle between **L** and **Q**.

The specular reflection component consists of several terms: D is the distribution function of the microfacet orientation, and F represents the Fresnel spectral reflectance [9] of the microfacets. G is the geometrical attenuation factor. D is assumed as a Gaussian distribution function with rotational symmetry about the surface normal **N** as

$$D(\varphi, \gamma) = \exp\left\{-\log(2)\varphi^2/\gamma^2\right\},$$
(2)

where the parameter γ is a constant that represents surface roughness. The Fresnel reflectance *F* is described as a nonlinear function with the parameter of the refractive index *n* (see [10]).

The unknown parameters in this model are the coefficient β , the roughness γ and the refractive index *n*. The reflection model is fitted to the measured spectral radiance factors by the method of least squares. In the fitting computation, we used the average radiance factors on wavelength in the visible range. We determine the optimal parameters to minimize the squared sum of the fitting error

$$e = \min \sum_{\theta_i, \theta_r} \left\{ \overline{Y(\lambda)} - \overline{S(\lambda)} - \beta \frac{D(\varphi, \gamma) G(\mathbf{N}, \mathbf{V}, \mathbf{L}) F(\theta_{\varrho}, n)}{\cos \theta_i \cos \theta_r} \right\}^2$$
(3)

where $Y(\lambda)$ and $S(\lambda)$ are the average values of the measured and diffuse spectral reference factors, respectively. The diffuse reflectance $S(\lambda)$ is chosen as a minimum of the measured spectral reflectance factors. The above error minimization is done over all angles of θ_i and θ_i .

The lower figure in Figure 5 shows the fitting results to the same sample IKD-20 shown in Figure 3, where a red solid curve indicates the fitted reflectance, and a black broken curve indicates the original measurement. The model parameters were estimated as β =1.31, γ =0.22, and *n*=1.90. The squared error was *e*=4.11. The upper figure in Figure 5 shows the 3D view of the fitting results at the incidence of 10 degrees. We have repeated the same fitting of the model to many skin samples with different material compositions of foundation.

A detailed comparison between two sets of curves in Figure 5 suggests that there is a certain discrepancy between the measured reflectances and the estimated ones by the model. The similar discrepancy occurs for all the other samples. These facts show the limitations of the model used for describing the set of complicated spectral reflectance curves by using a small number of parameters.



Figure 5 Fitting results of the Cook-Torrance model to the skin surface IKD-20.

PCA-based Analysis/Synthesis of Spectral Reflectance

We consider a new approach to creating the detailed shape of surface spectral reflectance of the skin with make-up foundation.

Analysis

First, we investigate the basic reflection property of the skin surface. The standard dichromatic reflection model [11] assumes that the surface reflection consists of two additive components, the body (diffuse) reflection and the interface (specular) reflection, which is independent of wavelength. The spectral reflectance (radiance factor) $Y(\theta_i, \theta_r, \lambda)$ of the skin surface is a function of the wavelength and the geometric parameters of incidence angle θ_i and viewing angle θ_r . The assumption means that the reflectance is expressed in a linear combination of the diffuse reflectance and the constant reflectance as

$$Y(\theta_i, \theta_r, \lambda) = C_1(\theta_i, \theta_r) S(\lambda) + C_2(\theta_i, \theta_r) , \qquad (4)$$

where the weights $C_1(\theta_i, \theta_r)$ and $C_2(\theta_i, \theta_r)$ are the geometric scale factors.

We test the adequacy of the standard dichromatic reflection model for the present skin surfaces. To perform this, the PCA analysis is applied to the whole set of spectral reflectance curves observed under different geometries of θ_i and θ_r . Assume that each spectral reflectance is sampled at *n* points with an equal interval $\Delta\lambda$ (say 5nm) in the range [400, 700nm]. Let $\mathbf{y}(\theta_i, \theta_r)$ and \mathbf{s} be *n*-dimensional column vectors, representing the observed reflectance $Y(\theta_i, \theta_r, \lambda)$ and the diffuse reflectance $S(\lambda)$, respectively. A singular value decomposition (SVD) of the set of $\mathbf{y}(\theta_i, \theta_r)$ provides that each observed reflectance can uniquely be expressed in a linear combination of the n orthogonal vectors as

$$\mathbf{y}(\theta_i, \theta_r) = \mu_1 v_1(\theta_i, \theta_r) \mathbf{u}_1 + \mu_2 v_2(\theta_i, \theta_r) \mathbf{u}_2 + \dots + \mu_n v_n(\theta_i, \theta_r) \mathbf{u}_n$$
$$= k_1(\theta_i, \theta_r) \mathbf{u}_1 + k_2(\theta_i, \theta_r) \mathbf{u}_2 + \dots + k_n(\theta_i, \theta_r) \mathbf{u}_n$$
(5)

where { μ_1 , μ_2 , ..., μ_n ($\mu_i > \mu_{i+1}$)} are the singular values of the observation matrix of all $\mathbf{y}(\theta_i, \theta_r)$, { \mathbf{u}_1 , \mathbf{u}_2 , ..., \mathbf{u}_n } are the left singular values, and { $v_1(\theta_i, \theta_r)$, $v_2(\theta_i, \theta_r)$..., $v_n(\theta_i, \theta_r)$ } are determined from the right singular values. Consider an approximate representation of the reflectances in terms of some component vectors chosen from \mathbf{u}_1 , \mathbf{u}_2 , ..., \mathbf{u}_n . The performance index of the chosen principle components is given by the percent variance $P(K) = \sum_{i=1}^{K} \mu_i^2 / \sum_{i=1}^{n} \mu_i^2$.

The above SVD technique is used for testing the standard dichromatic reflection model based on (4). The procedure is to examine the two assumptions of two-dimensionality and constant reflectance.

The reflectances of IKD-54 of (M=54, T=5) were measured at 81 viewing angles of -80, -78, ..., -2, 0, 2, ..., 78, 80 degrees and 13 incidence angles of 0, 5, 10, ..., 60 degrees. First, a 61×1053 reflectance matrix was constructed by sampling the spectral curves at 5nm intervals. Second, the 61 singular vectors \mathbf{u}_1 , \mathbf{u}_2 , ..., \mathbf{u}_{61} were obtained from SVD. The performance index then became P(1)=0.995 and P(2)=1.000. Therefore it is determined that the spectral reflectances are two-dimensional, which can be described only the two component vectors \mathbf{u}_1 and \mathbf{u}_2 . Third, the vectors \mathbf{u}_1 and \mathbf{u}_2 were fit to the unit vector \mathbf{i} using linear regression. In Figure 6 the nearly straight line shows the fitting result. The constant spectral curve is estimated as a spectral component of the data set. For two reasons, we conclude that the spectral reflectance of the skin surface has the property of the standard dichromatic reflection.



Figure 6 Test of the standard dichromatic reflection model.

Synthesis

Let us consider the estimation of spectral reflectances for various angles of incidence and viewing without observation. The constant reflectance 1 and the diffuse reflectance $S(\lambda)$ are used to represent the observed reflectances with sufficient accuracy. So it

is expected to represent any unknown reflectance in terms of the same component curves. The estimate can be expressed as a function of two parameters in the form

$$Y(\theta_i, \theta_r, \lambda) = \hat{C}_1(\theta_i, \theta_r) S(\lambda) + \hat{C}_2(\theta_i, \theta_r) , \qquad (6)$$

where $\hat{C}_1(\theta_i, \theta_r)$ and $\hat{C}_2(\theta_i, \theta_r)$ denote the estimates of the weighting coefficients on a pair of angles (θ_i, θ_r) .

In order to develop the estimation procedure, we analyze the weighting coefficients based on the observed data. Figure 7 shows the distributions of the coefficients $C_1(\theta_i, \theta_r)$ and $C_2(\theta_i, \theta_r)$ obtained from the measurements of IKD-54. Note that the diffuse coefficient C_1 varies in a limited range [0.8, 1.4], while the specular coefficient C_2 increases steeply when the angles of incidence and viewing become large. Again SVD is applied to the data set of those weighting coefficients. It is found that the weighting coefficients can be decomposed into two basis functions with a single parameter as

$$C_{1}(\theta_{i},\theta_{r}) = w_{11}(\theta_{i})v_{11}(\theta_{r}) + w_{12}(\theta_{i})v_{12}(\theta_{r})$$

$$C_{2}(\theta_{i},\theta_{r}) = w_{21}(\theta_{i})v_{21}(\theta_{r}) + w_{22}(\theta_{i})v_{22}(\theta_{r})$$
(7)

Figure 8 shows two sets of principal component curves (v_{11}, v_{12}) and (v_{21}, v_{22}) as a function of viewing angle θ_r . The performance indices for the two components are P(2)>0.995 in both cases. The weighs (w_{11}, w_{12}) and (w_{21}, w_{22}) for these principal component curves are depicted in a function of incidence angle in Figure 9.

Therefore the estimation of $\hat{C}_1(\theta_i, \theta_r)$ and $\hat{C}_2(\theta_i, \theta_r)$ for any unknown reflectance can be reduced into a simple form



Figure 7 Distributions of the coefficients obtained from the measurements of IKD-54.



Figure 8 Principal component curves ($v_{11},\,v_{12}$) and ($v_{21},\,v_{22}$) as a function of viewing angle.



Figure 9 Weighs ($w_{11},\,w_{12}$) and ($w_{21},\,w_{22}$)in a function of incidence angle.

$$\hat{C}_{1}(\theta_{i},\theta_{r}) = \hat{w}_{11}(\theta_{i})v_{11}(\theta_{r}) + \hat{w}_{12}(\theta_{i})v_{12}(\theta_{r})
\hat{C}_{2}(\theta_{i},\theta_{r}) = \hat{w}_{21}(\theta_{i})v_{21}(\theta_{r}) + \hat{w}_{22}(\theta_{i})v_{22}(\theta_{r}) ,$$
(8)

where $\hat{w}_{ij}(\theta_i)$ (*i*, *j*=1, 2) are determined by interpolating the coefficients at observation points such as $w_{ij}(0)$, $w_{ij}(5)$, ..., $w_{ij}(60)$. Thus, the spectral reflectance of the skin surface at arbitrary angular conditions is generated using the diffuse spectral reflectance $S(\lambda)$, two pairs of the principal component weights $v_{ij}(\theta_r)$ (*i*, *j*=1, 2), and two pairs of weights $w_{ij}(\theta_i)$ (*i*, *j*=1, 2). It should be noted that these basis data are all one-dimensional.

Experimental Results

We have examined the feasibility of the proposed PCA based method for surface-spectral reflectance estimation by using many skin samples with different material compositions of make-up foundation. The surface-spectral reflectance of each sample was measured at 13 incidence angles of 0, 5, ..., 60, and 81 viewing angles of -80, -78, ..., 0, ..., 78, 80 by the goniospectrophotometer. First, the diffuse spectral-reflectance was determined from the minimum reflectance among the whole spectral data. Second, the principal components of spectral reflectances were extracted from SVD of the whole spectral data. The adequacy of the standard dichromatic reflection model was then tested on the principal component. It is found that the surface reflection properties of all skin samples satisfy the conditions of the standard model. Third, the two-dimensional coefficient data were decomposed further by using SVD into the principal components of the coefficients on viewing angle and their weighting coefficients on incidence angle. Finally, the diffuse reflectance, the principal components, and the weights were synthesized to estimate the skin surface-spectral reflectance at arbitrary incidence and viewing directions. The unknown weighting coefficients were estimated by interpolating the known coefficients with a smooth curve.

Figure 10 shows the estimation results of surface-spectral reflectances for IKD-54 at six incidence angles, where a perspective view of reflectance is depicted as a function of wavelength and viewing angle. The black mesh indicates the original measurements, and the red one indicates the fitting results by the Cook-Torrance model. The blue mesh indicates the estimates by the proposed method, where the curved data in Figure 6, and Figures 8-9 were used in this estimation. A comparison among three kinds of meshes in Figure 10 suggests that the estimated reflectances are almost coincident with the measurements at all incident angles, while clear discrepancy occurs for the Cook-Torrance model at the angles of around 20-40 degrees.

For numerical evaluation, we calculated colorimetric error between the estimated spectral reflectance and the direct measurements in the whole spectral data.. The CIE Lab color difference under D65 is 3.55 for Cook-Torrance and 2.52 for PCA.

Figure 11 demonstrates color images for rendering appearance of the skin surface with IKD-54 under different viewing angles. We assume that the surface is illuminated from the direction of incidence 45 degrees with CIE D65. The proposed method provides accurate color reproduction, especially at large viewing angles, compared with the Cook-Torrance model.



Figure 10 Estimation results of surface-spectral reflectances for IKD-54.



Figure 11 Color images of skin surface with IKD-54 under different viewing angles.

Conclusion

This paper has analyzed the spectral reflection properties of skin surface with make-up foundation. Foundations with different material compositions were painted on a bio-skin. We showed the limitations of the previous mathematical models used for describing the complicated reflectance curves by a small number of parameters. A new approach based on PCA was proposed for describing the detailed shape of surface-spectral reflectance. All skin surfaces have the property of the standard dichromatic reflection. Then the observed reflectances could be represented by only two spectral components of a constant reflectance and a diffuse reflectance. Moreover, it was found that the weighting coefficients were decomposed into two basis functions with a single parameter. Thus, the spectral reflectance at arbitrary angular conditions could be estimated by synthesizing the diffuse spectral reflectance and several one-dimensional basis functions. Finally, the feasibility of the proposed method was examined using many skin samples with different material compositions of makeup foundation. Color images using the estimated spectral reflectances demonstrated the appearance of skin surfaces.

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