Geodesic Based Ink Separation for Spectral Printing

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Abstract

An ink separation algorithm is introduced for printing with 6 to 9 inks. A spectral gamut mapping algorithm is also introduced that projects an input reflectance onto the manifold of the printer spectral gamut space The ink separation, which is finding the best ink combination to reproduce a given reflectance, is done by applying an interpolation between printer gamut points neighboring a projected point point's geodesic location. The technique finds the best manifold projection using ISOMAP. The algorithm searches for the lowest dimensionality that holds the spectral information accurately. Using this method we were able to find a good ink combination given an input reflectance for both a 6-ink and 9-ink printer model.

Introduction

In comparison to standard color printing, spectral printing aims to reproduce a given reflectance spectrum rather than produce a metameric reflectance spectrum that simply matches a given color. Spectral printing aims to reduce a problem that can arise in metameric color printing which is that the reproduced color may match under one illuminant, but not match well under some other illuminant. Clearly, if the printed output reflectance matches the input reflectance, the printed color will match the input color under all illuminants.

Spectral printing requires a significantly larger number of inks than the standard CMYK ones, but this increases the computational complexity of printing algorithms in terms of both time and space. In particular, standard gamut-mapping algorithms map colors within a 3-dimensional space. Generally, their computational complexity increases rapidly with dimension, so that they become intractable for the gamut-mapping of spectra represented in, say, 11 dimensions. For example, a gamut-mapping algorithm that relies on the computation of the convex hull of the measured gamut will not work, since computing a d-dimensional convex hull of n points requires order $O(n^{**}floor(d/2)+1)$ operations. Bakke et al. [7] address this problem by reducing the dimensionality via principal components analysis and then computing the convex hull in up to 8 dimensions.

The first part of this paper introduces an ink separation method based on spectral data. This method uses interpolation in geodesic locations to find the best ink combination to match a spectral reflectance. McIntosh et al. [12] have previously applied the idea of interpolating over geodesic distances rather than Euclidean distances to improve an image segmentation algorithm. The second part of the paper a spectral gamut mapping algorithm is introduced based on manifold projection. The last part of the paper evaluates performance of the two models against some of the existing approaches. The result is presented in both Root Mean Squared of the spectral reproduction and ΔE_{94} under 11 different illuminations.

ISOMAP and Multidimensional Scaling

ISOMAP [2] is a nonlinear generalization of classical Multidimensional Scaling (MDS) [1]. MDS maps the input data to a lower dimensional space, subject to the constraint that pairwise distances between data points are preserved as much as possible. The main idea of ISOMAP is to perform MDS, not on the input space distances, but on the geodesic distances between points on the data manifold. The geodesic distances represent the shortest paths along the curved surface of the manifold. This can be approximated by a sequence of short steps between neighboring sample points. ISOMAP then applies MDS to the geodesic, rather than straight line, distances to find a low-dimensional mapping that preserves these pairwise distances.

Thin Plate Spline Interpolation

As is typical of interpolation methods, thin-plate spline (TPS) interpolation [10] constructs a function f that matches a given set of data values y_i, corresponding to a given set of data vectors $X_i = [X_{i,1}, X_{i,2}, … X_{i,D}]$ in the sense that $y_i = f(X_i)$. Xiong et al. extended the TPS model to N-dimensions and applied it to illumination estimation successfully [9].

For the spectral printing process, in this paper TPS is used to find a continuous function that maps between the set of inks and each of the output dimensions. For instance if the output spectral reflectance of an 8-ink printer is measured from 380nm to 730nm with a 10 nm sampling, TPS is used to create 36 separate functions mapping from the 8 input dimensions to each reflectance wavelength 380nm, 390nm, …. to 730nm individually

Geodesic Location and Ink Separation

Interpolation is a common approach to ink separation, and we use interpolation here. In general, an ink combination is interpolated as a weighted combination of ink formulas from nearby experimentally measured data points. The weights typically are derived from the distance of the point to be interpolated from its neighbors. The distance metric can be defined in many different ways. For instance, the distance between two spectral reflectances can be measured as the Euclidean distance between them. We propose interpolation based on geodesic distances on the gamut manifold.

Many spaces appear to have a high dimensionality in a linear space, but actually have lower intrinsic dimensionality. A good example is the Swiss Roll example of Tenenbaum et al. shown in Figure 1, along with its projection into 2 dimensions shown in Figure 2. Clearly, the geodesic distances on the 2D manifold are not the same as the direct Euclidean distances in 3-space.
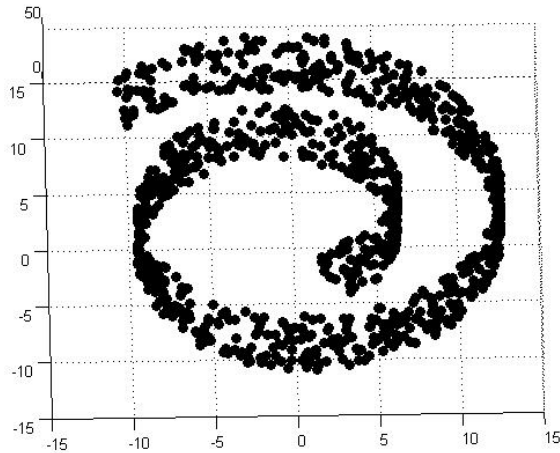
Figure 1: Swiss Roll representation in 3 dimensions

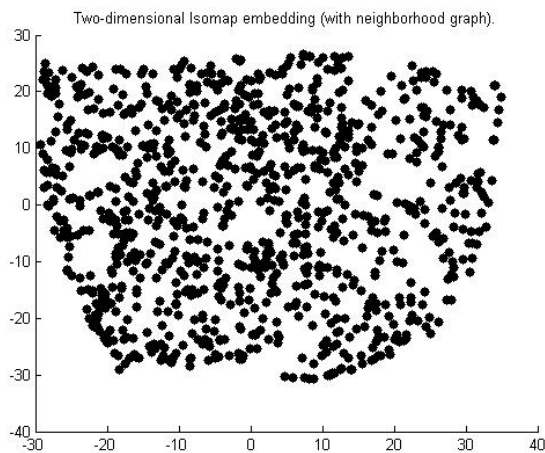Two-dimensional Isomap embedding (with neighborhood graph).

Figure 2: Un-folded Swiss Roll data into 2 dimensions using ISOMAP

The proposed ink separation method uses the geodesic distances between data points. The algorithm is as follows:

1. Given a training set of points (reflectances of print samples), the geodesic distances between the input reflectance (the reflectance to be printed) and all training points in the gamut are calculated
2. The geodesic distances are used in MDS to calculate the point locations in a space of lower dimension.
3. Thin Plate Spline interpolation is used based on the data point locations in the new space

Steps 1 and 2 are part of the standard ISOMAP algorithm [2]. ISOMAP makes the assumption that the Euclidean distances to points within the local neighborhood of a given point P approximate the corresponding geodesic distances. The geodesic distances to a point Q outside the local neighborhood is calculated

as the sum the distances between neighboring points along the shortest path from P to Q.

Spectral Gamut Mapping based on Manifold Projection

Bastani et al. [6] showed that almost 99% of the scene reflectances in spectral space fall outside of the printer gamut even if the printer ink sensitivity is optimized and we use as many as 12 inks. This means spectral gamut mapping becomes an important part of spectral reproduction work.

In this section we present a spectral gamut mapping algorithm based on manifold projection. The steps are as follows:

1. Given a printer gamut space, calculate the data point geodesic locations using ISOMAP
2. Transfer the input spectral reflectance using the same transformation
3. After the transformation, gamut mapping is then applied in the transformed space, which tends to have a lower dimensionality.

Evaluation Method

As a measure of the gamut mapping performance, we find the difference between a mapping using the above technique versus a mapping that iteratively searches hierarchically for the best closest match. In this algorithm a subdivision of ink combinations is created. Let the set of the subdivisions of ink space be M, where there is a spectral reflectance associated with each ink combination, m_i, in M. The Hierarchical Search (HS) method is:

1. Find the closest m_i spectrum to a given input point ⏼ in spectral space.
2. Create a grid of ink subdivisions around m_i with smaller ink variation.
3. Go back to step 1 until the grids are small enough. Then go to the next step.
4. Return spectral reflectance of m_i as the closest point.

The accuracy of the manifold projection algorithm is compared to the above HS search method.

Time and Space Complexity

There are several methods to calculate Geodesic distances between points given the distances between neighboring points. Most commonly Dijkstra's algorithm is used to find the shortest path between each point in the data set [2]. If there are M_p points representing the printer gamut, and M_D input points for ink separation, the time complexity of Dijkstra's algorithm based on the Fibonacci heap algorithm becomes $O(E + (M_p+M_D)Log(M_p+M_D))$, where E represents the number of edges between the points. The number of edges varies with the data set characteristics and diameter of the neighborhood around each data point. In practice it takes around 2.5 seconds to calculate the geodesic distances for 2000 points on an average compuer.

Experiments

Printer Gamut

To evaluate the gamut complexity of the printer, two printer gamuts were evaluated. The first one is based on spectral measurements of 900 patches printed with a 7-ink printer. We refer to this gamut as the "realistic" printer gamut. The second printer gamut is based on the synthetic printer model introduced by Tzeng et al. [3] and [4]. The model is used to predict the spectral reflectance resulting from a given ink combination. The following equations are used to predict the reflectance:

$$R_\lambda = (R_{\lambda\,paper}^{1/w} - \Box_{\lambda,mixture})^w \qquad (1)$$

$$\Box_{\lambda,\,mixture} = \Box c_i R_{i\lambda}$$

$$\Box_\lambda = R_{\lambda\,paper}^{1/w} - R_{\lambda,i}^{1/w}$$

Where $\Box_{i\lambda}$ is the reflectance of ink i at wavelength λ at maximum concentration. This model is used to generate data points. This printer gamut is referred to as the "synthetic" printer gamut. For 3, 6 and 9 ink models, 125, 4096 and 12000 data points were generated with the constraint that no more than 6 inks may be used at a time.

Ink Choices

The 7 inks used for the realistic printer gamut were Cyan, Magenta, Yellow, Light Cyan, Light Magenta, Black and Gray. For the synthetic printer gamut, spectral reflectances of 6 real inks were used. The inks were Orange (O), Cyan (c), Magenta (m), Yellow (y), Green (Gr), Violet (V) and Black (K), Light Magenta (LM) and Light Cyan (LC) for the 9-ink printer. Orange (O), Cyan (c), Magenta (m), Yellow (y), Green (Gr), Violet (V) were used for the 6-ink printer, and the 3-ink printer model used Cyan (c), Magenta (m), Yellow (y).

Printer Spectral Gamut Intrinsic Dimensionality

What are the intrinsic dimensionalities of the gamuts of the two printers? In terms of a linear model, Principal Component Analysis (PCA) provides one answer. However, in terms of a non-linear model, ISOMAP provides a second answer. If the answers differ, then we can conclude that the printer gamuts bend in a way that is similar to the Swiss Roll example. Figures 3 and 4 compare how the residual variance changes with increasing dimensionality for both PCA and ISOMAP. If PCA shows a higher dimensionality for the data set than what ISOMAP finds, then we can conclude that the underlying structure of the gamut is a lower-dimensional data set. Figure 3 and Figure 4 are based on the realistic gamut; Figures 5 and 6 are for the synthetic gamut.

The analysis shows that the printer gamuts are of lower (3 or 4 versus 5 or 6) intrinsic dimensionality than can be determined by linear PCA. As a result, we expect to be able to have obtain more

accurate ink separations using interpolation based on the distances between the ISOMAP-embedded locations of the reflectances.
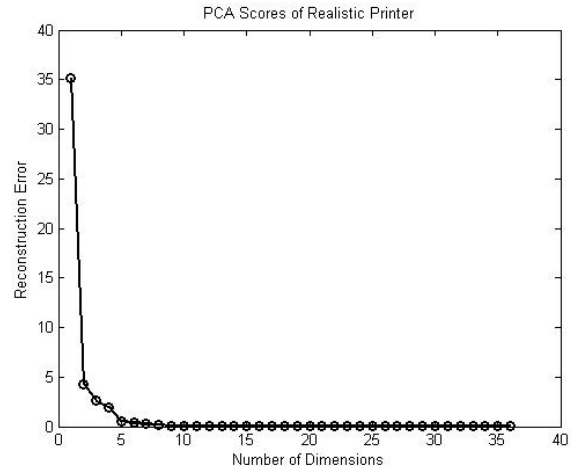

Figure 3: PCA residual variance for the realistic printer gamut space. The plot shows that the dimensionality of the 7-ink printer is around 5 dimensions
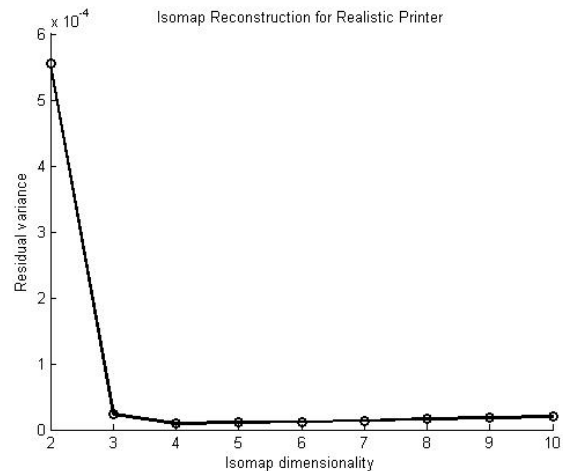

Figure 4: ISOMAP residual for the realistic gamut . The data shows that the underlying dimensionality of the gamut is around 3 .
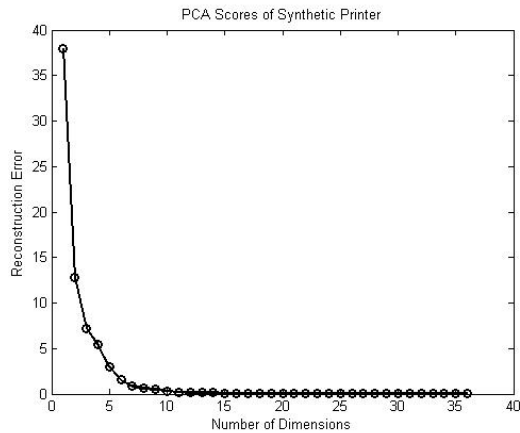
Figure 5: PCA residual variance for the synthetic printer gamut space. The scores show that the dimensionality of the 6-ink printer is around 6 or 7 dimensions
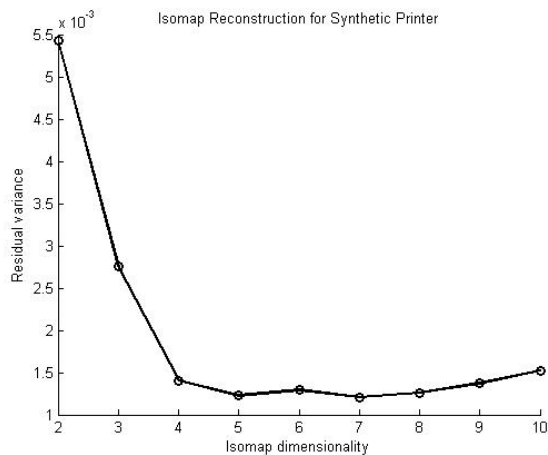

Figure 6: ISOMAP residual for the synthetic gamut . The data shows that the underlying dimensionality of the gamut is around 4 .

Metamerism Measure

The RMS (root mean square) difference between two spectral reflectances does not necessarily represent the difference that may be apparent to the eye. As an alternative measure, we use the maximum ΔE_{94} of the two spectral reflectances found under 11 different lights. The 11 illuminants used in this paper are from the Simon Fraser data base [5].

11 illumination types used for ΔE comparison
1. Sylvania 50MR16Q (12VDC)---A basic tungsten bulb
2. Sylvania 50MR16Q (12VDC) + Roscolux 3202 Full Blue filter
3. Solux 3500K (12VDC)--Emulation of daylight
4. Solux 3500K (12VDC)+Roscolux 3202---Emulation of daylight
5. Solux 4100K (12VDC)--Emulation of daylight
6. Solux 4100K (12VDC)+Roscolux 3202---Emulation of daylight
7. Solux 4700K (12VDC)--Emulation of daylight
8. Solux 4700K (12VDC)+Roscolux 3202---Emulation of daylight
9. Sylvania Warm White Fluorescent (110VAC)
10. Sylvania Cool White Fluorescent (110VAC)
11. Philips Ultralume Fluorescent (110VAC)

Table 1 The lights used in measuring the color variation of two similar reflectance spectra under different illuminants.

Results

Ink Separation Evaluation

To evaluate how well the ink separation technique works, we sampled the synthetic printer gamut uniformly in ink space, obtaining 2300 data points as a training data set. An additional 250 data points from inside the printer gamut were selected to represent the test sample. The test and training sets are disjoint.

The 250 test points are processed through the ink separation algorithm. The predicted ink combinations are then run through the printer model to predict the corresponding spectral reflectances. The predicted reflectances are then compared to the original input reflectances.

To evaluate the performance of the geodesic ink separation model, we compare its results to those obtained by doing the separation in linear space. Table 2 **Error! Reference source not found.**shows that there is a gain when the interpolation is based on the geodesic distances instead of the Euclidean distances.

Inks		Geodesic		Linear	
		RMS	ΔE_{94}	RMS	ΔE_{94}
	min	0.0011	0.236	0.0078	1.204
	mean	0.034	3.068	0.0335	6.33
3	max	0.134	8.2	0.0717	24.78
	min	0	0.005	0	0.006
	mean	0.0089	2.843	0.0298	3.379
6	max	0.0541	18.29	0.1339	20.45
	min	0	0.004	0	0.014
	mean	0.0081	2.617	0.0179	3.051
9	max	0.0487	14.1	0.1238	20.05

Table 2: Ink Separation evaluation based on Geodesic location and linear space locations. The errors reported are the minimum, mean, and max ΔE_{94} that occur under the 11 different illuminations, and the RMS difference between the spectra.
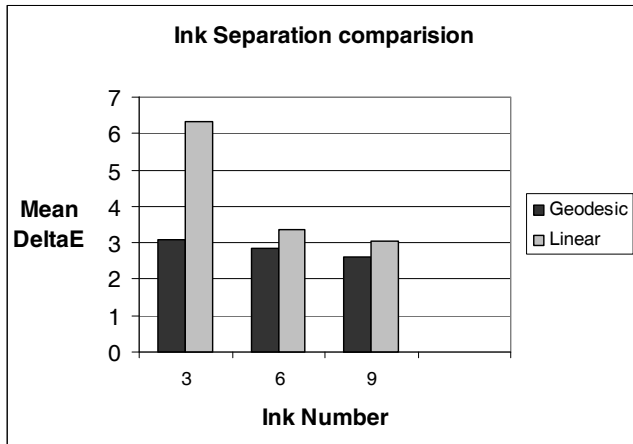
Figure 7: Ink separation methods evaluated in ΔE_{94} under 11 different illuminations. The data above shows average ΔE_{94} for the 11 illuminations

Inks		Isomap		Hierarchical	
		RMS	ΔE_{94}	RMS	ΔE_{94}
	min	0.019	0.38	0.09	2.1
	mean	0.085	9.89	0.2	29.38
3	max	0.2133	42.87	0.6	78.8
	min	0.005	0.2288	0.03	1.9
	mean	0.1128	12.23	0.18	23.5
6	max	0.416	56.74	0.45	60.2
	min	0.02	1.8	0.019	1.2
	mean	0.11	10.8	0.134	18.4
9	max	0.38	42.5	0.37	56.3

Table 3: Performance Evaluation of the two Spectral Gamut Mapping Approaches. ΔE_{94} represents average color variation between input reflectance and projected input reflectance under the 11 different illuminations.

Spectral Gamut Mapping Evaluation

Test Data

To test the gamut mapping algorithm, the scene reflectances from SFU data base were used. There are 1350 individual reflectances in the data base. Table 3**Error! Reference source not found.** shows the accuracy of the spectral reproduction when the proposed spectral-gamut mapping algorithm is used compared to the Hierarchical Search method in order to map the out-of-gamut points onto the gamut hull. The table shows that the proposed gamut mapping algorithms is as accurate as the hierarchical search algorithm or sometimes better.

It is important to keep in mind that the hierarchal search algorithm is not 100% accurate either. Some of the inaccuracy of the hierarchical search algorithm comes from the sampling resolution of each ink axis. The higher the sampling resolution is the more accurate the model. For this paper, hierarchical search had 6 levels, and at each level the sampling resolution for each axis was 5. For instance for a 3-ink system, at each search level there are 5^3 different ink combinations to choose from. Once the closest ink combination is selected (P_a), 5^3 samples are selected close to the point P_a. This process is repeated for 6 levels.
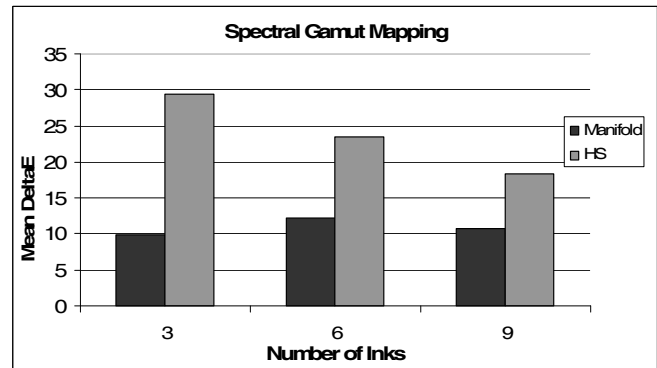

Figure 8: Comparison of the average ΔE_{94} performance for the hierarchical search versus the manifold spectral gamut mapping algorithm.

Conclusion

A spectral ink separation algorithm is introduced based on interpolation using the geodesic distances between neighboring points. A spectral gamut mapping algorithm is also introduced using ISOMAP manifold transformation.

The performance of the ink-separation model was evaluated for both a 6-ink and a 9-ink printer using a synthetic printer model. The experimental tests show that the accuracy of interpolation, and thus of the resulting ink separation, improves if the calculation is done using geodesic distances. The accuracy of the technique is very close to the best that can be done via a long iterative search over all printer ink combinations.

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