Color Gamut Mapping Algorithm for Preserving Spatial Ratios

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Abstract

This paper proposes a calculation algorithm for a spatial gamut mapping technique for preserving the spatial ratios in an original image. McCann suggested the effectiveness for preserving ratios of colors at adjacent edges in the gamut mapping procedure. He also proposed a mapping algorithm based on the Retinex with two input images which are an original image with large gamut and a best image with limited gamut of the reproduction media. The proposed algorithm provides a concrete algorithm to obtain the reproduction image with preserving spatial ratios from only the input image by using the relaxation labeling technique. Experimental results show that the proposed method can work well for a Mondrian pattern.

Introduction

Gamut mapping is an important problem in color management, and has been one of the most active areas of color imaging research. The problem of the gamut mapping is of interest in achieving an optimum color reproduction whenever a translation from one device to another device is requested. The design of the optimal technique thus involves a suitable trade-off among image attributes such as contrast, luminance detail, vividness, and smoothness. In order to map the out of gamut colors to the colors within the gamut of an output device, several gamut mapping algorithms (GMAs) have been proposed in the literature. Morovic and Luo have made an exhaustive survey in Refs. [1] and [2].

The first generation of GMAs was based on non-adaptive point-wise processing. The ICC color management is based on this first generation. The next step has been to investigate the selection of an appropriate GMA directly to the image gamut instead of the input device gamut. To further improve GMAs, it has been advocated that preservation of the image details is a very important issue for perceptual quality [3]. Recently, Spatial Gamut Mapping Algorithms (SGMAs), which take into account the spatial and color characteristics of the image, have been proposed. With such algorithms, two pixels of the same color in an input image might map to different colors in the output image, depending on the spatial characteristics in their respective spatial neighborhood. Mayer and Barth first introduced this concept using homomorphic filtering to separate low and high spatial frequency channels, and then to apply global dynamic range compression only to the low frequency channel [4]. Then some SGMAs have been proposed for several years at the IS&T/SID *Color Imaging Conference* [5]-[8].

In Refs.[6] and [9], McCann used the principles of Retinex theory to develop an iterative gamut mapping which attempts to preserve ratios of colors at adjacent pixels. The algorithm using spatial comparisons can generate in-gamut reproductions that look more like the original, because they employ the benefits of human color constancy processing. However, this method aims at the image quality improvement and requires two input images which are an original image with large gamut and a best image with limited gamut of the reproduction media.

Our work has been strongly influenced by McCann's work and we propose a new calculation algorithm to obtain the reproduction image with preserving spatial ratios from only an input image without another image. In this study, we formulate the spatial gamut mapping problem as a combinatorial optimization problem and an appropriate mapping result can be derived by using the relaxation labeling technique which is one of the optimization algorithms.

GMA to Preserve Spatial Ratios

Let us consider a Mondrian pattern shown in Fig.1. This pattern was introduced in Ref.[9] for the demonstrations of color appearance. Each color pentagon is called an *area* in this paper, and 17 areas from "A" to "Q" exist on this pattern as shown in Fig. 2(a). The boundary of the adjacent area is called an *edge* which is numbered from 1 to 32 as shown in Fig. 2(b).

McCann suggests preserving spatial ratios while applying a gamut mapping procedure [6],[9]. The basic idea is to preserve the ratios at adjacent edges for each XYZ component in the original image, while projecting onto the target gamut as a constraint. This problem can be formalized as an optimization problem as follows:

minimize $\Delta R = average_{i,j}$

$$\sqrt{\left(1-\frac{X_{\text{Rep}}^{(i)}/X_{\text{Rep}}^{(j)}}{X_{\text{Ori}}^{(i)}/X_{\text{Ori}}^{(j)}}\right)^2+\left(1-\frac{Y_{\text{Rep}}^{(i)}/Y_{\text{Rep}}^{(j)}}{Y_{\text{Ori}}^{(i)}/Y_{\text{Ori}}^{(j)}}\right)^2+\left(1-\frac{Z_{\text{Rep}}^{(i)}/Z_{\text{Rep}}^{(j)}}{Z_{\text{Ori}}^{(i)}/Z_{\text{Ori}}^{(j)}}\right)^2}, \quad (1)$$

where, $\left(X_{\text{Ori}}^{(i)},Y_{\text{Ori}}^{(i)},Z_{\text{Ori}}^{(i)}\right)$ and $\left(X_{\text{Rep}}^{(i)},Y_{\text{Rep}}^{(i)},Z_{\text{Rep}}^{(i)}\right)$ mean the tristimulus values defined in the 1931 CIE report at area *i* in an original input image and the gamut mapped reproduction image, respectively. $X^{(j)}, Y^{(j)}$ and $Z^{(j)}$ are the tristimulus values of an adjacent area *j* of the area *i*.

Subject to: $\left(X_{\text{Rep}},Y_{\text{Rep}},Z_{\text{Rep}}\right)$ exist within the gamut of the reproduction media.

The optimization problem in Eq.(1) is an ill-posed problem, because there are infinite solutions which satisfy the following equation under the given subject:

$$\left(X_{\text{Rep}}^{(i)},Y_{\text{Rep}}^{(i)},Z_{\text{Rep}}^{(i)}\right)=\kappa\left(X_{\text{Ori}}^{(i)},Y_{\text{Ori}}^{(i)},Z_{\text{Ori}}^{(i)}\right),\forall i, \quad (2)$$

where κ is a scaling factor in real number. However, those optimum solutions might cause a serious problem. If only one pixel greatly away besides the gamut exists, a remarkable decrease in luminance and the chroma is caused, because the optimum solution strongly compresses the whole pixels for moving in the gamut. Such a phenomenon can happen easily in a real scene by the noise. Thus, the global optimum solutions in Eq.(2) is not practicable. A local optimum solution might be an appropriate

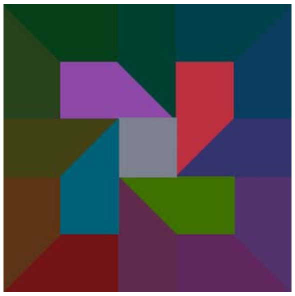
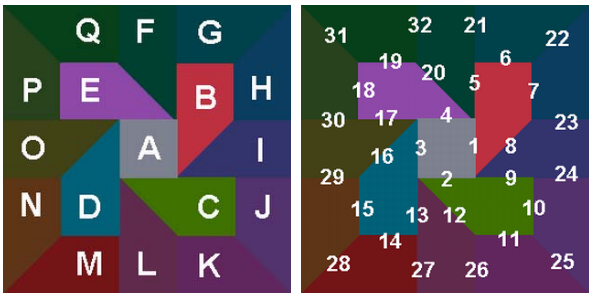


Figure 1 *Original Mondrian pattern [9].*



(a) Area name *(b) Edge number*

Figure 2 *Definition of Area name and Edge number.*

solution. We propose an optimization algorithm for deriving a local optimum solution in the next section.

Gamut Calculation by Relaxation Labeling

In this section, we propose a gamut calculation algorithm using the relaxation labeling [10].

Relaxation labeling is a technique mainly used in the various fields of image processing as the optimization technique for solving labeling problems [11]. The structure of relaxation labeling is motivated by two basic concerns: 1) the decomposition of a complex computation into a network of simple "myopic," or local, computations; and 2) the requisite use of context in resolving ambiguities. By using the relaxation method, it is possible to evaluate the adjacent ratios by compatibility coefficients which will be defined in Eq.(7). In a labeling problem, one is given:

1) a set of objects;
2) a set of labels for each object;
3) a neighbor relation over the objects; and
4) a constraint relation over labels at pairs (or n-tuples) of neighboring objects.

A given set of objects is denoted by $O=\{o_i\}_{i=0}^{I-1}$. In this paper, the objects correspond to areas in Mondrian patterns. In Fig.1, I=17 objects exist in the pattern. A set of possible labels for each object o_i is denoted by $\Lambda_i=\{\lambda_k^{(i)}\}_{k=0}^{K-1}$. Labels correspond to possible Y stimulus which are K-th quantized discrete values in the range [0,100]. We want to find the most matched Y stimulus $\lambda_{k*}^{(i)}$ for each o_i. The relaxation labeling can derive the solution by reducing local ambiguities using contextual information.

Let $P_{i(k)}^{(t)}$ denote t-times developed probability for classification of object o_i into $\lambda_k^{(i)}$, and require that

$$0 \le P_{i(k)}^{(t)} \le 1, \qquad \forall i,k\ , \tag{3}$$

$$\sum_{k=0}^{K-1} P_{i(k)}^{(t)} = 1, \qquad i=0,1,\ldots,I-1\,. \tag{4}$$

Then iterative processing starts and the probabilities updates to minimize the objective function in Eq.(1) by the following iterative equations:

$$P_{i(k)}^{(t+1)} = \frac{P_{i(k)}^{(t)} \times q_{i(k)}^{(t)}}{\sum_{k'} P_{i(k')}^{(t)} \times q_{i(k')}^{(t)}}\,. \tag{5}$$

Compatibility function (CF) $q_{i(k)}^{(t)}$ denotes the neighborhood contribution defined as follows:

$$q_{i(k)}^{(t)} = \sum_{o_j \in N_i} \left\{ \max_{\lambda_l^{(j)}} r_{ij(k,l)} \times P_{j(l)}^{(t)} \right\}, \tag{6}$$

where N_i denote a set of neighboring objects of $o_j \in N_i$ into possible labels $\Lambda_j=\{\lambda_l^{(j)}\}_{l=0}^{L-1}$. In Fig. 2(a), in the case $o_i=$"A", $N_i=\{$"B","C","D","E"$\}$. The constant $r_{ij(kl)}$ denotes the real-valued compatibility coefficient which represents how the classification of o_i into $\lambda_k^{(i)}$ is compatible with the classification of o_j into $\lambda_l^{(j)}$. If the neighboring objects support the classification of o_i into $\lambda_k^{(i)}$, the CF $q_{i(k)}^{(t)}$ must be high in the framework of the relaxation labeling. In this paper, the compatibility coefficient $r_{ij(kl)}$ is formulated for evaluating ratios on Eq.(1) among adjacent colors as follows:

$$r_{ij(kl)} = \max\left(0, \alpha - \left|1 - \frac{\lambda_k^{(i)} / \lambda_l^{(j)}}{Y_{\text{ori}} / Y'_{\text{ori}}}\right|\right), \qquad \alpha > 0\,, \tag{7}$$

where, α is a sensitive parameter for evaluating the difference of the ratio. Equation (7) becomes the maximum value α, when the ratio is the same between the original image and the reproduction image. When the ratio is too different between them, it becomes the minimum value 0, and it is formulated so that it may change linearly. In generally, we set α=2.

Equation (7) evaluates the objective function in Eq.(1) locally. They are iterative procedures that aim at reducing local ambiguities and achieving global consistency through a parallel exploitation of contextual information. The process in Eq.(5) is performed iteratively until the process converges or until a certain termination condition is satisfied. After T-times iterative processing, a Y stimulus $\lambda_{k*}^{(i)}$ with the maximum probability $P_{i(k*)}^{(T)}$ for each area x_i is selected as the solution:

$$Y_{\text{Rep}}^{(i)} = \lambda_{k*}^{(i)} \tag{8}$$

Then $X_{\text{Rep}}^{(i)}$ and $Z_{\text{Rep}}^{(i)}$ can be calculated with preserving ratios as follows:

$$\begin{cases} X_{\text{Rep}}^{(i)} = X_{\text{Org}}^{(i)} \cdot \left(Y_{\text{Rep}}^{(i)} / Y_{\text{Org}}^{(i)}\right) \\ Z_{\text{Rep}}^{(i)} = Z_{\text{Org}}^{(i)} \cdot \left(Y_{\text{Rep}}^{(i)} / Y_{\text{Org}}^{(i)}\right) \end{cases} \tag{9}$$

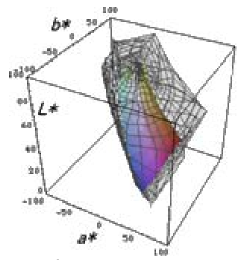


Figure 3 Color gamut of the DLP projector (solid) and sRGB (wire).

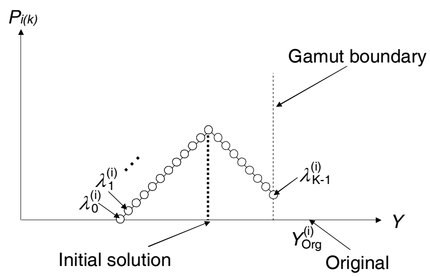


Figure 4 Setting of initial probabilities.

Since the probabilistic relaxation gives local optimum solution, the result depends on the initial probabilities $P_{i(k)}^{(0)}, \forall i,k$. The initial probability is considered in the next section.

Experimental Results

In order to verify the performance, we have tested the proposed algorithm by using the Mondrian pattern in Fig.1.

Gamut mapping to narrow gamut - Experiment 1 -

In the first experiment, we used an HP sb21 DLP projector as the target gamut. The color gamut in CIELAB space is shown in Fig.3 comparing with sRGB gamut. As shown in the figure, the gamut volume of the DLP projector is small.

In this experiment, we gave initial solution within a gamut for each area by simple clipping algorithm. Here, all colors outside of the gamut moved toward a single focal point (L^*,a^*,b^*)=(50,0,0) in CIELAB color space. If the color of an area exist within the gamut beforehand, the original color becomes the initial solution. Then initial probabilities $P_{i(k)}^{(0)}$ were given to become the maximum at the initial solution, and they were given other labels within the gamut to decrease linearly from the initial solution. Figure 4 shows how to set the initial probabilities. In the case that the original intensity exists within the gamut, the proposed algorithm may work not only gamut compression but also gamut expansion. In this experiment, we set K=100.

Figure 5 shows the reproduction images. Fig. 5(a) shows the image reproduced by the initial solution, i.e. T=0. As shown in the figure, the contrast of Fig. 5(a) seems to be different from the original image in Fig. 5(f). So, the spatial ratio has not been preserved. From Figs. 5(b) to 5(e) show the reproduction pattern by the proposed algorithm depending on the number of iteration t. As shown in the figure, Fig. 5(e) is an excellent reproduction result compared with Fig 5(a) within the narrow gamut.

In order to verify the performance objectively, we used two spatial metrics. One is ΔR in Eq.(1) in originally defined by McCann [9], and another is ΔL in CIELAB space under standard D65 illumination analogous to ΔR as follows:

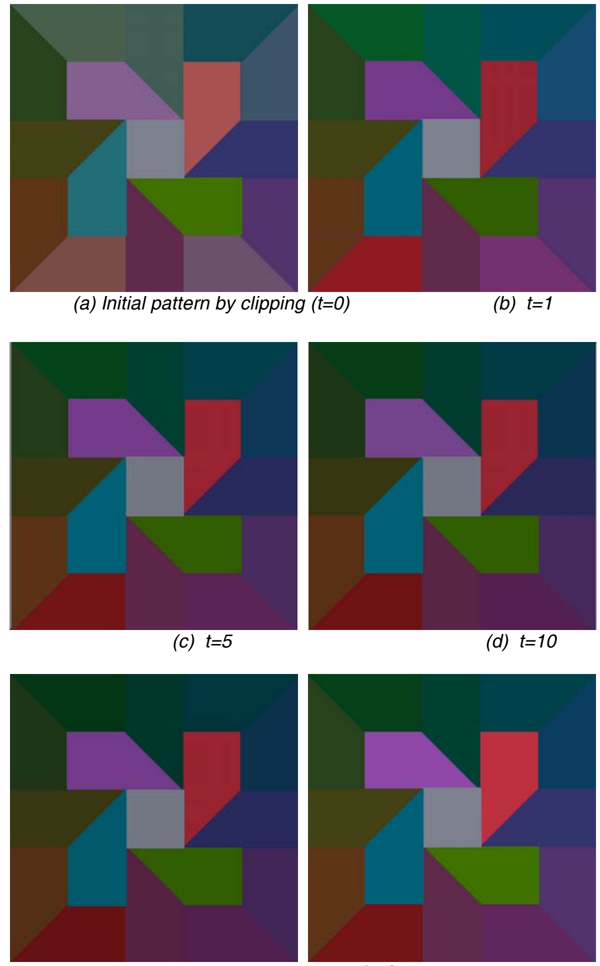
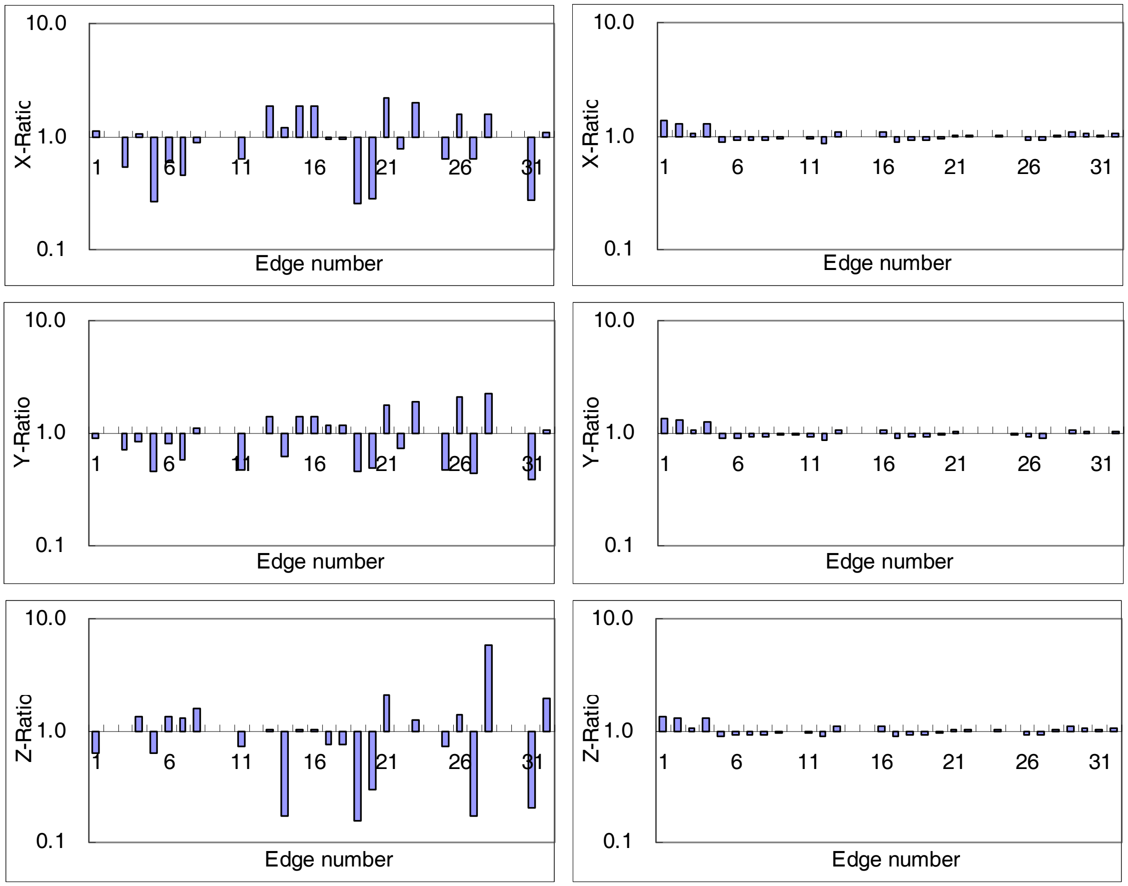

(a) Initial pattern by clipping (t=0) *(b) t=1*
(c) t=5 *(d) t=10*
(e) t=100 *(f) Original image in Fig.1*

Figure 5 Gamut mapping results in Experiment 1 by our algorithm.

$$\Delta L = average_{ij}\left|1-\frac{L^{*(i)}_{\text{Rep}}/L^{*(j)}_{\text{Rep}}}{L^{*(i)}_{\text{Ori}}/L^{*(j)}_{\text{Ori}}}\right| \quad (10)$$

where $L^{*(i)}_{\text{Rep}}$ and $L^{*(j)}_{\text{Rep}}$ are luminance values of area i and its neighboring area j in the reproduction image, respectively. $L^{*(i)}_{\text{Ori}}$ and $L^{*(j)}_{\text{Ori}}$ are luminance values of area i and its neighboring area j in the reproduction image, respectively. If all ratios of the reproduction image for 32 edges in Fig. 2(b) are the same as ones of the original image, both metrics become 0. Therefore, smaller value of each metric is better.

Table 1 shows results by the spatial metrics for evaluating Figs. 5(a) and 5(e). Both results of metrics fall close to 0 by the proposed gamut mapping. Figure 6 shows results for the 32 edge ratios (e.g. $\left(X^{(i)}_{\text{Rep}}/X^{(j)}_{\text{Rep}}\right)/\left(X^{(i)}_{\text{Ori}}/X^{(j)}_{\text{Ori}}\right)$). The results X, Y and Z are plotted in Fig. 6. Results of initial solution show that the area comparisons are very different for the initial solution and original. On the other hand, all the comparisons of $X^{(i)}/X^{(j)}$ (top), $Y^{(i)}/Y^{(j)}$ (middle) and $Z^{(i)}/Z^{(j)}$ (low) fall very close to 1.0.



(a) Initial pattern *(b) Reconstruction pattern by the proposed algorithm (t=100)*

Figure 6 *Results for 32 edge XYZ ratios in Experiment 1.*

Figure 7 shows $1-\left(\left(L*^{(i)}_{\mathrm{Rep}}\big/L*^{(j)}_{\mathrm{Rep}}\right)\Big/\left(L*^{(i)}_{\mathrm{Ori}}\big/L*^{(j)}_{\mathrm{Ori}}\right)\right)$ for the 32 edges. This graph shows how edge contrast for luminance preserves by the gamut mapping. Both average and maximum values fall close to 0.

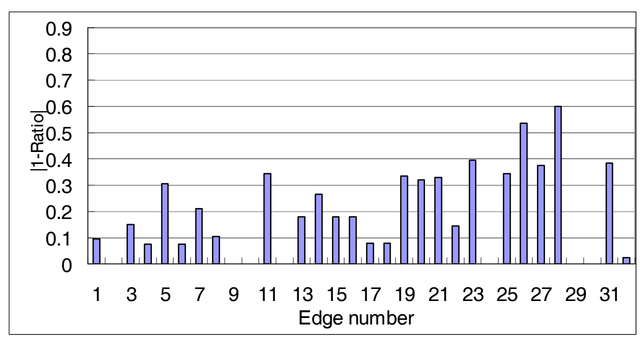
Table 1 Evaluation for spatial ratio in Experiment 1.

	ΔR	ΔL
Initial pattern	0.80	0.18
Reproduction pattern	0.13	0.03

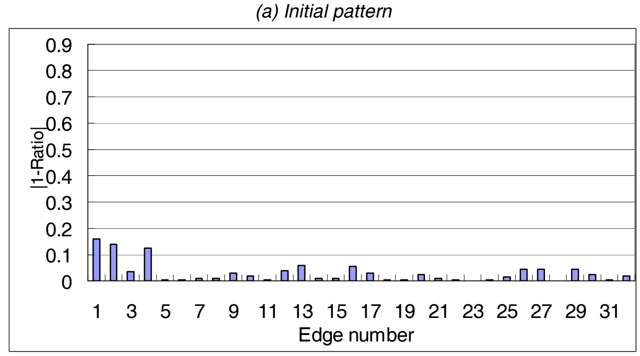
Gamut mapping to wide gamut - Experiment 2 -

In the next experiment, we used an EIZO ColorEdge 221 monitor with Adobe RGB gamut as the target gamut. The color gamut in CIELAB space is shown in Fig.8. As shown in the figure, the gamut volume of the monitor is large. We used the image "COPY B" in Ref.[9] as the initial solution. The image gamut of the initial pattern is larger than the original image.

Figure 9 shows the reproduction images. Fig. 9(a) shows the image reproduced by the initial solution, i.e. T=0. As shown in the figure, the contrast of Fig. 9(a) seems to be remarkably different from the original image in Fig. 9(f). So, the spatial ratio has not been preserved. From Figs. 9(b) to 9(e) show the reproduction pattern by the proposed algorithm depending on the number of iteration t. As shown in the figure, Fig. 9(e) is an excellent



(a) Initial pattern



(b) Reproduction pattern by the proposed algorithm

Figure 7 *Results for 32 edge luminance ratios in Experiment 1.*

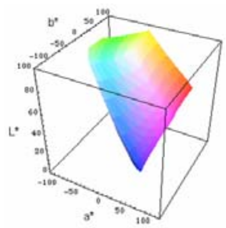
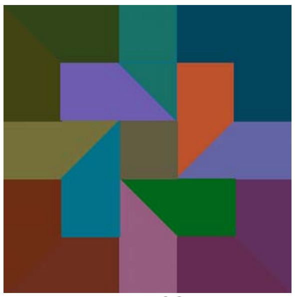
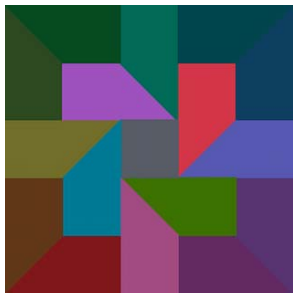


Figure 8 *Color gamut of the LCD monitor (Adobe RGB).*

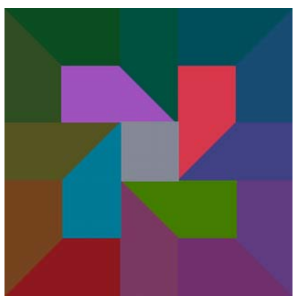


(a) Initial pattern "COPY B" [9] (t=0)

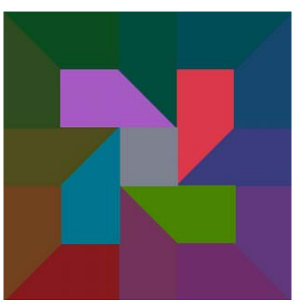


(b) t=1

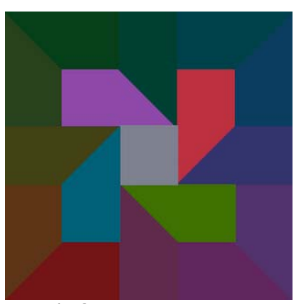
(c) t=5



(d) t=10



(e) t=100



(f) Original image in Fig.1

Figure 9 *Gamut mapping results in Experiment 2 by our algorithm.*

reproduction result compared with Fig 9(a) within the narrow gamut. Comparing with Fig. 5(e), the reproduction image in Fig. 9(e) is brighter, but almost the same sensation can be obtained among Fig. 1, Fig. 5(e) and Fig. 9(e).

Table 2 shows results by the spatial metrics. In this case, both results of the metrics also fall close to 0 by the proposed gamut mapping. Figs.10 and 11 show the spatial ratio in X-Y-Z and L^*, respectively. These results show that the proposed algorithm can be used also for the adjustment to preserve the spatial ratio after the conversion is performed by conventional pixel-based gamut mapping algorithms. The proposed algorithm is based on an iterative processing. We show the convergence property in Fig.12. In this figure, the change of the spatial ratio ΔL for increasing of the number of iteration t is plotted. Figures 12(a) and (b) show the results for the DLP and LCD, respectively. In both cases, initial values are different, but the ratios decrease monotonically and begin to converge around t=15.

Table 2 Evaluation for spatial ratio in Experiment 2.

	ΔR	ΔL
Initial pattern	1.28	0.31
Reproduction pattern	0.05	0.01

Conclusions

This paper has proposed a calculation algorithm for spatial gamut mapping to preserve ratios at edges, and verified the effectiveness of the proposed algorithm through experiments using a Mondrian pattern. Our algorithm can derive a practical solution by the relaxation labeling algorithm. Experimental result on DLP and LCD devices using a Mondrian indicated that our algorithm can work well as a SGMA. The practical application for natural images is a future problem.

Ackowledgments

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Author Biography

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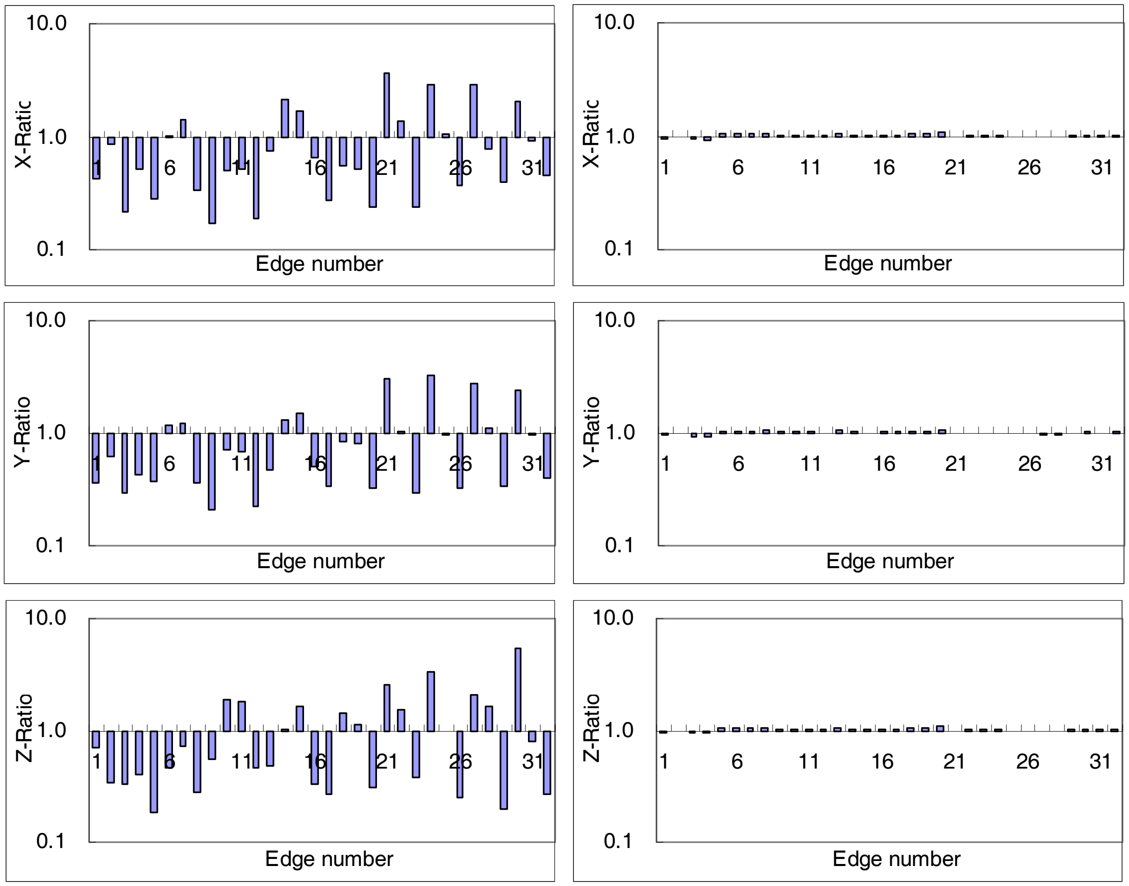
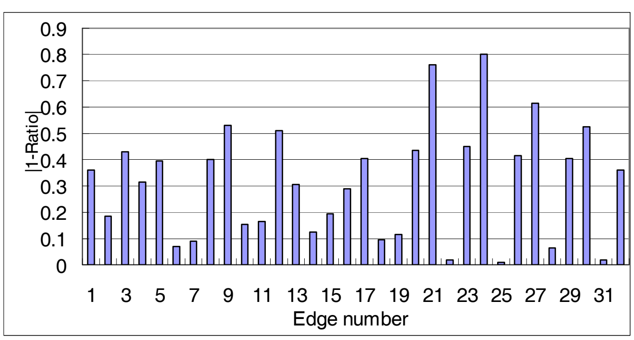


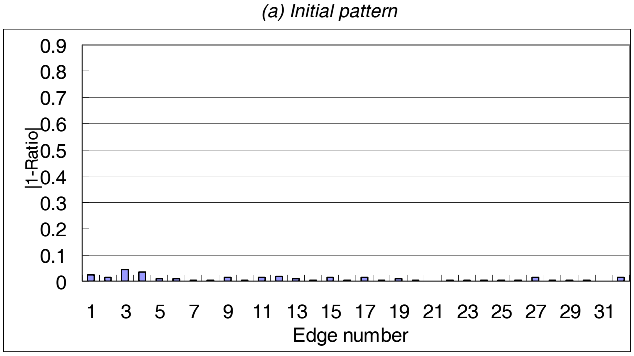
(a) Initial pattern

(b) Reconstruction pattern by the proposed algorithm

Figure 10 *Results for 32 edge XYZ ratios in Experiment 2.*

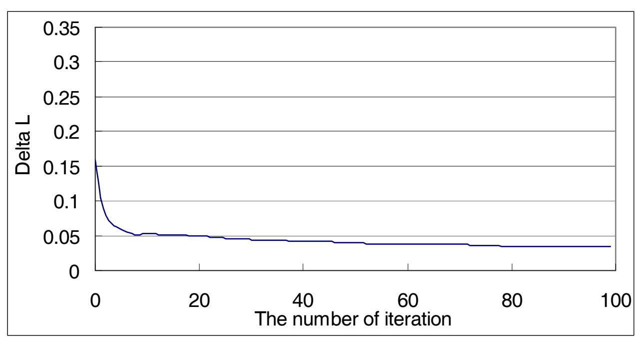


(a) Initial pattern

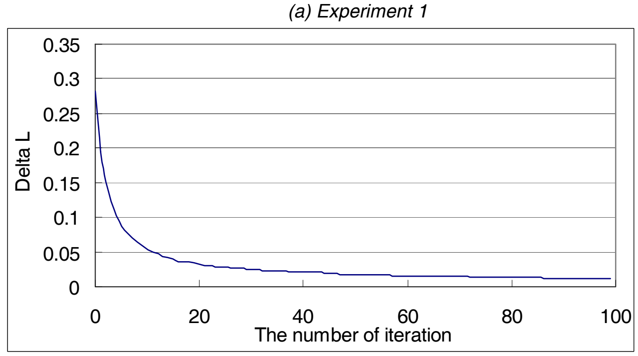


(b) Reproduction pattern by the proposed algorithm

Figure 11 *Results for 32 edge luminance ratios in Experiment 2.*



(a) Experiment 1



(b) Experiment 2

Figure 12 *The relation between ΔL and the number of iteration.*