Color Correction by Considering the Distribution of Metamers within the Mismatch Gamut

Philipp Urban[1], Roy S. Berns[1], Rolf-Rainer Grigat[2];
[1] Munsell Color Science Laboratory, Chester F. Carlson Center for Imaging Science,
Rochester Institute of Technology, 54 Lomb Memorial Drive, Rochester, New York 14623
[2] Hamburg University of Technology, Vision Systems, Harburger Schloßstraße 20, 21079 Hamburg, GERMANY

Abstract

Color correction describes the transformation process between device RGB values and CIEXYZ or CIELAB values. This mapping is in general not unique, because the spectral sensitivities of most of the devices do not satisfy the Luther condition and the acquisition and viewing light sources have a different power spectrum. Therefore, there exists a set of colors with different reflectance spectra which result in the same device RGB response (device metamerism), but leads to different tristimuli for an observer under the viewing light source. To determine an optimal mapping between a given device RGB and a CIELAB color, the distribution of metamers in a metamer mismatch gamut has to be characterized in the viewing CIELAB space. We present a novel method by estimating the distribution of metamers within the mismatch gamut using a Monte Carlo method. The main idea is the construction of a basic collection of metameric blacks (used by the Monte Carlo method) that is calculated by using a representative set of reflectance spectra and performing principle component analysis (PCA) within the black space of the device. The transformation of the sum of a fundamental metamer for the sensor response and the basic collection in the CIELAB color space leads to a point cloud with a centroid approximating the center of gravity of the mismatch gamut. This point is the optimal color correction in the sense of the smallest mean error.

Introduction

Color correction describes the transformation process between device RGB values and an device independent color space of an observer (e.g. CIEXYZ, CIELAB). In each metamer color reproduction system this is the first color transformation after image acquisition. The mapping is in general not unique, because the spectral sensitivities of most of the devices do not satisfy the Luther-Ives condition [1][2] and the acquisition and viewing light sources have a different power spectrum. Therefore, there exists a set of colors with different reflectance spectra which result in the same device RGB response (device metamerism), but have different color appearances for an observer under the viewing light source (metamer mismatch gamut). For this reason color correction is a classical one-to-many transformation problem.

In recent years various color correction methods have been proposed. Besides common target based methods like linear transformation into the CIEXYZ color space using a simple 3x3 matrix or multi-order polynomial regression into the CIEXYZ color space, Hardeberg [3] and Koenig [4] proposed multi-order polynomial regression into the CIELAB color space after initially performing different transformation steps to consider the nonlinear relationship of human color vision to the intensity of the RGB values. They achieved a distinct improvement of error rates. Vrhel et al. [5] used neural networks, Koenig [4] proposed a matrix method that was robust to noise; Hung [6] used LUT-interpolation and extrapolation; and Finlayson [7] proposed a constrained least square-regression to preserve the white point. This list of target based methods is not exhaustive.

Other methods use the model of a linear image acquisition system for color correction. Finlayson, et al. [8] characterized the metamer mismatch gamut by an enclosing cube in the CIEXYZ color space using linear programming and chose the center of gravity of this cube for color correction. The results outperform the standard target based methods. Urban, et al. [9] used also a linear programming technique to calculate a metamer boundary descriptor matrix that characterizes the metamer mismatch gamut within the CIELAB color space. The center of gravity of the matrix entries had been chosen for color correction. Other approaches used the linear image acquisition model to calculate a matrix based transformation function [10][11][12].

The aim of this paper is to determine an optimal mapping between a given device RGB value and a device independent color in the sense of the minimal mean color difference. Our approach is to estimate the density distribution of metamers inside the metamer mismatch gamut (see [13][14][15]) within an observer's perceptual color space (i.e. the device independent color space) using a Monte Carlo method and choose the center of gravity of the metamers considering this density distribution for color correction. The main idea is the construction of a basic collection of device metameric blacks which is calculated by using a representative set of reflectance spectra and a PCA within the black space of the device. For a RGB sensor response we calculate a fundamental metameric spectrum and add the spectra of the basic collection according to the well-known metameric black method. Each of the resulting spectra leads by construction to the given RGB value. Transforming the whole set into the perceptual color space of the observer yields to a point cloud which density distribution is assumed as a good approximation of the density distribution of metamers within the metamer mismatch gamut.

This method improves the performance of target and regression based methods especially in the area of saturated colors.

The Metamer Mismatch Gamut

The following text is a short introduction to metamer mismatch gamuts. A detailed description can be found in [10][11][12].

If all spectra are sampled at N equi-spaced wavelength a linear acquisition system can be described algebraically as

$$c = SL_a r + = \Omega_a r + \varepsilon \quad (1)$$

where $c = (R, G, B)$ is the sensor response, S is a $3 \times N$ matrix which contains the channel sensitivities as row vectors, L_a is a $N \times N$ diagonal matrix with the radiant spectrum of the acquisition illuminant along the diagonal, r is a $N \times 1$ vector of the reflectance sample and is additive noise. The matrix product of the sensor response matrix S and the illuminant matrix L_a is the acquisition lighting matrix Ω_a.

By means of the lightning matrix Ω_a the device black space is defined as follows

$$\text{Kernel}(\Omega_a) := \{r \mid \Omega_a r = 0\} \quad (2)$$

The *device black space* contains all spectra which sensor responses are zero, i.e. black. The set of all device metameric spectra which lead to the sensor response c can be derived as follows

$$R_c = \{r \mid r = f_c + \text{Kernel}(\Omega_a)\} \cap R_{\text{all}} \quad (3)$$

where f_c is the so called *fundamental metameric spectrum* of the sensor response c and R_{all} is the space of all natural reflectance spectra. Each reflectance spectrum with sensor response c can be used for f_c. To calculate such a spectrum from the sensor response c, various methods can be used, e.g. pseudoinverse, Wienerinverse or principle eigenvector method. The intersection with R_{all} is to ensure physically useful spectra which are positive, bounded and smooth.

In an analogous manner to the sensor response the observer's tristimulus value $o \in$ CIEXYZ can be described

$$o = ML_v r = \Omega_v r \quad (4)$$

where $o = (X, Y, Z)$ is the CIEXYZ tristimulus, M is a $3 \times N$ matrix which contains the CIE $\bar{x}$, $\bar{y}$, $\bar{z}$ color matching function as row vectors, L_v is a $N \times N$ diagonal matrix with the radiant spectrum of the viewing illuminant along the diagonal, r is a $N \times 1$ vector of the reflectance sample and Ω_v is the observer lighting matrix. Due to the common difference of acquisition and viewing illuminant and the non-compliance of the Luther-Ives condition, device metameric spectra lead to different tristimuli. The space of all tristimuli resulting from device metameric spectra for the sensor response c is called metamer mismatch gamut

$$M_c = \Omega_v R_c \quad (5)$$

Each tristimulus within M_c could be the tristimulus that occur for an observer if he is looking on the acquired sample with reflectance spectrum r under the viewing illuminant L_v.

Unfortunately, only one tristimulus is the correct one and it is impossible to determine the real tristimulus given only the sensor response c. Since the density of metamers within the metamer mismatch gamut is not equally distributed [14], we can increase the chance to select the right tristimulus by choosing the one with the largest density. More related to practice is a selection that minimizes the mean color difference to all other colors within the metamer mismatch space considering the density of the metamers. In this context an inspection of metamer mismatch gamuts in perceptual color spaces e.g. CIELAB where the perceived color difference is defined as the Euclidean metric has considerable advantages.

The density distribution of metamers chages their skewness and kurtosis due to the non-linear relationship of the color space transformations.

The following method uses a Monte Carlo calculation to determine the density distribution of metamers within the metamer mismatch gamut. Basis of this Monte Carlo calculation is a representative set of reflectances. In our simulation we use 1269 spectra of Munsell color chips, available on the website of the University of Joensuu, Finland (`http://spectral.joensuu.fi`). If the set of acquired spectra is known, e.g. an output of a calibrated printer, than these spectra can be used instead of the Munsell set, to improve the results.

The Color Correction Method

The proposed method can be subdivided into three parts:

1. Constructing of the basic collection of device metameric blacks (once)
2. Calculation of the fundamental metamer (for each sensor response)
3. Monte Carlo calculation (for each sensor response)

In the following text we denote the representative set of reflectance spectra by $r_1, \ldots, r_n$. Furthermore we use principle component analysis [16][17] of different datasets to calculate their representative spectra. For the spectra $x_1, \ldots, x_m \in \mathbb{R}^N$ the function PCA returns a real valued, unitary $N \times N$ matrix X containing as columns vectors the characteristic spectra of the set ordered according to the size of their singular values

$$X = \text{PCA}(x_1, \ldots, x_m) \quad (6)$$

Constructing a Basic Collection of Device Metameric Blacks (once)

The basic collection of device metameric blacks is a set of representative spectra that are within Kernel(Ω_a). We calculate these spectra using a representative set of reflectance spectra in three processing steps:

1. In the first step we calculate the characteristic spectra of the representative set of reflectance spectra

$$U = \text{PCA}(r_1, \ldots, r_n) \quad (7)$$

2. In the second step we calculate a basis of the black space Kernel(Ω_a) which adopts the information of the representative set of reflectance spectra inside the black space. For this purpose

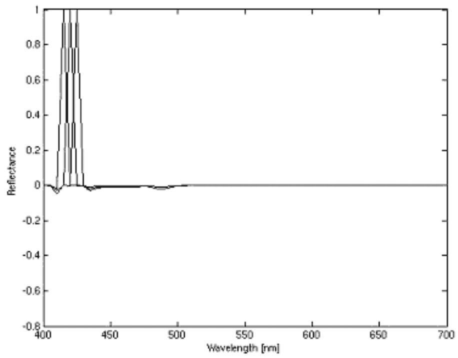

Figure 1. *Orthonormal basis spectra of the black space for a Leica camera with acquisition illuminant CIE D50. Here: 3 column vectors of matrix V.*

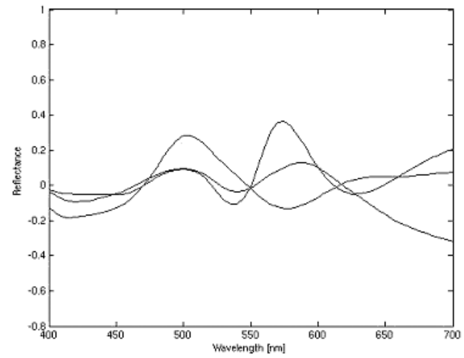

Figure 2. *Orthonormal basis spectra of the black space for a Leica camera with acquisition illuminant CIE D50. Here: The first 3 characteristic spectra $u_1^{\Omega_a}, \ldots, u_3^{\Omega_a}$ calculated for 1269 Munsell color chips as representative spectral set inside the device's black space.*

we calculate at first an arbitrary basis of $\mathrm{Kernel}(\Omega_a)$ by using e.g. the singular value decomposition of Ω_a^T

$$\Omega_a^T = VDW^T \tag{8}$$

The column vectors $v_4, \ldots, v_N$ of V which corresponds to singular values equal zero are a orthonormal basis of $\mathrm{Kernel}(\Omega_a)$ (see Figure 1). To calculate a basis which contains the basic information of the representative set of reflectance spectra within the first basis elements, we add the first three column vectors $u_{i_1}, u_{i_2}, u_{i_3}$ of U to $v_4, \ldots, v_N$, which are linearly independent to $v_4, \ldots, v_N$. According to the theorem of basis completion the resulting N vectors $u_{i_1}, u_{i_2}, u_{i_3}, v_4, \ldots, v_N$ are a basis of $\mathbb{R}^N$, and the matrix

$$B = (u_{i_1}, u_{i_2}, u_{i_3}, v_4, \ldots, v_N) \tag{9}$$

is invertible. Than we calculate the coefficients for all reflectance spectra of the representative spectral set respective the basis $u_{i_1}, u_{i_2}, u_{i_3}, v_4, \ldots, v_N$:

$$Q = B^{-1}R \tag{10}$$

where $R = (r_1, \ldots, r_n)$. The column vectors of $Q = (q^1, \ldots, q^n)$ are the desired coefficients. We can decompose each reflectance spectrum r_i of the representative set into a vector r_i^p which is a linear combination of the three characteristic spectra of the representative set $u_{i_1}, u_{i_2}, u_{i_3}$ and a vector $r_i^{\Omega_a}$ which lies inside the black space of the device

$$r_i = Bq^i = \sum_{j=1}^{3} q_j^i u_{i_j} + \sum_{j=4}^{N} q_j^i v_j = r_i^p + r_i^{\Omega_a} \tag{11}$$

To calculate the characteristic spectra of the representative set of reflectance spectra inside the black space of the device we have to perform a PCA of the set $r_1^{\Omega_a}, \ldots, r_n^{\Omega_a}$:

$$U^{\Omega_a} = \mathrm{PCA}(r_1^{\Omega_a}, \ldots, r_n^{\Omega_a}) \tag{12}$$

The first k ($k < N-2$) column vectors $u_1^{\Omega_a}, \ldots, u_k^{\Omega_a}$ of the matrix U^{Ω_a} are the first k most significant characteristic spectra of the representative set of reflectance spectra within the black space of the device (see Figure 2).

3. Using this preliminary work we can define the basic collection by means of a decomposition of each reflectance spectrum in the representative spectral set

$$r_i = \sum_{j=1}^{3} q_j^i u_{i_j} + \sum_{j=1}^{N-3} g_j^i u_j^{\Omega_a} \tag{13}$$

The basic collection $r_1^{\Omega_a}, \ldots, r_n^{\Omega_a}$ is defined by the first k terms of the second sum, i.e.

$$\widehat{r}_i = \sum_{j=1}^{k} g_j^i u_j^{\Omega_a} \tag{14}$$

The spectra $r_i^{\Omega_a}$ of the basic collection are smooth, they are inside the black space of the device and contain the principle black space information of the representative set.

Calculation of the fundamental metamer (for each sensor response)

Given the sensor response $c = (R, G, B)$ the fundamental metamer can be calculated as follows

$$f_c = A(\Omega_a A)^{-1} c \tag{15}$$

where $A = (u_{i_1}, u_{i_2}, u_{i_3})$ is the matrix containing the first three characteristic spectra which are linear independent to the black space.

The Monte Carlo Method (for each sensor response)

For the Monte Carlo calculation we consider only bounded ($r \leq 1$) and positive ($r \geq 0$) spectra and add the fundamental metamer f_c to ensure that the set is not empty. The resulting set has n_c ($n_c \leq n+1$) elements (see Figure 4)

$$S_c = \{f_c + \widehat{r}_i \mid 0 \leq f_c + \widehat{r}_i \leq 1,\ i = 1, \ldots, n\} \cup \{f_c\} \tag{16}$$

After transforming the spectra of S_c to the CIELAB color space for the viewing illuminant we take the mean value of the resulting point cloud for color correction:

$$\frac{1}{n_c} \sum_{r \in S_c} \mathscr{L}(\Omega_v r) \tag{17}$$

where $\mathscr{L}$ is the color space transformation from CIEXYZ to CIELAB.

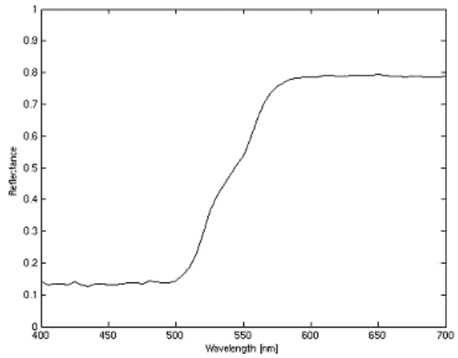

Figure 3. *A Munsell spectrum*

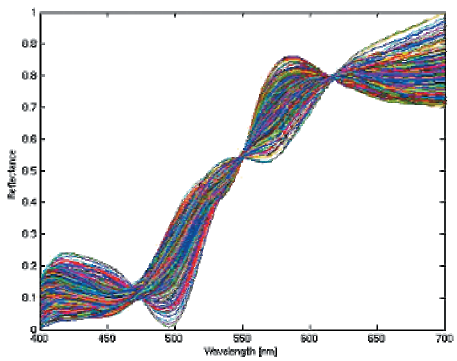

Figure 4. *Spectral set S_c. Using the Leica Camera (see Figure 9) under the acquisition illuminant CIE D50 these spectra result in the same sensor response as the spectrum in Figure 3.*

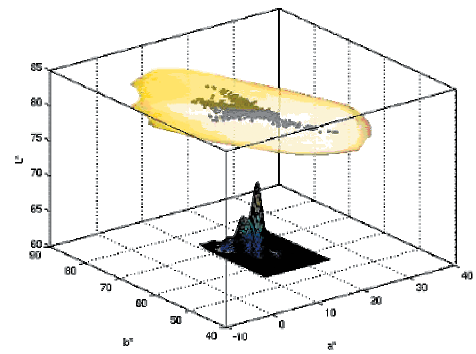

Figure 5. *Point cloud resulting from the Monte Carlo calculation within the mismatch gamut (determined by the MBD method [9]). The points were calculated from the response of the Leica camera resulting from the reflectance spectrum in Figure 3 for the acquisition illuminant CIE D50 and viewing illuminant CIE F11. The projected density function below shows a distinct maximum as well as in all of the investigated cases.*

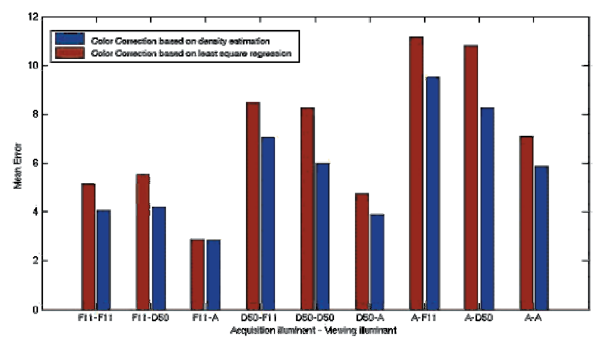

Figure 6. *Mean ΔE^*_{ab} errors for all reflectance spectra of the Vrhel database.*

Experimental Results

We have tested our method by means of simulation experiments using a Leica camera with sensitivities shown in Figure 9, left. The sensitivities have been measured using a calibrated monochromator. Each of the combinations of the illuminants CIE A, CIE C, CIE F11 is used as acquisition and viewing illuminants. As mentioned before, we use the 1269 Munsell color chips as our representative set of reflectance spectra and four characteristic spectra of the black space (i.e. k = 4). We compare our method with a simple least square regression matrix method that use as virtual target the same Munsell spectral reflectance set. As test spectra we use the reflectance spectra of the Vrhel database [18]. The results are shown in Figure 6, 7 and 8.

Discussion and Conclusion

The results demonstrate the effectiveness of this novel approach, particularly since the verification data, the Vrhel set, have spectra quite dissimilar to the calibration data, the Munsell Book of Color. The density estimated color correction achieves distinctive smaller ΔE^*_{ab} errors compared with the target based linear least square method. The behavior of the standard deviation of ΔE^*_{ab} errors is similar. The magnitude of error strongly depends on the special combination of acquisition and viewing illuminant. Though both methods have a completely different structure, their relative behavior considering acquisition and viewing illuminant is very similar: For acquisition illuminant A and viewing illuminant D50 or F11 the error rates are high. For acquisition illuminant F11 and viewing illuminant the error rates for both methods are very small. The reason of these similarities is the size of the metamer mismatch gamut which is on average conspicuous smaller for F11-A combination compared to A-D50 or A-F11. The density estimated color correction outperforms the regression based for each combination of acquisition and viewing illuminants. For illuminant combinations result in small metamer mismatch gamuts the error differences are small but for large metamer mismatch gamuts the error rates of the density estimated color correction are noticeable smaller.

Acknowledgements

The authors thank the Deutsche Forschungsgemeinschaft (German Research Foundation) for the sponsorship of this project.

References

[1] R. Luther. Aus dem gebiet der farbreizmetrik. *Zeitschrift für technische Physik*, 8:540–558, 1927.

[2] H. E. Ives. The transformation of color-mixture equations from one system to another. *J. Franklin Inst.*, 16:673–701, 1915.

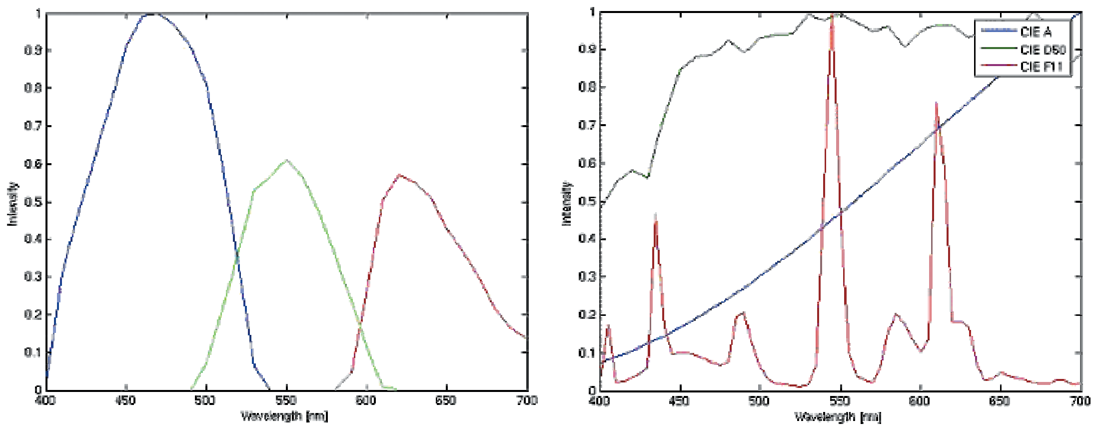

Figure 9. Left: Sensitivities of the Leica camera. Right: Acquisition and viewing illuminants for simulation experiments.

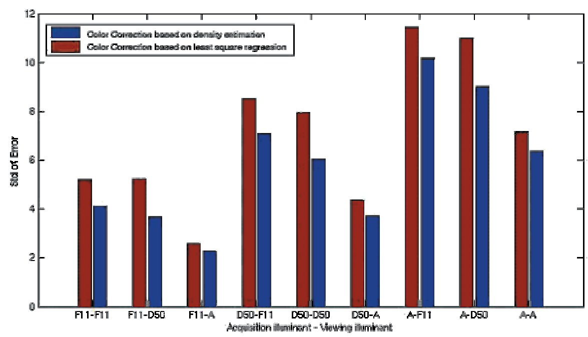

Figure 7. Standard deviation of ΔE^*_{ab} errors for all reflectance spectra of the Vrhel database.

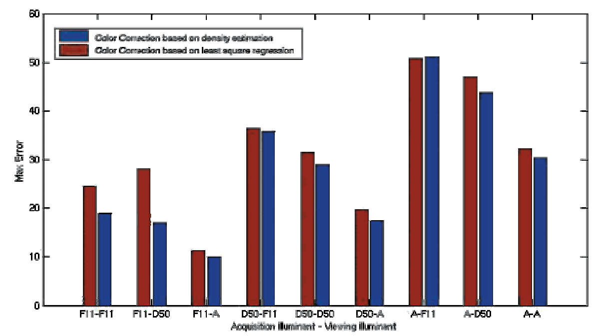

Figure 8. Max ΔE^*_{ab} errors for all reflectance spectra of the Vrhel database.

[3] J. Y. Hardeberg. *Acquisition and reproduction of colour images: colorimetric and multispectral approaches*. PhD thesis, Ecole Nationale Supérieure des Télécommunications, France, 1999.

[4] F. König. *Die Charakterisierung von Farbsensoren*. PhD thesis, Rheinisch-Westfälische Technische Hochschule Aachen, Germany, 2001.

[5] M. J. Vrhel and H. J. Trussell. Color scanner calibration via a neural network. In *Proceedings IEEE ICASSP-99*, volume 6, pages 3465–3468, Phoenix, Arizona, 1999.

[6] P.-C. Hung. Colorimetric calibration for scanners and media. In *Proceedings of SPIE*, volume 1448, pages 164–174, San Jose, Ca, 1991.

[7] G. D. Finlayson and M. S. Drew. Constrained least-squares regression in color spaces. *Journal of Electronic Imaging*, 6:484–493, 1997.

[8] G. D. Finlayson and P. M. Morovic. Metamer constrained color correction. *Journal of Imaging Science and Technology*, 44:295–300, 2000.

[9] P. Urban and R.-R. Grigat. The Metamer Boundary Descriptor Method for Color Correction. *Journal of Imaging Science and Technology*, 49:417–429, 2005.

[10] M. S. Drew and B. V. Funt. Natural metamers. In *CVGIP: Image Understanding*, pages 139–151, 1992.

[11] G. D. Finlayson and P. Morovic. Intensity constrained errorless colour correction. In *IS&T/SID*, pages 106–110, Scottsdale Ariz., 2004.

[12] G. D. Finlayson and P. Morovic. Error less colour correction. In *ICPR 2004*, pages 181–185, Cambridge, UK, 2004.

[13] G. Wyszecki and W.A. Stiles. *Color Science: Concepts and Methods, Quantitative Data and Formulae*. John Wiley & Sons, inc., 2 edition, 2000.

[14] W.A. Stiles, G. Wyszecki, and N. Ohta. Counting metameric object-color stimuli using frequency-limited spectral reflectance functions. *Journal of the Optical Society of America*, 67:779–784, 1977.

[15] N. Ohta and G. Wyszecki. Theoretical chromaticity mismatch limits of metamers viewed under different illuminants. *Journal of the Optical Society of America*, 63:327–333, 1975.

[16] I. T. Jolliffe. *Principal Component Analysis*. Springer, 2 edition, 2002.

[17] D.-Y. Tzeng and R. S. Berns. A Review of Principal Component Analysis and Its Applications to Color Technology. *Color Research and Application*, 30:84–98, 2005.

[18] M.J. Vrhel, R. Gershon, and L.S. Iwan. Measurement and analysis of object reflectance spectra. *Color Research and Application*, 19:4–9, 1994.

Author Biography

Philipp Urban received his M.S. degree in Mathematics from the University of Hamburg in 1999 and his Dr. degree in the field of color science from the Hamburg University of Technology in 2005. From 1999 until 2006 he was part of the research group "Vision Systems" at the Hamburg University of Technology and worked for Ratio Entwicklungen GmbH (ICC-member) where he developed color managing systems. Since 2006 he is a visiting scientist at the Munsell Color Science Laboratory at the Rochester Institute of Technology. His research interests are color science and multispectral imaging.