Spectral Gamut Mapping and Gamut Concavity

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Abstract

A spectral gamut-mapping algorithm is introduced that works well for printers with a large number of inks. It finds the best mapping onto the convex hull of the printer spectral gamut while preserving color defined in CIE XYZ as much as possible. The technique employs a non-negative least-square fit. Since the gamut-mapping algorithm depends on the common assumption that the gamut is convex, an experimental study of the degree of gamut concavity is conducted. It finds that there is a significant amount of concavity, and that that the degree does not appear to change much as the number of inks is increased. Finally, the performance of the gamut-mapping algorithm and gamut coverage in spectral space is compared for 3-, 4-, 5- and 6-ink printers using both synthetic ink models and real ink data.

Introduction

In comparison to standard color printing, spectral printing aims to reproduce a given reflectance spectrum rather than produce a metameric reflectance spectrum that simply matches a given color. Spectral printing aims to reduce a problem that can arise in metameric color printing which is that the reproduced color may match under one illuminant, but not match well under some other illuminant. Clearly, if the printed output reflectance matches the input reflectance, the printed color will match the input color under all illuminants.

Spectral printing requires a significantly larger number of inks than the standard CMYK ones, but this increases the computational complexity of printing algorithms in terms of both time and space. In particular, standard gamut-mapping algorithms map colors within a 3-dimensional space. Generally, their computational complexity increases rapidly with dimension, so that they become intractable for the gamut-mapping of spectra represented in, say, 11 dimensions. For example, a gamut-mapping algorithm that relies on the computation of the convex hull of the measured gamut will not work since computing a d-dimensional convex hull of n points requires order *O(n**floor(d/2)+1)* operations. Bakke et al. [12] address this problem by reducing the dimensionality via principal components analysis and then computing up the convex hull in up to 8 dimensions.

The first part of this paper introduces a spectral gamut-mapping algorithm that projects an out-of-gamut spectrum onto the printer gamut's convex hull without having to calculate the hull explicitly. It is based on a non-negative least-squares solution to a set of constraints that Finlayson et. al. [1] originally proposed as part of a color constancy method. The computational requirements of the proposed gamut-mapping algorithm are reasonable for the higher number of dimensions required. A modification to the algorithm is then presented that maps out-of-gamut spectra to the closest spectra on the gamut's convex hull subject to the constraint that it preserve XYZ tristimulus values.

The second part of the paper evaluates the validity of the assumption that the printer gamut is convex. To what extent is the printer gamut concave, and does the degree of concavity vary as a function of the number of inks? Experimental results on the concavity of the gamuts of 3 through 6 inks are presented.

Spectral Gamut Mapping Using Non-Negative Least Squares

For a point ρ inside a convex gamut, it can be represented as a convex combination of other points, q_i, within the gamut:

$$\rho = \Sigma\, \alpha_i q_i, \quad \alpha_i \geq 0, \quad \Sigma\alpha_i = 1 \qquad (1)$$

The α_i's are weights, and the restrictions on the weights ensure that ρ does not lie outside the convex hull of the q_i.

For a point ρ outside a convex gamut, we can find the closest point to ρ lying on the convex hull of the gamut by finding α_i minimizing the distance e:

$$e = |\, \rho - \Sigma\, \alpha_i q_i\, |2, \quad \alpha_i \geq 0, \quad \Sigma\alpha_i = 1 \qquad (2)$$

Finlayson et. al. [1] showed that (2) can be rewritten to include a weight W as an extra dimension in the input data, and that the revised equations can then be solved by standard non-negative least squares. Their derivation is as follows.

$$q_i' = [q_i\ W] \qquad (3)$$

$$e' = |\, \rho - \Sigma\, \alpha_i q'_i\, |2 \qquad (4)$$

Re-writing e' yields

$$e' = e + W^*(1 - \Sigma\, \alpha_i) \qquad (5)$$

The advantage of (5) is that it can be minimized by non-negative least squares. Choosing a large value for W emphasizes the second term in (5), thereby enforcing the constraint $\Sigma\alpha_i = 1$.

Spectral gamut mapping means mapping a spectrum that lies outside the printer gamut onto a printable spectrum. For a spectrum represented as a point, ρ, minimizing (5) finds the closest point on the gamut's surface, in other words it finds the closest printable spectrum. The spectrum is described as a linearly weighted combination of other printable spectra, spectra that are within the printer gamut.

This proposed gamut-mapping algorithm is easy to implement and the computation is relatively fast the dimensionality of spectra. The space and time requirements grow with the number of input data points. However, Bastani et. al. [2] have shown that by sampling ink space intelligently, the number of points required to represent a gamut space can be reduce by as much as 95%.

Color-Preserving Spectral Gamut-Mapping

Spectral gamut mapping using equation (5) maps to the closest printable spectrum in spectral space, but there is no guarantee that this new spectrum will have the same color (for a fixed illuminant) as the original spectrum. It would be preferable to have a spectral gamut-mapping algorithm that maps an out-of-gamut spectrum to the closest in-gamut spectrum subject to the constraint that it preserves color as well. This can be accomplished by modifying the algorithm so that the projection onto the gamut is in a direction perpendicular to CIE XYZ space. Doing so preserves the XYZ coordinates as much as possible. A similar approach was used by Chau et al. [11] in preserving CIE XYZ values (fundamental components).

Let U represent the principal component basis of the x-bar, y-bar, and z-bar color matching functions. The basis vectors in U can be sorted in order of decreasing variance.

Let P be the set of spectra in the printer gamut *{p1, p2, ...}*, then

$P_u = P\underline{U}$

represents the gamut in U space sorted in terms of decreasing variance in XYZ space. After this linear transformation, the first 3 coordinates of P_u represent the tristimulus values of spectra in the printer gamut. Applying the gamut-mapping algorithm described above to the weighted P_u yields the closest spectrum on the hull of the printer gamut that least changes XYZ.

Can we assume the spectral printer gamut is convex?

Many existing gamut-mapping algorithms [3], [4] and [5], including the LabPQR spectral gamut-mapping algorithm [6], map an out-of-gamut point onto the convex hull of the printer gamut. The assumption is that the gamut is convex. Is this assumption is valid and how much accuracy is lost by assuming a convex printer space?

Algorithms such as Alpha Shape [7] can measure concavity of a space in a low-dimensional space (3D), but cannot be used in high-dimensional spectral space. As a measure of gamut concavity, we find the difference between a mapping onto the convex hull versus a mapping that does not rely on convexity. In particular:

1. Gamut map spectra onto the hull gamut using a gamut-mapping algorithm (e.g., the one described above, LabPQR, etc.)
2. Map the spectra onto the manifold of the gamut space by searching for the closest ink spectra. It is important that the search does not rely on the convexity assumption.
3. Measure the difference between mapped spectra obtain in steps 1 and 2.

For searching the closest gamut reflectance, an algorithm based on hierarchical search in ink space is used. In this algorithm a subdivision of ink combinations is created. Let the set of the subdivisions of ink space be *M*, where there is a spectral reflectance associated with each ink combination, m_i, in *M*. The algorithm is as follows:

1. Find the closest m_i spectrum to a given input point ρ in spectral space.
2. Create a grid of ink subdivisions around mi with smaller ink variation.
3. Go back to step 1 until the grids are small enough. Then go to the next step.
4. Return spectral reflectance of m_i as the closest point.

How to Measure Metamerism

RMS (root mean square) difference between two spectral reflectances does not necessarily represent the difference that may be apparent to the eye. As an alternative measure, we use the maximum deltaE_{94} of the two spectral reflectances found under 11 different lights. The 11 illuminants used in this paper are from the Simon Fraser data base [10].

Table 1 The lights used in measuring the color variation of two similar reflectance spectra under different illuminants.

	11 illumination types used for delta E comparison
1.	Sylvania 50MR16Q (12VDC)---A basic tungsten bulb
2.	Sylvania 50MR16Q (12VDC) + Roscolux 3202 Full Blue filter
3.	Solux 3500K (12VDC)--Emulation of daylight
4.	Solux 3500K (12VDC)+Roscolux 3202---Emulation of daylight
5.	Solux 4100K (12VDC)--Emulation of daylight
6.	Solux 4100K (12VDC)+Roscolux 3202---Emulation of daylight
7.	Solux 4700K (12VDC)--Emulation of daylight
8.	Solux 4700K (12VDC)+Roscolux 3202---Emulation of daylight
9.	Sylvania Warm White Fluorescent (110VAC)
10.	Sylvania Cool White Fluorescent (110VAC)
11.	Philips Ultralume Fluorescent (110VAC)

Experiments

Printer Model

To make empirical testing easier a printer model introduced by Tzeng et. al. [8], [9] is used to predict the spectral reflectance resulting from a given ink combination. The following equations are used to predict the reflectance:

$$R_\lambda = (R_\lambda{}^{1/w}{}_{paper} - \psi_{\lambda,mixture})^w \quad (6)$$

$$\psi_{\lambda,\ mixture} = \Sigma c_i R_{i\lambda}$$

$$\psi_\lambda = R_\lambda{}^{1/w}{}_{paper} - R_{\lambda,i}{}^{1/w}$$

Where $\psi_{i\lambda}$ is the reflectance of ink *i* at wavelength λ at maximum concentration.

Ink Choices

Three sets of inks are used in this study. One set is based on reflectances of real inks, and the other two sets are synthetic ink reflectances. Both synthetic and actual measurement data are used to make the result less dependent on a specific ink selection.

Below are the 3 types of inks used:

1. Spectral reflectance of real pigmented inks. The following 7 inks were used: Orange (O), Cyan (c), Magenta (m), Yellow (y), Green (Gr), Violet (V) and Black (K).
2. Synthetic square-wave reflectances with no overlap.

3. Synthetic square-wave reflectances with 30% overlap.

Figure 1 illustrates the 3 square-wave synthetic inks with no overlap.

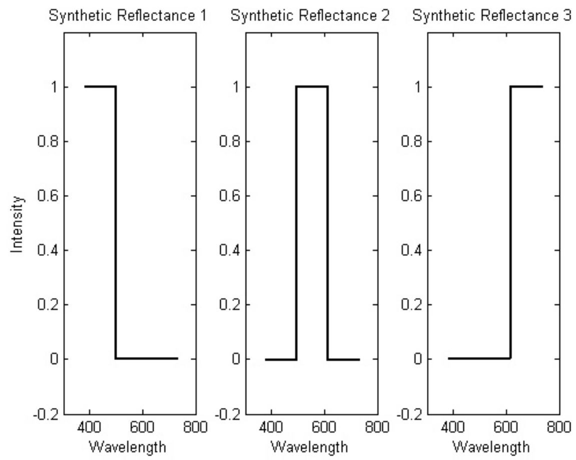

Figure 1: The 3 non-overlapping square-wave reflectances used with the LP model. The x-axis represent the wavelength of each reflectance. Y-axis represent intensity of the spectrum

Test Data

To test the gamut mapping algorithm, the scene reflectances from SFU data base were used. There are 1350 individual reflectances in the data base.

Results

Figure 2 and Figure 3 show the accuracy of the spectral reproduction when the proposed spectral-gamut mapping algorithm is used to map the out-of-gamut points onto the gamut hull. The table shows that, as one would expect, when the number of inks is increased, reproduction improves.

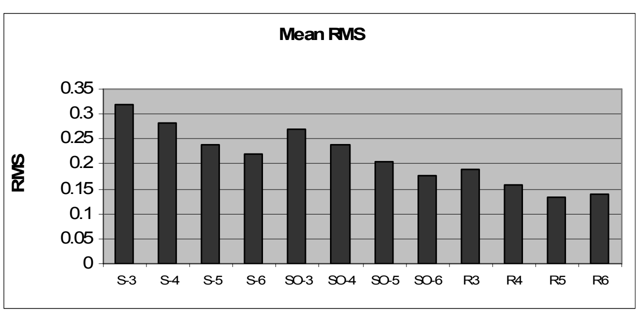

Figure 2: Average RMS of spectral reproduction accuracy. All out-of-gamut spectra are mapped to the gamut hull using the proposed spectral-gamut mapping algorithm. The RMS entries are the differences between the input spectrum and the reproduced spectrum. S-3, represents Synthetic 3-ink printer, SO-3 is for Synthetic ink with overlap and, Ri's represent the real inks discussed earlier.

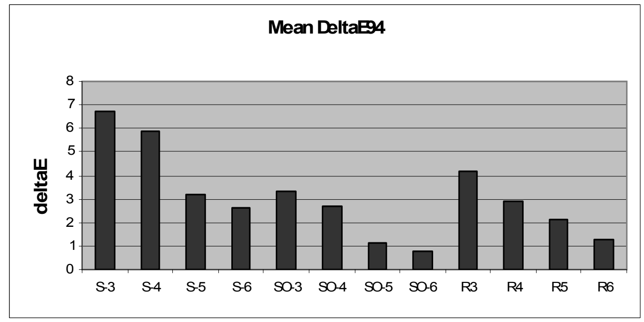

Figure 3 Average Delta E for the differences between the input spectrum and the reproduced spectrum. S-3 represents Synthetic 3-ink printer, SO-3 is for Synthetic ink with overlap, and Ri's represent the real inks discussed earlier.

Figure 4 and Figure 5 show the difference between the closest spectrum to the input spectrum found by projection onto the convex hull versus finding it via the proposed search algorithm. Within the limits of the search tolerance, the search projection method is optimal in the sense that it finds the closest point on the hull surface whether or not the surface is concave. The difference in the spectra obtained by the two methods is a measure of the convexity of the printer gamut.

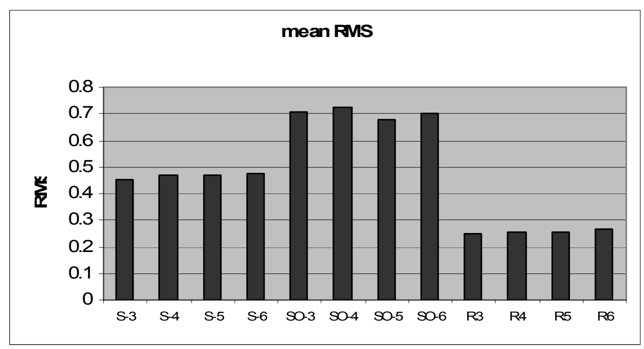

Figure 4: Averaged RMS between proposed gamut mapping and running a search. The data represents distances between mapping onto a convex, versus a potentially concave, gamut surface

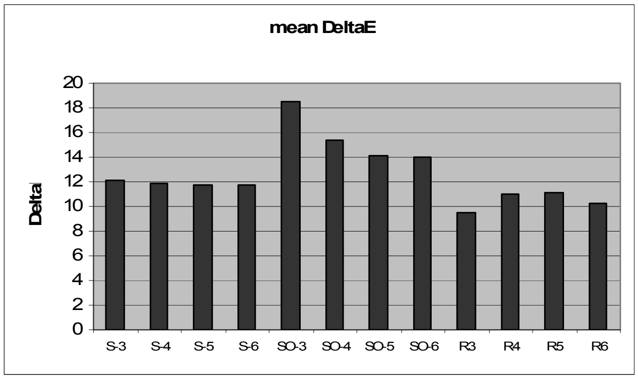

Figure 5: Averaged RMS between proposed gamut mapping and running a search

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Author Biographies

Behnam Bastani completed his Masters degree at Simon Fraser University in Computing Science in 2005, where his research was focused on gamut mapping and characterization of digital color displays. He joined Hewlett-Packard Company in 2004, where his research is on designing auto-calibration models for high-end ink-jet printers. He is also a PhD candidate at Simon Fraser University where his research focus is in Spectral Printing and Ink Separation.

Brian Funt received the Ph.D. degree in computer science from the University of British Columbia, Vancouver, BC, Canada, in 1976. He is a Professor with the School of Computing Science, Simon Fraser University, Vancouver, where he has been since 1980. He has published widely in color, color constancy, retinex, color calibration, and illumination estimation.