# A Colour Gamut Based on Reflectance Functions

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#### Abstract

A new gamut [1] to represent object colours was developed by the authors. Its boundary is defined by CIELAB L\*,  $C_{ab}$ \* and  $h_{ab}$  under D50/2° conditions. This paper describes an extension of this colour gamut to be reflectance based. Examples are given to illustrate the advantages to use spectral based rather than colorimetric based colour gamut.

#### INTRODUCTION

Colour gamut is an important aspect for designing imaging devices. It is hoped that these devices can reproduce as many as the real world colours, either natural or artificial. Hence, it is typical to plot colour gamuts of imaging devices by comparing their performances. Figure 1 gives a typical example to compare the colour gamuts of three types of mobile displays. There have been debates on which space or attributes are more suitable for comparing the gamuts of imaging devices. The most important information obviously is the percentage of the real world colours which can be reproduced.



Figure 1. Colour gamuts of three mobile displays

With this in mind, Pointer [2] and ISO TC130 [3] members developed gamuts of object colours. A recent study by Li *et al* [1] presented in this conference defines a new colour gamut. This gamut is considered to be more reliable than the previous ones [2,3] because it was derived based on a comprehensive set of collection including 78044 reflectance functions measured by modern spectrophotometers. The data come from 17 data sets including many different types of surfaces representing the world of colours. Some issues for developing reliable colour gamut were raised in their study. It was found that ISO gamut is unrealistically too large to represent object colours as illustrated in Figure 2 for CIELAB  $h_{ab}=200^{\circ}$  plane, where the diamond, circle and cross curves represent the ISO, Pointer's and the new gamut boundaries, respectively, and the triangles and crosses are the data from the ISO SOCS (printer) data [4] and the other colours accumulated [1] within the hue angles of 195° to 205°. It can be seen clearly that comparing with the data accumulated, ISO colour gamut boundary is much too large and the Pointer's gamut is too small. The new gamut lies between the ISO and Pointer's gamuts and fits well to the most outliners of data cluster. The same trend can be found in all the other hue planes. Figure 2 also shows that some colours plotted in triangles in the dark saturated region are located outside the three gamuts and they form an isolated group. All these colours come from the ISO SOCS (Printer) data set. These were removed when developing the new colour gamut [1]. Possible reasons were also discussed in reference [1] to explain the large size of ISO gamut.



Figure 2. Triangles and crosses represent ISO SOCS (Printer) data and the other data sets plotted in CIELAB L\*C\* plane at h=200°. The diamond, circle and cross curves represent the boundaries of the ISO, the Pointer's and the new gamuts.

All the above three gamuts were defined by  $L^*$ ,  $C_{ab}^*$  and  $h_{ab}$  values either under CIE C/2° or D50/2° conditions. However, these gamuts cannot be used when the other illuminant or 10° observer is required. One possible solution is to use a chromatic adaptation transform (CAT) to transform from the source to destination conditions. However, CAT is not designed for this kind of applications, because it cannot accurately predict colours on the gamut boundary as found by Brill [5]. An ideal solution is to define a colour gamut based on reflectance functions, which is the main motivation of this paper.

## DESCRIBING A COLOUR GAMUT BASED ON REFLECTANCE FUNCTIONS

This section describes the development of a gamut boundary based on reflectance functions. The comprehensive reflectance

data sets accumulated earlier [1] and the new gamut boundaries were used. The new gamut was defined by colour coordinates of  $L^*$ ,  $C_{ab}^*$ ,  $h_{ab}$  under D50/2° conditions. There are 36 hue divisions from 0° to 360° at 10° interval, and 19 lightness divisions from 5 to 95 at 5 unit interval.

Using the colour coordinates of the new colour gamut, reflectance functions were generated from the reflectance data sets accumulated earlier [1]. Generating reflectance based on a given set of colour coordinates such as  $L^*$ ,  $C_{ab}^*$ ,  $h_{ab}$  or X, Y, Z has a long history. However, the problem has never been solved satisfactorily. The difficulty is that the number of equations is less than the number of unknowns. Thus the (inverse) problem is not well defined. Hence, more constraints such as basis vectors [6], smoothest conditions [7] and colour constancy [8] are introduced to reduce the dimensionality of the problem.

The procedure to generate the reflectance function is given below:-

Stage 1: to transform the given boundary coordinates in  $L^*$ ,  $C_{ab}^*$ ,  $h_{ab}$  under D50/2° conditions to tristimulus values X, Y, Z. Let  $p^T = (X, Y, Z)$  and W (n by 3) be the weighting table and f (n by 1) be the desired reflectance function or vector, then we have

$$p = W^T f$$

(1)

Stage 2: to use the singular value decomposition or PCA approach for deriving the basis vectors  $\mathcal{E}^{(1)}, \mathcal{E}^{(2)}, \dots, \mathcal{E}^{(n)}$ . The main difference for searching the basis vectors from the literature is that we build the basis vectors using a subset of reflectance functions accumulated. Any function from the subset has a hue angle close to  $h_{ab}$ , the hue angle of the given coordinates. Under this constraint it is expected that the reconstructed reflectance function f is close to real. It is expected that

 $\varepsilon^{(1)}, \varepsilon^{(2)}, \dots, \varepsilon^{(m)}$  with m being much less than n reflect fully the whole characteristics of the subset of reflectance functions. Hence the reconstructed f will satisfy

$$f = E\alpha \tag{2}$$

where

$$E = (\mathcal{E}^{(1)}, \mathcal{E}^{(2)}, \cdots, \mathcal{E}^{(m)}), \text{ and } \alpha \in \mathbb{R}^m.$$
(3)

Stage 3: to apply the smoothness condition. Let G be the smoothness operator [7], so that the smoothest condition is given below:

$$\frac{Min}{f \in R^n} \|Gf\|^2 = \frac{Min}{\alpha \in R^m} \|GE\alpha\|^2$$
(4)

Stage 4: to use the boundary constraints

$$0 \le f = E\alpha \le 1 \tag{5}$$

Stage 5: to apply the colour constancy constraint [8]. All considerations above lead to a constrained least squares problem:

Minimise  $|| BE\alpha ||^2 + s || GE\alpha ||^2$ 

Subject to:  $0 \le E\alpha \le 1$ , and  $p = W^T E\alpha$ 

The matrix B is given in reference [8]. Here s is the scaling factor which balances the smoothness of the reflectance and the colour inconstancy index.

Following the above procedure, a full set of reflectance functions was generated for describing the colour gamut. Figures 3a and 3b show some reflectance functions generated with hues from  $0^{\circ}$ ,  $10^{\circ}$ , to  $350^{\circ}$  at L\* of 60 and 30, respectively.



Figure 3. Reflectance functions of the new gamut corresponding to 36 hue angles from 0° to 350° at a 10° interval with L\* of a) 60 and b) 30.



Figure 4. Reflectance Gamuts of object colours for illuminant (a) D50, (b) A, (c) F2 and (d) F110f 2° observer.

## THE ADVANTAGES FOR APPLYING THE COLOUR GAMUT BASED ON REFLECTANCE FUNCTIONS

Since the gamut is defined by reflectance functions, the 3-D colour gamut can be accurately specified at any combination of illuminant and standard colorimetric observer. Figure 4 shows the 3-D colour gamuts for D50/2, A/2, F2/2 and F11/2 conditions, representing typical graphic art, domestic, office and shop lightings.

Figure 5 shows the comparisons of colour gamuts between the object colours (thin curve) and a particular colour printer (thick curve) under (a) D50/2, (b) A/2, (c) F2/2 and (d) F11/2. These are engineering information which can be used for quantifying colour gamut of an imaging device and for comparing imaging devices' gamuts from different makers. The closer to the gamut of object colours, the better performance of the device will be. The numerical figures can be provided, e.g. the gamut volume or the projected area in a particular illuminant/observer, or the ratio of those between the object and device gamuts.

Another example in using the colour gamut of real object colours is to evaluate the colour rendering property of CIE illuminant simulators. The colour gamuts in Figure 4 represent the volumes of colour space that object colours can be realised under 4 illuminants. The physical measure to specify colour gamut is volume, which can be used to represent the quality of colour rendering. The volume of the 3-D colour gamut can be calculated by the sum of volumes of tetrahedrons, which have a common vertex, located in the middle of neutral axis (i.e.,  $L^*=50$ ). Thus, the colour volume can be used as a new colour rendering index. Table 1 lists the colour rendering performance of six D65 simulators in terms of V, SV and CIE CRI. The former is the gamut volume computed for each simulator. The Shared Volume (SV) value is computed from the shared 3-D gamut volume between the simulator tested and the reference CIE illuminant D65. The latter is the widely used colour rendering index (CRI) recommended by CIE [9]. Note that the CRI of a D65 simulator will never be greater than 100, which means the D65 illuminant is the reference illuminant. However, when using V measure, certain simulators' volumes may be greater than that of illuminant D65. Hence, it has a larger volume than that of D65 illuminant. When the SV is used, all the values will not be more than 100. Many studies showed that the CRI underestimates the yellow+blue white

LED or RGB white LED sources. This was confirmed by a psychophysical experiment [10] conducted in our group.



Figure 5. 2-D The projected colour gamuts of object colours (thin curve) and a digital photo printer (thick curve) under Illumiants D50(a), A(b), F2(c), and F11(d)

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Illuminant and simulators	V	SV	CIE CRI
CIE D65 illuminant	100	100	100
7-phosphor Fluorescent Lamp	100.5	99.2	97.6
Filtered Tungsten source	99.3	98	96.6
4-phosphor Fluorescent Lamp	85.4	83.2	74.8
6-cluster LED source	101.7	99.9	98.8
Yellow+blue white LED	93.4	89.6	81.5
4-cluster LED plus a fluorescent lamp	101.8	99.5	97.2

Table 1: The performance of D65 simulators in terms of CIE CRI, V and SV measures

The results in Table 1 reflect this fact, i.e. although the ranks of the simualtors investigated are the same for all 3 measures, V and SV results gave a much higher colour rendering performance than that of CIE CRI for the yellow+blue LED source.

## CONCLUSION

Following our previous work [1] in the development of a colour gamut for object colours under D50/2° conditions, a new

gamut was developed based on reflectance functions. Most importantly, it can be used in any combination of illuminant/observer conditions. It has several advantages over the conventional colorimetric based colour gamuts in various industrial applications. Some examples were given.

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