# Spectral encoding / decoding using *LabRGB*

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## Abstract

*Spectral encoding / decoding methods using unique eigenvectors and physically meaningful values were explored. Three new methods such as, TrW6 consisting of 6 unique trigonometric functions, Lab2 consisting of two CIELAB , LabRGB consisting of CIELAB and RGB , were derived and compared against the traditional eigenvectors method. It was found that TrW6 and LabRGB showed almost the same accuracy as the traditional eigenvectors method. By using LabRGB , color characteristics can be estimated by only looking at its encoding signals and we do not have to exchange eigenvectors beforehand for exchanging a different population of object colors. LabRGB can be applied not only to spectral imaging but also to traditional tri-chromatic imaging world, so its usage is unlimited.*


## Introduction

Spectral encoding / decoding using eigenvectors is a well-known method since a long time ago. For example, spectral distribution can be written as a linear combination of 6 eigenvectors, such as;

$$\rho(\lambda) = \sum_{i=1}^{6} w_i \cdot e_i(\lambda) \quad (1)$$

where, $\lambda$ is wavelength, $\rho(\lambda)$ is spectral reflectance of an object color, $e_i(\lambda)$ is $i$ -th eigenvector and, $w_i$ is a weighting factor of the $i$ -th eigenvector.

In the present study, 6 eigenvectors from $e_1(\lambda)$ through $e_6(\lambda)$ were obtained first by principal component analysis applied to a population of object colors as bellow.

a) Generating 1000 object colors using the pseudo-object color condition [1].
b) Calculating eigenvectors by principal component analysis.
c) Verifying estimation error.

An example of eigenvectors is shown in Fig. 1.

The pseudo-object color condition is a convenient method to generate spectral reflectance of pseudo-object colors with an assumption of less than 3% variations from the average reflectance of neighboring samples on an object's reflectance spectrum for 10 nm step data.

First, spectral reflectance estimation was made on an object color by multiple regression analysis using Eq. 1 with known eigenvectors and unknown weighting factors (called $W6$ ).

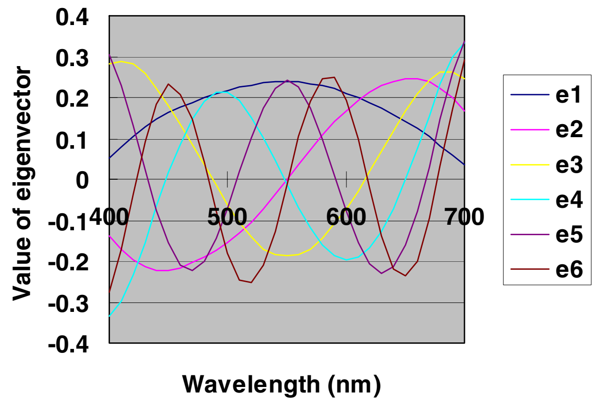


***Figure 1. Example of eigenvectors***

Figure 2 shows spectral reflectance estimation for one color from the Macbeth color chart and Fig. 3 shows the standard deviation of reflectance estimation as a function of wavelength for 1000 pseudo-objects colors.

An encoding / decoding method above using eigenvectors has the least estimation error. On the other hand, eigenvectors cannot be defined uniquely, because they depend on a sample selection of population.

Also, signals of encoding / decoding method using eigenvectors have no physical meaning. So it is difficult to directly estimate either a shape of spectral reflectance, or color characteristics of an original object color. It is therefore not easy to verify an encoding / decoding process. Furthermore, it cannot be applied to current tri-chromatic imaging systems directly.

As such, several challenges have been made on an encoding / decoding method described above. A recently reported one is *LabPQR* [2]. *LabPQR* is a concept of the encoding which has three dimensions ( *CIELAB* [3]) to represent the colorimetric characteristics of a color under a specific illuminant and additional dimensions ( *PQR* ) to describe the metameric black spectrum of a spectral power distribution [4]. The intention of *LabPQR* is to convey physical values so that an encoding signal can be used to estimate an original object color. Several variations of the *PQR* aspects of *LabPQR* have been described in the literature [2], [5] including those based on a population of samples or those based on fundamental spectral stimuli [4]. Although the *LabPQR* concept has well introduced, an applicable algorithm is not available yet.

The present paper investigates and deliver an actual algorithm of the *LabPQR* concept to the real world and describes encoding / decoding methods which have unique, well-defined eigenvectors, physically meaningful encoding values, and are cable of handling both spectral imaging and current tri-chromatic imaging equipments.

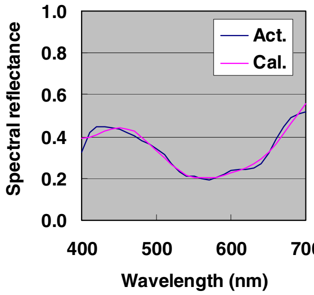


***Figure 2. Spectral reflectance estimation using W6***

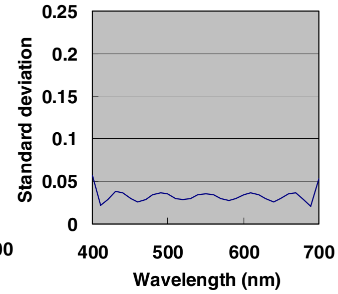


***Figure 3. Standard deviation of spectral reflectance estimation using W6***

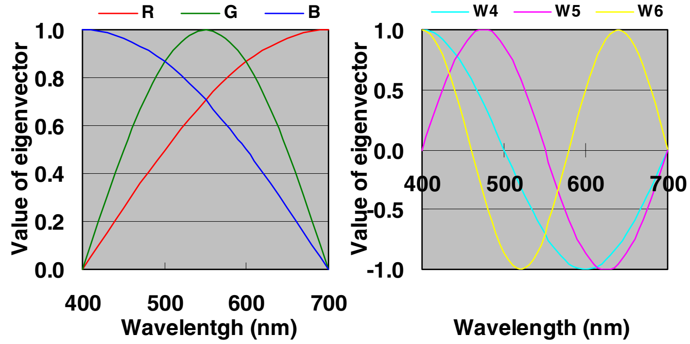


***Figure 4. Eigenvectors using trigonometric function***

## Eigenvectors using trigonometric function

There are several physically meaningful values such as $RGB$ and $L^*a^*b^*$. Along them, we here use the $RGB$ values. An eigenvector set can be chosen as shown in Eq. 2. The first three eigenvectors are roughly designed to represent $RGB$ spectral distribution curve and the last three eigenvectors cover higher frequency.

$$\left.\begin{aligned}
e_1(\lambda) &= \sin\left(\frac{1}{2}\pi\frac{\lambda-\lambda_{\min}}{\lambda_{\max}-\lambda_{\min}}\right)\\
e_2(\lambda) &= \cos\left(\frac{1}{2}\pi\frac{\lambda-\lambda_{\min}}{\lambda_{\max}-\lambda_{\min}}\right)\\
e_3(\lambda) &= \sin\left(\pi\frac{\lambda-\lambda_{\min}}{\lambda_{\max}-\lambda_{\min}}\right)\\
e_4(\lambda) &= \cos\left(\frac{3}{2}\pi\frac{\lambda-\lambda_{\min}}{\lambda_{\max}-\lambda_{\min}}\right)\\
e_5(\lambda) &= \sin\left(2\pi\frac{\lambda-\lambda_{\min}}{\lambda_{\max}-\lambda_{\min}}\right)\\
e_6(\lambda) &= \cos\left(\frac{5}{2}\pi\frac{\lambda}{\lambda_{\max}-\lambda_{\min}}\right)
\end{aligned}\right\} \qquad (2)$$

With this eigenvector set, color characteristics can be estimated by weighting factors of the first three eigenvectors. The shape of the trigonometric eigenvectors is shown in Fig. 4. Spectral reflectance estimation was made using an equation obtained by substituting Eq. 2 into Eq. 1 (called $TrW6$). Figure 5 shows a spectral reflectance estimation for one color from the Macbeth color chart and Fig. 6 shows the standard deviation of spectral reflectance estimation as a function of wavelength for 1000 pseudo-object colors. Overall Standard deviation of 1000 pseudo-object colors were 0.0335 in $W6$ and 0.0365 in $TrW6$, and a difference between those were only 0.3%, so that an almost equivalent accuracy can be obtained.

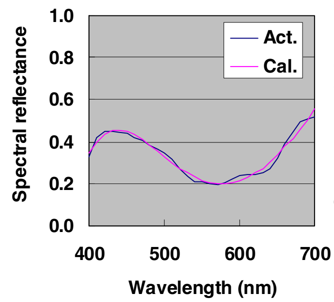


***Figure 5. Spectral reflectance estimation using TrW6***

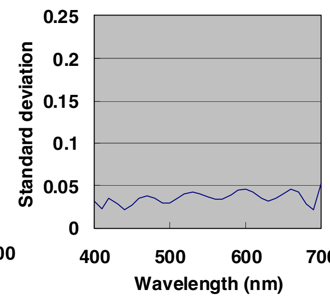


***Figure 6. Standard deviation of spectral reflectance estimation using TrW6***

In Eq. 2, frequency multiplier of the trigonometric function of $e_4(\lambda) \sim e_6(\lambda)$ were selected from all combination of up to 4 by a 1/2 step. There were some other combination of the frequency multiplier, which gives a standard deviation of spectral reflectance estimation better than $TrW6$ and surprisingly even better than $W6$ as well. The present selection was made by its simplicity, a balance along the wavelength and a reasonable accuracy.

## Lab2

The next two encoding /decoding methods are to use two sets of $CIELAB$ [3] values (called $Lab2$) and a combination of $CIELAB$ and $RGB$ (called $LabRGB$). $Lab2$ is to use two different sets of $CIELAB$ values corresponding to two illuminants such as $D65$ and $A$ for encoding signals. Spectral reflectance estimation was made then by using $CIEXYZ$ [3] values $X_{D65}$, $Y_{D65}$, $Z_{D65}$, $X_A$, $Y_A$, $Z_A$.

Equation-3 contains 6 simultaneous equations with six unknown weighting factors $w_1 \sim w_6$. In Eq. 3, $E_{65}(\lambda)$, $E_A(\lambda)$ are the spectral energy distributions of illuminant $D65$ and $A$, and $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ are the color matching functions.

$$
\left.
\begin{aligned}
X_{D65} &= \sum_{i=1}^{6} w_i \int e_i(\lambda)\cdot E_{65}(\lambda)\cdot \bar{x}(\lambda)d\lambda \\
Y_{D65} &= \sum_{i=1}^{6} w_i \int e_i(\lambda)\cdot E_{65}(\lambda)\cdot \bar{y}(\lambda)d\lambda \\
Z_{D65} &= \sum_{i=1}^{6} w_i \int e_i(\lambda)\cdot E_{65}(\lambda)\cdot \bar{z}(\lambda)d\lambda \\
X_{A} &= \sum_{i=1}^{6} w_i \int e_i(\lambda)\cdot E_{A}(\lambda)\cdot \bar{x}(\lambda)d\lambda \\
Y_{A} &= \sum_{i=1}^{6} w_i \int e_i(\lambda)\cdot E_{A}(\lambda)\cdot \bar{y}(\lambda)d\lambda \\
Z_{A} &= \sum_{i=1}^{6} w_i \int e_i(\lambda)\cdot E_{A}(\lambda)\cdot \bar{z}(\lambda)d\lambda
\end{aligned}
\right\} \quad (3)
$$

By solving Eq.3 for $w_1 \sim w_6$, the original object color can be readily obtained. With this method, the decoding was done, and Fig. 7 shows a spectral reflectance estimation for one color from the Macbeth color chart and Fig. 8 shows the standard deviation of spectral reflectance estimation as a function of wavelength for 1000 pseudo-object colors.

$X_{D65}$, $Y_{D65}$, $Z_{D65}$, $X_A$, $Y_A$, $Z_A$ in *Lab*2 have a physical meaning, so that feature of an object color can be estimated without decoding into spectral reflectance curve. On the other hand, the standard deviation of spectral reflectance estimation is worse near the both ends of the wavelength scale. This is due to the luck of power in the x, y, z-bar equations at the ends. So the colorimetric accuracy is quite independent of spectral accuracy there.

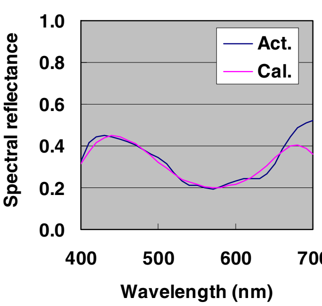


***Figure 7. Spectral reflectance estimation using Lab2***

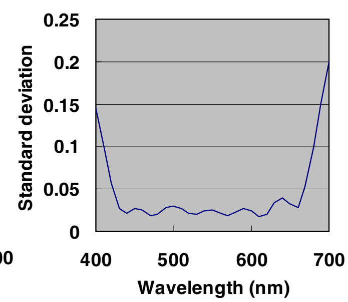


***Figure 8. Standard deviation of spectral reflectance estimation using Lab2***

## LabRGB

*LabRGB* uses a combination of *CIELAB* and *RGB*. Encoding is done by the following steps.

a) Calculate *CIEXYZ* and *CIELAB* values of a spectral reflectance $\rho(\lambda)$

b) Substitute Eq. 2 into Eq. 1 and calculate 6 weighting factors $w_1 \sim w_6$ for 6 eigenvectors in Eq. 2

where $w_1$, $w_2$, $w_3$ roughly represent $R$, $G$, $B$ component respectively. *LabRGB* values, obtained by the above steps, are used as encoding signals.

Decoding is done by the following steps.

a) Calculate an estimation of *CIEXYZ* values $\hat{X}\hat{Y}\hat{Z}$ using only $w_1$, $w_2$, $w_3$ as,

$$
\left.
\begin{aligned}
\hat{X} &= \sum_{i=1}^{3} w_i \int e_i(\lambda)\cdot E(\lambda)\cdot \bar{x}(\lambda)d\lambda \\
\hat{Y} &= \sum_{i=1}^{3} w_i \int e_i(\lambda)\cdot E(\lambda)\cdot \bar{y}(\lambda)d\lambda \\
\hat{Z} &= \sum_{i=1}^{3} w_i \int e_i(\lambda)\cdot E(\lambda)\cdot \bar{z}(\lambda)d\lambda
\end{aligned}
\right\} \quad (4)
$$

b) Calculate $w_4$, $w_5$, $w_6$ from the original $XYZ$ values and estimated $\hat{X}\hat{Y}\hat{Z}$ values using Eq. 5

$$
\left.
\begin{aligned}
X-\hat{X} &= \sum_{i=4}^{6} w_i \int e_i(\lambda)\cdot E(\lambda)\cdot \bar{x}(\lambda)d\lambda \\
Y-\hat{Y} &= \sum_{i=4}^{6} w_i \int e_i(\lambda)\cdot E(\lambda)\cdot \bar{y}(\lambda)d\lambda \\
Z-\hat{Z} &= \sum_{i=4}^{6} w_i \int e_i(\lambda)\cdot E(\lambda)\cdot \bar{z}(\lambda)d\lambda
\end{aligned}
\right\} \quad (5)
$$

c) Substituting obtained $w_4$, $w_5$, $w_6$ and known $w_1$, $w_2$, $w_3$ into Eq. 1

With this method, the decoding was done, and Fig. 9 shows a spectral reflectance estimation for one color from the Macbeth color chart and Fig. 10 shows the standard deviation of spectral reflectance estimation as a function of wavelength for 1000 pseudo-object colors.

*LabRGB* consists of two classes of physical attributes, and its eigenvectors *TrW*6 are unique trigonometric functions. Overall Standard deviation of 1000 pseudo-object colors were 0.0389 in *LabRGB*, and difference between *W*6 and *LabRGB* were small (only 0.54%), so as *TrW*6. Therefore an almost equivalent accuracy can be obtained.

## Comparison of the encoding / decoding methods

Table 1 shows the four encoding / decoding methods described above with the three different populations of object colors. *W*6 eigenvectors were calculated from 1000 pseudo-object colors, so *W*6 showed the best result for that population of object colors. Both *TrW*6 and *LabRGB* were better than *W*6 for 24 Macbeth colors and 49776 SOCS colors [6]. It can be said that *TrW*6 and *LabRGB* performance are almost equivalent to *W*6 performance.

Table 2 and Table 3 show the standard deviation of colorimetric estimation error using 1000 pseudo-object colors and 49776 socs colors. Illuminant *D*65 was used in *LabRGB* encoding / decoding calculation, so the combination of *LabRGB* and observation illuminant *D*65 shows minimum colorimetric estimation error (almost zero). Furthermore *LabRGB* also worked better than *W*6 with observation illuminant *D*50 and *A*. It is because *W*6 minimizes spectral estimation error and *LabRGB* minimizes colorimetric estimation error.

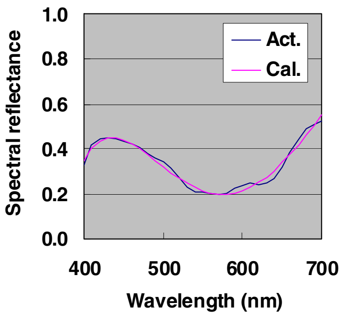


**Figure9. Spectral reflectance estimation using LabRGB**

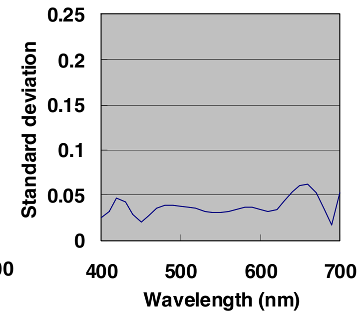


**Figure 10. Standard deviation of spectral reflectance estimation using LabRGB**

Table1. Comparison of the spectral reflectance estimation overall standard deviation (ratio)

| *Object colors* | *Encoding/decoding methods* | | | |
|---|---|---|---|---|
| | *W6* | *TrW6* | *Lab2* | *LabRGB* |
| *24 Macbeth colors* | *0.0255* | *0.0206* | *0.0464* | *0.0222* |
| *1000 pseudo-object colors* | *0.0335* | *0.0365* | *0.0746* | *0.0389* |
| *49776 SOCS colors* | *0.0247* | *0.0216* | *0.0622* | *0.0226* |

## Minimizing colorimetric estimation error

According to Table 3, the colorimetric estimation error in *LabRGB* is about 1.8 from observation illuminant *A* to *D*65 , which is about 3600 K in color temperature range. So, colorimetric estimation error of less than one in $\Delta Eab$ unit can be achieved over the same observation color temperature range by selecting illuminant color temperature of *LabRGB* encoding / decoding calculation to disperse colorimetric estimation error.

Table 4 shows the standard deviation of colorimetric estimation error using 49776 socs colors. 5 different illuminants were used in *LabRGB* encoding / decoding calculation. $\Delta Eab$ are less than one for all observation illuminants in *LabRGB* encoding / decoding calculation using 4000 K black body radiation (blue text, indicated as BBR 4000 K in Table 4), while sum of $\Delta Eab$ is minimum in *LabRGB* encoding / decoding calculation using illuminant *D*50 (red text in Table 4).

*D*50 is better than BBR 4000 K, because *D*50 is well defined common illuminant, spectrally about equal energy distribution and it gives minimum of sum of $\Delta Eab$ .

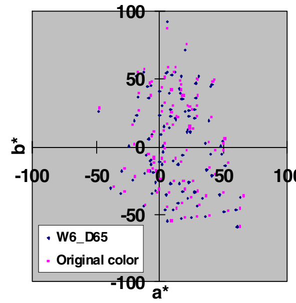


**Figure11. W6 colorimetric estimation error under illuminant D65**

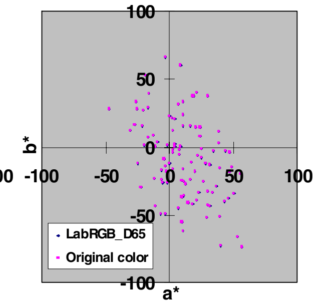


**Figure 12. LabRGB colorimetric estimation error under illuminant D65**

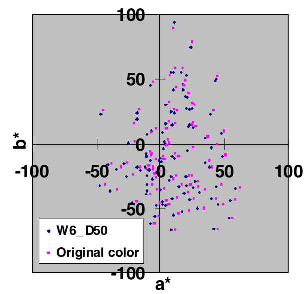


**Figure13. W6 colorimetric estimation error under illuminant D50**

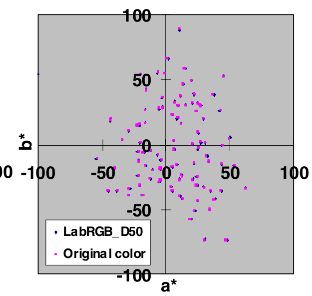


**Figure 14. LabRGB colorimetric estimation error under illuminant D50**

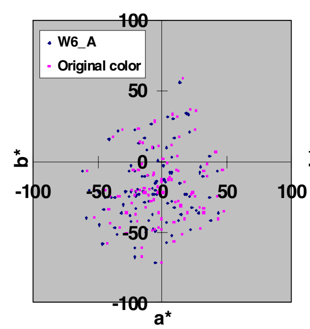


**Figure15. W6 colorimetric estimation error under Illuminant A**

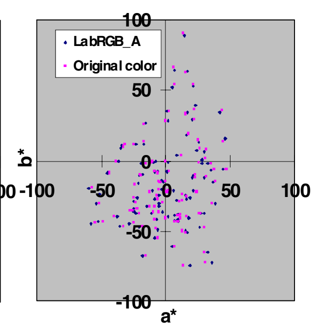


**Figure 16. LabRGB colorimetric estimation error under Illuminant A**

Table2. Comparison of colorimetric estimation error (standard deviation of 1000 pseudo-object colors $\Delta Eab$ )

| *Encoding/decoding methods* | *Observation illuminants* | | |
|---|---|---|---|
| | *D65* | *D50* | *A* |
| *W6* | *1.5540* | *1.8730* | *2.4700* |
| *LabRGB* | *0.0006* | *0.2906* | *1.1416* |

**Table3. Comparison of the colorimetric estimation error (standard deviation of 49776 socs colors $\Delta Eab$ )**

| *Encoding/decoding methods* | *Observation illuminants* | | |
|---|---|---|---|
| | *D65* | *D50* | *A* |
| *W6* | *3.0773* | *3.3676* | *4.1670* |
| *LabRGB* | *0.0009* | *0.4757* | *1.7336* |

**Table4. Comparison of the colorimetric estimation error (standard deviation of 49776 socs colors $\Delta Eab$ )**

| *Illuminants used in LabRGB* encoding/decoding calculation | *Observation illuminants* | | | *Sum of* $\Delta Eab$ |
|---|---|---|---|---|
| | *D65* | *D50* | *A* | |
| *A* | 1.8429 | 1.4902 | 0.0010 | 3.3340 |
| *BBR 4000 K* | 0.8720 | 0.6652 | 0.9558 | 2.4930 |
| *BBR 4500 K* | 0.6693 | 0.6061 | 1.2160 | 2.4914 |
| *D50* | 0.4755 | 0.0008 | 1.3588 | 1.8351 |
| *D65* | 0.0009 | 0.4757 | 1.7336 | 2.2103 |

## Conclusion

Spectral encoding / decoding methods using unique eigenvectors and physically meaningful values were explored.

It was found that the unique trigonometric functions $TrW6$ can be used as eigenvectors without any or with negligible loss of accuracy. $Lab2$, consisting of two $CIELAB$ values, had standard deviation of spectral reflectance estimations worse near the both ends of the wavelength, this was due to the luck of power in the x, y, z-bar equations at the ends. $LabRGB$ consisting of $CIELAB$ and $RGB$ and showed almost the same performance as the traditional eigenvectors $W6$. By using $LabRGB$, we do not have to worry about a population of object colors each time, and we can send / receive encoding signals without exchanging eigenvectors beforehand. $LabRGB$ consists of physically meaningful attributes, so that color characteristics can be estimated by only looking at its encoding signals. $LabRGB$ can be applied not only to spectral imaging but also to traditional tri-chromatic imaging world, so its usage is unlimited. Future plan is to apply $LabRGB$ to a multi-spectral imaging system and implement a performance evaluation.

## References


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## Author Biography

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