Analysis of Mean Square Errors of Spectral Reflectances Recovered by Multispectral Cameras

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Abstract

Acquisition of the spectral reflectances of imaged objects through the use of sensor responses is important for color reproduction of images under various illuminations. The accuracy of the recovered spectral reflectances depends not only on the spectral sensitivities of sensors but also on the noise present in a device. For the first time, the mean square errors (MSE) between the measured and recovered spectral reflectances were separated into noise dependent and noise independent MSE based on the proposed model. Experiments were performed to study the influence of the bit-depth of the analog to digital conversion and of the sampling intervals over the visible wavelength on the MSE. It is shown that the sum of the theoretically calculated noise dependent and independent MSE agrees fairly well with the MSE of the spectral reflectances recovered by multispectral cameras. It is also confirmed that the noise dependent MSE increases with a decrease in the bit-depth and that the noise independent MSE increases with an increase in the sampling intervals.

The separation of the MSE into noise dependent and independent MSE gives a powerful tool for the accurate image acquisition.

Introduction

Acquisition of the spectral reflectances of imaged objects through the use of sensor responses is important for color reproduction of images under various illuminations. There have been various investigations to recover the accurate spectral reflectances of objects through the use of pixel values[1-4]. Therefore the development of the evaluation model for the image acquisition devices aimed at recovery of spectral reflectances is important for optimizing a set of spectral sensitivities. One of the authors (N.S.) already proposed a model for the evaluation of the image acquisition devices aimed at recovery of spectral reflectances and showed the appropriateness of the formulation by applying it to multispectral image acquisition systems[5]. It is well known that the accuracy of the recovered spectral reflectances depends not only on the spectral sensitivities of a set of sensors but also on the noise present in the image acquisition system. Therefore a new model for the estimation of the noise variance of an image acquisition system has been proposed by the author[6], and the estimates were used for the evaluation of the multispectral cameras[5] and for the accurate recovery of spectral reflectances[2]. The noise must be defined to include all sensor response errors that are originated not only from the CCD itself but also from the errors in measuring the spectral characteristics of sensors, illuminants and spectral reflectances, and also from quantization errors to solve the inverse problems such as the spectral reflectance recovery. The noise defined above is denoted as the system noise in this manuscript.

For the first time, the mean square errors (MSE) between the measured and the recovered spectral reflectances is separated into noise dependent and independent MSE by using the proposed model[6]. It is shown that the sum of the theoretically calculated noise dependent and independent MSE agrees fairly well with the MSE of the spectral reflectances recovered by multispectral cameras and that the separation provides us a powerful tool for the analysis of the system noise and the optimization of the image acquisition systems.

Separating Model

In this section, a brief sketch for the derivation of the noise analysis model for a color image acquisition system is described. A vector space notation is useful in the problems. In this approach, the visible wavelengths from 400 to 700 nm are sampled at the constant intervals and the number of the samples is denoted N. A sensor response vector from a set of color sensors for an object with an $N \times 1$ spectral reflectance vector $\mathbf{r}$ can be expressed by

$$\mathbf{p} = \mathrm{SL}\mathbf{r} + \mathbf{e} , \tag{1}$$

where $\mathbf{p}$ is an $M \times 1$ sensor response vector from the M channel sensors, S is an $M \times N$ matrix of the spectral sensitivities of sensors in which a row vector represents a spectral sensitivity, L is an $N \times N$ diagonal matrix with samples of the spectral power distribution of an illuminant along the diagonal, and $\mathbf{e}$ is an $M \times 1$ additive noise vector. The noise $\mathbf{e}$ represents the system noise[6] described above. The system noise is assumed to be signal independent, zero mean and uncorrelated to itself. For abbreviation, let $S_L = SL$. The MSE of the recovered spectral reflectances $\hat{\mathbf{r}}$ is given by

$$\mathrm{MSE} = E\left\{\left\|\mathbf{r} - \hat{\mathbf{r}}\right\|^2\right\} , \tag{2}$$

where $E\{\bullet\}$ represents the expectation. If the Wiener estimation is used to recover a spectral reflectance $\hat{\mathbf{r}}$, then $\hat{\mathbf{r}}$ is given by

$$\hat{\mathbf{r}} = R_{SS} S_L^T \left(S_L R_{SS} S_L^T + \sigma_e^2 I\right)^{-1} \mathbf{p} , \tag{3}$$

where T represents the transpose of a matrix, R_{SS} is an autocorrelation matrix of the spectral reflectances of samples that will be captured by a device, and σ_e^2 is the noise variance used for the estimation. Substitution of Eq.(3) into Eq.(2) leads to [6]

$$\mathrm{MSE} = \sum_{i=1}^{N} \lambda_i - \sum_{i=1}^{N}\sum_{j=1}^{\beta} \lambda_i b_{ij}^2 + \sum_{i=1}^{N}\sum_{j=1}^{\beta} \frac{\sigma_e^4 + \kappa_j^{v^2}\sigma^2}{\left(\kappa_j^{v^2} + \sigma_e^2\right)^2} \lambda_i b_{ij}^2, \tag{4}$$

where, λ_i is the eigenvalues of Rss , and b_{ij} , κ_i^v and β represent j-th row of the i-th right singular vectors, the singular value and the rank of a matrix $S_L V \Lambda^{1/2}$, respectively, σ^2 is the actual system noise variance, V is a basis matrix and Λ is an $N \times N$ diagonal matrix with positive eigenvalues λ_i along the diagonal in decreasing order. It is easily seen that the MSE is minimized when $\sigma_e^2 = \sigma^2$ by differentiating Eq.(4) with respect to σ_e^2 , and the MSE is given by

$$\mathrm{MSE} = \sum_{i=1}^{N} \lambda_i - \sum_{i=1}^{N}\sum_{j=1}^{\beta} \lambda_i b_{ij}^2 + \sum_{i=1}^{N}\sum_{j=1}^{\beta} \frac{\sigma^2}{\kappa_j^{v^2} + \sigma^2} \lambda_i b_{ij}^2 . \tag{5}$$

The first and second terms in the right hand side of the Eq.(5) give the noise independent errors MSE_{free} and the third term gives the noise dependent errors MSE_{noise}, and they are described as:

$$\mathrm{MSE}_{free} = \sum_{i=1}^{N} \lambda_i - \sum_{i=1}^{N}\sum_{j=1}^{\beta} \lambda_i b_{ij}^2 , \tag{6}$$

and

$$\mathrm{MSE}_{noise} = \sum_{i=1}^{N}\sum_{j=1}^{\beta} \frac{\sigma^2}{\kappa_j^{v^2} + \sigma^2} \lambda_i b_{ij}^2 . \tag{7}$$

The MSE_{free} is determined by the spectral sensitivities of a set of sensors, the spectral power distribution for the recording illumination and the spectral reflectances of objects being imaged. The MSE_{noise} depends not only on the factors described above but also on the system noise variance and on the singular values of a matrix $S_L V \Lambda^{1/2}$. It is very important to note the MSE_{noise} are suppressed when the singular values are larger than the value of the system noise variance[7], i.e., $\kappa_i^v >> \sigma^2$.

Let us give a brief sketch for the model of the system noise variance estimation (for more detail, see ref.6). If we let the noise variance $\sigma_e^2 = 0$ for the Wiener filter in Eq. (3), then the MSE($\sigma_e^2 = 0$) is derived as (by letting $\sigma_e^2 = 0$ in Eq. (4))

$$\mathrm{MSE}\left(\sigma_e^2 = 0\right) = \sum_{i=1}^{N} \lambda_i - \sum_{i=1}^{N}\sum_{j=1}^{\beta} \lambda_i b_{ij}^2 + \sum_{i=1}^{N}\sum_{j=1}^{\beta} \frac{\sigma^2}{\kappa_j^{v^2}} \lambda_i b_{ij}^2 . \tag{8}$$

From Eq.(8) the system noise variance $\hat{\sigma}^2$ can be estimated by

$$\hat{\sigma}^2 = \frac{\mathrm{MSE}\left(\sigma_e^2 = 0\right) - \mathrm{MSE}_{free}}{\Sigma_{i=1}^{N} \Sigma_{j=1}^{\beta} \dfrac{\lambda_i b_{ij}^2}{\kappa_j^{v^2}}} . \tag{9}$$

Therefore, the system noise variance σ^2 can be estimated using Eq. (9), since the MSE_{free} and the denominator of Eq. (9) can be computed if the surface reflectance spectra of objects, the spectral sensitivities of sensors and the spectral power distribution of an illuminant are known. The MSE($\sigma_e^2 = 0$) can also be obtained by the experiment using Eqs. (2) and (3) by applying the Wiener filter with $\sigma_e^2 = 0$ to the sensor responses. Therefore, Eq. (9) gives a method to estimate the actual noise variance σ^2 .[6]

As stated above, the value of the MSE is minimized when the noise variance used for the Wiener estimation σ_e^2 is equal to the actual system noise variance σ^2 . From this fact, the noise variance which minimizes the MSE (hereafter the noise variance is called as optimal noise variance σ_{opt}^2) considered as the system noise variance; i.e., the values of σ_{opt}^2 can be obtained by changing noise variance σ_e^2 in Eq.(3), and then the σ_e^2 that gives minimum value of the MSE in Eq(2) is considered as the σ_{opt}^2 . Therefore the accuracy of the estimated system noise variance is confirmed by comparing the estimates $\hat{\sigma}^2$ with the optimal noise variance σ_{opt}^2 which minimizes the MSE[6].

Experimental Results

Experimental Procedures

The set of seven spectral sensitivities of the multispectral camera were optimized based on the colorimetric evaluation model proposed by the author (N.S.)[8] and the camera was assembled by using seven interference filters (Asahi Spectral Corporation) in conjunction with a monochrome video camera (Kodak KAI-4021M). The spectral sensitivities were measured over wavelength from 400 to 700 nm at 1-nm intervals and the characteristics are shown in Fig.1. Image data from the video camera were converted to 16-bit-depth digital data by an AD converter. The illuminant that simulates daylight (Seric Solax XC-100AF) was used for the image acquisition. The spectral power distribution of the illuminant measured by the spectroradiometer (Minolta CS-1000) from 400 to 700 nm at 1 nm sampling interval and is presented in Fig.2. The spectral reflectances of the Gretag Macbeth ColorChecker (24colors) were measured by the spectrophotometer (Shimadzu UV-3100PC) from 400 to 700 nm at 1-nm intervals. The color chart was illuminated from the direction of about 45 degree to the surface normal, and the images were captured by the camera from the normal direction. Spatial correction was performed to minimize the effect of any spatial non-uniformity in illumination and sensitivities of the pixels of a CCD. Since the spectral sensitivities of a camera depend on the camera gain, the

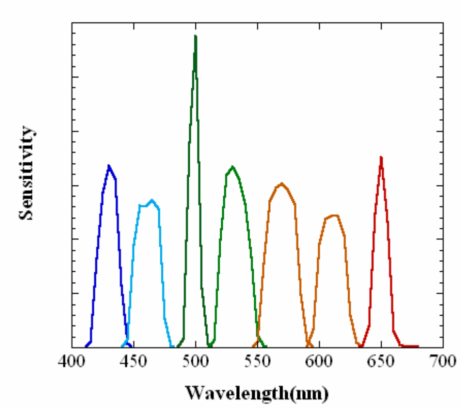

Figure 1. Spectral sensitivities of the multispectral camera used in this experiment.

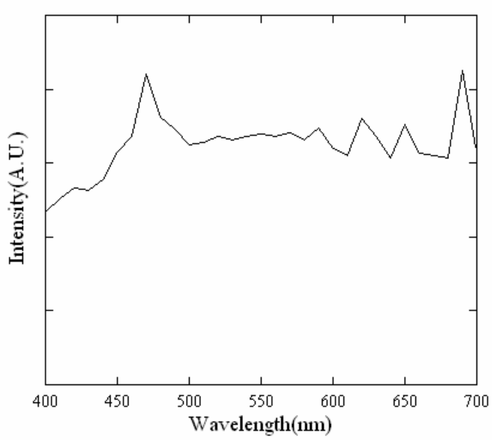

Figure 2. Spectral power distribution of the illuminant used image acquisition.

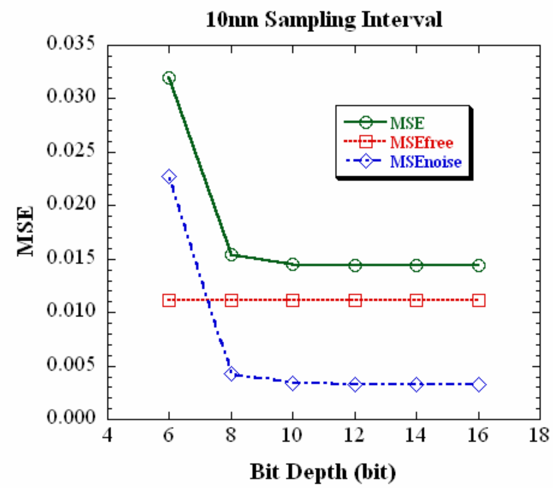

Figure 3. MSE, MSEfree and MSEnoise as a function of bit depth at 10 nm sampling interval.

camera's responses to a color computed by using the measured spectral sensitivities of the sensors, the illuminant and the surface reflectance of the color dose not equal to the actual sensor responses. Therefore, the sensitivities were calibrated using an achromatic color in the charts. In this work, the constraint is imposed on the signal power as given by $\rho = \mathrm{Tr}\left(S_L R_{SS} S_L^T\right)$(that is corresponds to averaged signal power), where relation of $\rho = 1$ was used so that the estimated system noise variance can be compared for different sensor sets. The bit-depth of the image data were changed from 16 to 6 bit by deleting the least significant bit and the sampling intervals of the wavelength were changed from 2, 5, 10 to 20 nm. Since the values of the MSE depend on the sampling intervals, the values of the MSE were normalized to 10 nm sampling interval; i.e., if the 2 nm sampling interval is used, the MSE is divided by 5.

By using various combinations of sensors from six to seven in Fig.1, the system noise variance was estimated by the methods described above for each combination of the sensors and the estimates were used to calculate the MSE_{noise} using Eq.(7). Then the estimated noise variance was also used to recover the spectral reflectances by using Eq,(3), and the MSE of the recovered spectral reflectances was computed by the use of Eq.(2).

Experimental Results

Experiments were performed to recover spectral reflectances by the multispectral cameras with various combinations of sensors and the typical examples of the calculated noise dependent and independent MSE are summarized in Table, where the 16-bit-depth and 10 nm sampling interval were used in the experiments.

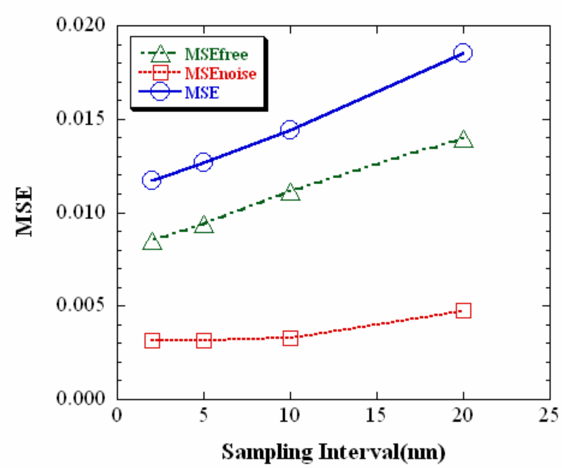

Figure 4. MSE, MSEfree and MSEnoise as a function of sampling interval at 16-bit-depth of analog to digital conversion.

Table Experimental results for various combinations of sensors at 10 nm sampling intervals and 16-bit-depth of analog to digital conversion

Sensors	MSEfree	MSEnoise	MSEfree + MSEnoise	MSE	$\hat{\sigma}^2$	$\hat{\sigma}^2_{opt}$
1234567	0.011106	0.003270	0.014376	0.014391	8.06e-005	8.30e-005
123456	0.041403	0.003614	0.045017	0.044878	5.25e-005	8.56e-005
123457	0.015649	0.003838	0.019487	0.019722	1.05e-004	4.97e-005
123467	0.013908	0.003314	0.017223	0.017296	9.86e-005	8.94e-005
123567	0.013716	0.003724	0.017440	0.017581	1.11e-004	8.69e-005
124567	0.014026	0.003866	0.017892	0.017979	1.00e-004	8.75e-005
134567	0.013114	0.003161	0.016275	0.016065	8.71e-005	1.53e-004
234567	0.014755	0.003428	0.018183	0.018078	9.47e-005	1.19e-004

The numbers in the left column of the table represent the combination of sensors used for image acquisition, i.e., the sensors were numbered from one (the shortest peak wavelength in sensitivities) to seven (the longest peak wavelength) in Fig.1. From the results, it is confirmed that the estimated noise variance $\hat{\sigma}^2$ agrees fairly well with the optimal noise variance σ_{opt}^2 and that the sum of the theoretically calculated MSE_{free} and MSE_{noise} agrees fairly well with the MSE of the spectral reflectances recovered by the Wiener estimation with the estimated noise variance. It may be concluded that the estimated noise variance $\hat{\sigma}^2$ is sufficient to estimate the MSE_{noise}. The minimum value of the MSE_{free} is obtained for seven sensors and the maximum value of the MSE_{free} is obtained for the six sensors that is the combination of "123456". Although the values of the MSE_{noise} depend on the combination of sensors but the values are almost the same in these experiments.

Figure 3 shows the influence of the quantization errors on the MSE_{free}, MSE_{noise} and MSE by changing the bit-depth of the quantization. The results show that the values of the MSE_{free} is constant over the bit-depth and that the quantization errors from 10bit to 16bit do not contribute to the noise dependent errors MSE_{noise} but the quantization errors below 8 bit influence on the MSE_{noise} and it also contributes to the increase in the MSE of the recovered spectra reflectances. It is confirmed that the 10bit-depth AD conversion is good enough for this image acquisition system. The experimental results on the influence of the sampling intervals on the MSE_{free}, MSE_{noise} and MSE is shown in Figure 4. From the experimental results the values of the MSE increase with an increase in the sampling intervals. The values of the MSEfree also increase with an increase in sampling intervals, on the other hand MSEnoise slightly increases from 10 nm to 20 nm sampling intervals. Therefore, it is concluded that the increase in the MSE with an increase in sampling intervals is mainly attributed to an increase in the MSEree. From the experimental results, the conditions of the 10 nm sampling intervals and 10-bit-depth analog to digital conversion are sufficient for accurate spectral information acquisition in this system, and the efforts for decreasing the MSEree are required in this system.

Conclusions

The MSE of the recovered spectral reflectances was separated into the noise dependent and independent MSE by using the proposed model and the sum of the theoretically calculated noise independent and dependent MSE agrees fairly well with the MSE of the spectral reflectances recovered by the Wiener estimation through the use of sensor responses. The separation of the MSE into noise dependent and independent MSE is useful for analyzing the noise present in an image acquisition device and also for optimizing the image acquisition system.

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