

Beyond Simple Additivity: A Two-Step Parametric Model for Digital Displays

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Abstract

Simple additivity is often used as basic model for digital display characterization. However, such a simple model cannot all the time satisfy the needs of demanding color management applications.

This paper presents an enhanced method to characterize the XYZ to RGB transform of a digital display. This parametric method exploits the independence between the luminance variation of the electro-optic response and the colorimetric responses for certain display types.

3-DMD projectors are the reference for high grade color management. However, the model is more generally applicable to Digital Displays (and not only projectors), including Single DMDs, CRTs, LCDs, etc, if the independence condition is satisfied.

While the problem to solve is a 3D to 3D transformation (from XYZ to RGB), the proposed parametric model is the composition of a 2D transform followed by a 1D transform. The 2D transform manages the chromatic aspects and, in succession, the 1D transform manages the Luminance variations.

This parametric digital model is applicable in the field of Color Management, with the objective to calibrate Digital Displays and apply a reference look such as a film look.

Introduction

Application context

In cinematographic post-production, digital processing of images - called Digital Intermediates (DI) – replaces more and more (and in some contexts *has* replaced) the traditional film workflow. Digital post-production requires the preview of DIs with a reproduction of colors, dynamics and resolution comparable to the final film projection [1].

To reproduce colors on Digital Displays, the simple additivity¹ model has revealed its limits and more elaborated models need to be developed to satisfy the application requirements in term of color quality. This paper presents a parametric model, based on display physics, and compares it to the simple linear model (3x3 matrix) [2][3] as well as to a more complex non-linear model (Splines and Tetrahedral) [3][4] for different display configurations.

Technical problems to solve

The technical problem to be solved is to find a reliable transform between the digital domain (e.g. RGB Values) and the visual domain (e.g. XYZ² Values) for an entire color space. Typically when R, G and B have a dynamic of $n = 8, 10$ or 12 bits, a correspondence must be found between RGBs and XYZs for $N = (2^n)^3$ triplets (N is over 10^9 for $n = 10$ bits).

For each candidate XYZ, the corresponding RGB must be estimated. This RGB values used as input to the Digital Display shall generate the required XYZ color with a sufficient accuracy for the application.

The problem to solve is to find a parametric model, which from (RGB-XYZ) measurements on a restricted set of colors will allow to efficiently compute a RGB triplet for each XYZ candidate.

Goal of the Parametric Digital Model

The goal of a characterization model is to approximate a color transform between the input and the output of a visualization device. In this discussion we use the RGB and the XYZ color spaces. RGB is the device input signal space and XYZ is the CIE1931 standard observer 2° color space. The transform addressed is the XYZ to RGB transform, sometimes named “inverse transform”. This inverse transform is the one used to map a look (for example a film look) onto a characterized display.

The presented parametric digital model applies to Digital Displays where some assumptions can be made about the way colors are generated (see section *Context of Applicability*).

The main advantage of the model is the ability to decompose a 3D problem into a 2D + 1D transform. The model would be simpler to implement and faster to compute for the same precision. Another advantage is the reduction of the characterization data set to be measured, therefore reducing the measurement time.

Main ideas

While in essence the characterization color transforms map an input 3D space into an output 3D space, the developed parametric model exploits knowledge on the Digital Displays.

Most Digital Displays are composed of two groups of components for the visual signal generation:

- Light intensity management components
- Color management components

¹ Simple additivity: adding up weighted light contributions of color primaries

² XYZ: CIE1931 standard observer tristimulus coordinates

The following table gives a view of what these components are for various displays:

Display type	Light Intensity	Color Management
3-DMD ³ Digital Projector	DMD On and Off states modulation	Lamp spectrum + Color filters on each path + Optics transmission+ Screen response
Single Chip Projector	DMD On and Off states modulation	Lamp spectrum, Color Wheel + Optics transmission+ Screen response
CRT	Electronic beam power modulation	Phosphors response
LCD	Liquid Crystal modulation	Pixilated color filters
PLASMA	Sub-fields addressing	Phosphors response

Table 1. Display types

The proposed model is based on the observation that, physically, the chromatic response (and thus, also the spectral repartition) is independent from the light intensity level.

The main idea of the model is to exploit these physical characteristics to modify a 3D to 3D problem into first a 2D transform followed by a 1D transform. The 2D transform will manage the Chromatic aspects and, in succession, the 1D transform will manage the Luminance variations.

Context of applicability

The Chromaticity / Luminance independence hypothesis is only valid for reasonable quality displays with a controlled setup. It is sometimes required to disable some disturbing signal processing features to reach this situation.

For example: For a Single-DMD projector at a given white level, the spectral response is determined by the lamp and its spectrum, the color wheel, the optics transmission and the screen characteristics. It does not depend on the DMD modulation. However, the possibility of using a white segment in the color wheel must be disabled to reach a controlled colorimetric situation.

Specific features of some displays will prevent or disturb the use of the presented model.

Here are some examples:

Display type / Feature	Cause of non-applicability
PLASMA / Automatic Power Limitation	For bright images, a Plasma display will moderate its output luminous power for electrical consumption reasons. The model will be applicable only below this "electrical protection" level.
LCD / Double Modulation	The double modulation is first by the backlight and second by the liquid crystal (for example for 'dynamic black' or HDR ⁴). This modifies the electro-optic response curve of the display, temporally and sometimes locally.
Single DMD / White peaking	Some Single DMD displays include a white segment in their color wheel for luminous output increase. This segment is managed internally as a fourth color channel, mixing up color proportions. This feature should be disabled to use the model

Table 2. Applicability

³ DMD: Digital Micro-mirror Device, a Texas Instrument technology.

⁴ HDR: High Dynamic Range

Model Description

The model description will start with a paragraph defining the color data set measured to characterize the display (section *Characterization data set*), then some notations and normalizations are defined (section *Notations and normalizations*) and then the proposed 2D and 1D transforms are derived (section *Inverse transform: XYZ to RGB*).

Characterization data set

A set of color measures is necessary to characterize a projector. These measures give the *RGB-XYZ* correspondence for a restricted number of *RGB* triplets. The model controls the generalization of this data set to the entire input color space.

The data set has a part of the measures used for determining the 2D transform and a part for the 1D transform.

The measurement data set so includes two subsets:

- The "L" data subset: a grey ramp for the luminance variation characterization
- The "C" data subset: samples on three color planes for colorimetry characterization

The "L" data subset covers the whole range of luminance variations, while the "C" data subset covers the whole range of color variations. Figure 1 represents this data set. The black dashed line is the "L" data subset and the three colored planes are the "C" data subset.

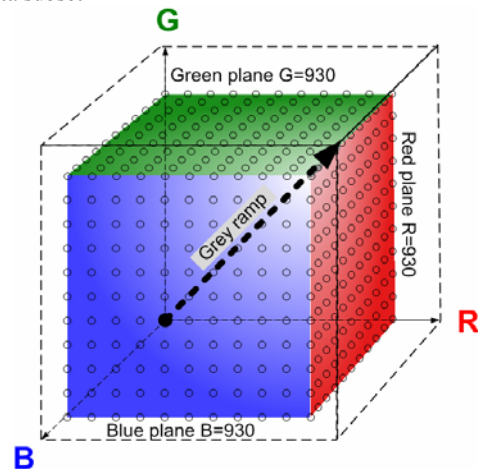


Figure 1. Characterization data set

For the model testing, a specific data set has been defined: RGB varying from 0 to 1023 in the application context, its characteristics are the following:

The luminance "L" data subset is a uniform grey ramp from 0 to 1023 with step 11.

The chromaticity "C" subset is composed of 3 planes in the *RGB* cube defined by:

$$\text{"C" subset} = \begin{cases} \text{Sampling of plane } R = REF \\ \text{Sampling of plane } G = REF \\ \text{Sampling of plane } B = REF \end{cases}$$

$REF=930$ was chosen and the sampling of planes *R*, *G* and *B* is uniform with step 93 (11×11 samples). Double points and the triple point are pruned.

Notations and normalizations

The first point is to guarantee the correct data exploitation. Most particularly, to cancel light leakage effect at low levels, the black XYZ_{black} level (corresponding to input $R=G=B=0$) is subtracted to all XYZ measures.

This done, the above hypothesis of independent light intensity variation / color variation can be expressed by using a normalized representation for RGB on one side and for XYZ on the other side. While various choices are possible, the following have been made in the context of the XYZ to RGB transform model.

XYZ normalization:

S , x and y are defined as:

$$S = X + Y + Z \quad x = \frac{X}{S} \quad y = \frac{Y}{S}$$

A XYZ triplet can univocally be represented by the triplet (x, y, S) . x and y are the same as the CIE 1931 x and y chromaticity coordinates.

RGB normalization:

Σ , r and g are defined as:

$$\Sigma = R + G + B \quad r = \frac{R}{\Sigma} \quad g = \frac{G}{\Sigma}$$

A RGB triplet can univocally be represented by the triplet (r, g, Σ) .

The light intensity vs color independence hypothesis can be expressed by:

1. (r, g) depend only on (x, y) , independently of S and Σ .

Or, said another way, a given (x, y) color has a stable proportion of R , G and B for every value of S .

2. For a given (x, y) , Σ depends only on S .

Inverse transform: XYZ to RGB

While both the forward - RGB to XYZ – and the inverse - XYZ to RGB – transforms are of interest to characterize a display, the inverse transform is of higher importance when we want to render accurately specific colors on the considered display. This inverse transform is often combined with, in example, a forward film emulation transform to create a film look on the target display. The inverse transform - XYZ to RGB - is used to predict what RGB input is required to generate a given XYZ color on the screen.

The following paragraphs describe first the colorimetric computation (section 2D transform) and then the luminance computation (section 1D transform).

Colorimetric computation / 2D transform

The goal of the XYZ to RGB transform is to determine a $(RGB)_{out}$ output triplet corresponding to a $(XYZ)_{in}$ input triplet.

In the XYZ to RGB transform, an input XYZ triplet is first normalized into the corresponding (x, y, S) triplet. After XYZ normalization, the measured data set provides a collection of sextuplets: (x, y, S, R, G, B) .

The 2D transform is in two steps:

- **First step:** Building a 2D mesh
- **Second step:** Interpolating in this mesh

The **first step** of the processing is to build a 2D mesh in the (x, y) plane, entirely paving the useful part of this space with triangles. The vertices of these triangles are the measured points (of the “C” data set) identified by their x and y coordinates.

This mesh is stored as reference during the characterization phase. Each node of the mesh stores four elements: $S=X+Y+Z$ value and the corresponding R , G and B values from the characterization data set.

Figure 2 represents such mesh: each (x,y) point stores the corresponding $(SRGB)$ data measured and acquired for the “C” data set.

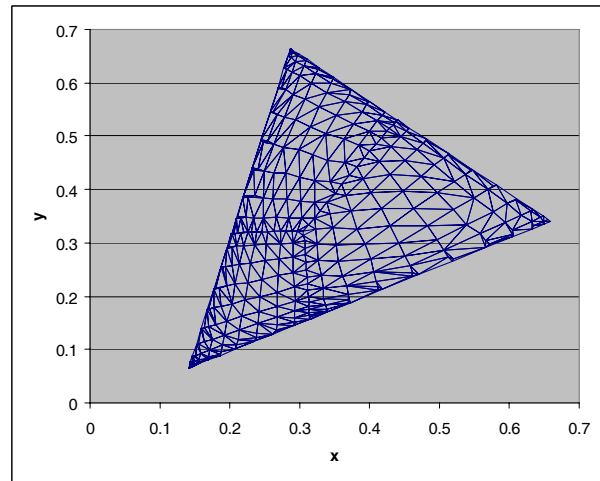


Figure 2. 2D mesh in the (x,y) plane. Each vertex stores $(SRGB)$ information.

The **second step** consists in interpolating S , R , G and B for an input $(XYZ)_{in}$.

From the above reference mesh, a $(SRGB)_{REF}$ quadruplet can be interpolated for each input $(XYZ)_{in}$ triplet:

- For this, first x_{in} and y_{in} are computed from $(XYZ)_{in}$.
- Then the triangle containing this (x_{in}, y_{in}) point is found in the reference mesh.
- Thirdly, the S_{REF} , R_{REF} , G_{REF} and B_{REF} values for this point are interpolated from the values stored at the vertices of the found triangle.

The interpolation in the mesh triangles is basically a “flat” (or linear) interpolation implemented using as weighting factors the surface of the sub-triangles created by the insertion of the input point.

The $(SRGB)_{REF}$ quadruplet is the output of the 2D transform.

This quadruplet represents the color point belonging to one of the three “reference” planes and having the same (x, y) coordinates as the $(XYZ)_{in}$ input color.

Below Figure 3 shows an example of such interpolation for the G component:

The white lines indicate the color gamut. The red, green and blue circles are respectively the R , G and B points in the “reference” planes.

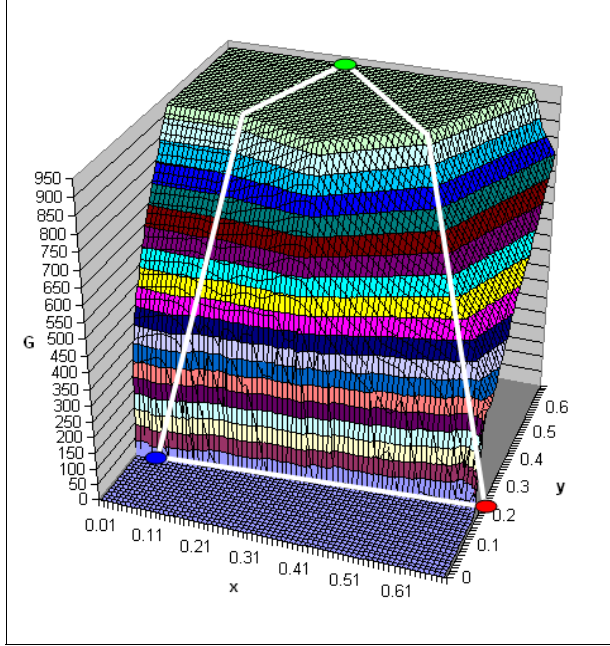


Figure 3. G component interpolation (3D view)

Luminance computation / 1D transform

Position of the problem:

We are looking for the color point $(RGB)_{out}$ corresponding to $(XYZ)_{in}$.

The above 2D transform allows to determine a point $(RGB)_{REF}$ of the RGB cube giving a visual color of (x_{in}, y_{in}) chromaticity identical to the $(XYZ)_{in}$ input point.

For this $(RGB)_{REF}$ point the $S_{REF} = X_{REF} + Y_{REF} + Z_{REF}$ value is known as interpolated by the 2D transform process.

As stated above, the colorimetry independence with regard to luminance implies that all colors sharing the same (x, y) have the same proportions of R, G and B in their composition.

$(RGB)_{REF}$ and $(RGB)_{out}$ share the same (x_{in}, y_{in}) coordinates and consequently have the same proportions of R, G and B.

In other words, $(RGB)_{out}$ and $(RGB)_{REF}$ are linked by a multiplicative factor:

$$(RGB)_{out} = k \cdot (RGB)_{REF} \Leftrightarrow \begin{cases} R_{out} = k \cdot R_{REF} \\ G_{out} = k \cdot G_{REF} \\ B_{out} = k \cdot B_{REF} \end{cases}$$

As $(RGB)_{out}$ corresponds to (x_{in}, y_{in}, S_{in}) and $(RGB)_{REF}$ corresponds to $(x_{in}, y_{in}, S_{REF})$, the factor k only depends on S_{in} and S_{REF} .

It is this luminance dependence we want to determine in the following paragraph.

Luminance dependence:

The curves on Figure 4 represent the response to a Red, Green, Blue and White ramp on a Digital Projector.

- The horizontal axis is $S = X + Y + Z$ in Cd/m^2 (or nits).
- The vertical axis is $\Sigma = R + G + B$.

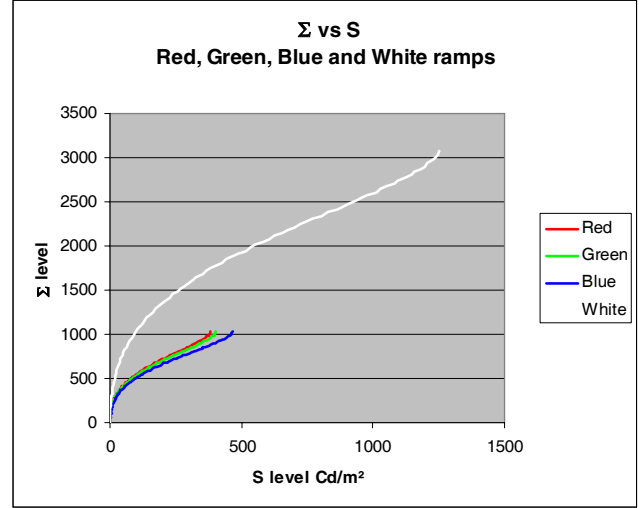


Figure 4. Sum of RGB vs. sum of XYZ for Red, Green, Blue and White ramps

Similar curves are obtained for other input color ramps with constant RGB proportions.

The principles of independence between light intensity and color explained in the above paragraph can be verified on this example by normalizing these curves by their values at a given level. This level can be the maximum of Σ , or preferably a proportion of this maximum to avoid saturation problems. Practically we are using $(Max(\Sigma), 930)/1023$ in order to have the same reference level as in the mesh construction above.

Normalizing consists in computing for each curve:

$$\Sigma_{REF} = (Max(\Sigma) \cdot 930) / 1023 \quad \Sigma_n = \frac{\Sigma}{\Sigma_{REF}} \quad S_n = \frac{S}{S_{REF}}$$

Where S_{REF} is the S level corresponding to Σ_{REF}

Normalization gives the following curves:

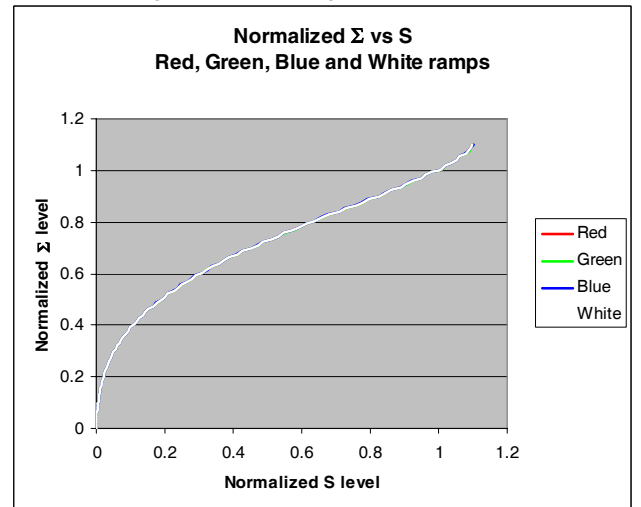


Figure 5. Normalized Σ vs S curves

These curves are superimposed as predicted in the model.

From this, a common 1D transform can be identified linking normalized Σ (i.e. Σ_n) to normalized S (i.e. S_n).

Naming f the function representing this common 1D transform, we have:

$$\Sigma_n = f(S_n)$$

This 1D transform can be represented by a number of measurement points with a linear or higher order interpolation, or by a mathematical function representing this curve.

RGB_{out} computation:

The input sum S_{in} and the quadruplet (S_{REF} , R_{REF} , G_{REF} , B_{REF}) resulting from the 2D transform are exploited to find the RGB_{out} triplet corresponding to the XYZ_{in} input point.

First a normalized S_n value is computed by dividing S_{in} by S_{REF} :

$$S_n = \frac{S_{in}}{S_{REF}}$$

Secondly, $\Sigma_{out} = R_{out} + G_{out} + B_{out}$ is computed using the 1D transform:

$$\Sigma_{out} = f(S_n),$$

$\Sigma_{REF} = R_{REF} + G_{REF} + B_{REF}$ is evaluated from the quadruplet.

Finally, R_{out} , G_{out} , and B_{out} are computed by:

$$R_{out} = \frac{R_{REF} \cdot \Sigma_{out}}{\Sigma_{REF}} \quad G_{out} = \frac{G_{REF} \cdot \Sigma_{out}}{\Sigma_{REF}} \quad B_{out} = \frac{B_{REF} \cdot \Sigma_{out}}{\Sigma_{REF}}$$

RGB_{out} is the output of the model corresponding to the XYZ_{in} input point.

Results

Instrumentation and Measures

Measurements have been performed using an automated application specifically developed for projectors and direct view displays characterization.

Using X-RITE / Gretag-Macbeth's Eye-One Probe as a measurement device, this application can automatically measure characterization and verification datasets of several hundreds of values, and apply the parametric characterization model and the verification procedures.

Verification method and error indicators

The verification "V" data subset is composed of a 5x5x5 uniform sampling of the RGB cube. With 10 bit dynamic, R , G and B take all combinations of the five following values:

$$\{104, 312, 520, 728, 936\}.$$

Verification consists in measuring the actual XYZ of each RGB_{in} triplet of the "V" data subset. Therefore, we have a list of 125 triplet pairs:

$$\{RGB_{in} \rightarrow XYZ_{meas}\}$$

With the parametric model, we can estimate RGB for each measured XYZ_{meas} . We so have a wider correspondence list:

$$\{RGB_{in} \rightarrow XYZ_{meas} \rightarrow RGB_{est}\}$$

From this list, we can compute an RGB error between the corresponding RGB input and RGB estimate:

$$\{RGB_{error} = dist^5(RGB_{in} - RGB_{est})\}$$

A global error $\Delta_{RGB\%}$ can be computed by taking the average of these relative RGB errors over the entire verification data set ($N=125$ measures):

$$\Delta_{RGB\%} = \frac{1}{Max(RGB)} \frac{1}{N} \sum_{i=1}^N RGB_{error}^6$$

However, as the RGB color space is far from being uniform in terms of color difference visibility, we prefer to evaluate also the error in the visual domain. For this we use the CIE1976 $L^*a^*b^*$ color space that is meant to be more uniform. So, the estimated RGB list is measured and we obtain a four triplets list of the form:

$$\{RGB_{in} \rightarrow XYZ_{meas} \rightarrow RGB_{est} \rightarrow XYZ_{est}\}$$

We can then compute an XYZ error between the initial XYZ measure (XYZ_{meas}) and the XYZ measure of the RGB_{est} estimation (XYZ_{est}). For this, respective $L^*a^*b^*$ colors Lab_{meas} and Lab_{est} are computed from XYZ_{meas} and XYZ_{est} using the standard formulae (recalled in [5]). The global error in this space will be:

$$Avg.\Delta E_{Lab} = \frac{1}{N} \sum_{i=1}^N dist(Lab_{meas}^i - Lab_{est}^i)$$

The model comparison of the next paragraph uses these global measures for each display and each characterization model used. The maximum of ΔE_{Lab} ($Max.\Delta E_{Lab}$) error over the verification set is given as complementary indicator as well as the 95th percentile value of ΔE_{Lab} ($95^{th}.\Delta E_{Lab}$ = value below which 95% of errors fall).

Model comparison

To evaluate the quality of the proposed parametric model, a reference needs to be chosen. It is the purpose of this paper to show that a better quality than a simple linear model (3x3 matrix) and more complex non-linear model (Splines and Tetrahedral) can be achieved. For the simple additive model, the reference chosen is, for example, the IEC 61966 standard [6] proposing to link RGB and white normalized XYZ (noted X'Y'Z') by a constrained 3x3 matrix:

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = S_{3 \times 3} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

$$\text{where } S_{3 \times 3} = \begin{pmatrix} x_R/y_R & x_G/y_G & x_B/y_B \\ 1 & 1 & 1 \\ z_R/y_R & z_G/y_G & z_B/y_B \end{pmatrix} \begin{pmatrix} S_R & 0 & 0 \\ 0 & S_G & 0 \\ 0 & 0 & S_B \end{pmatrix}$$

With S_R , S_G , S_B , solution of the equation:

$$\begin{pmatrix} x_R/y_R & x_G/y_G & x_B/y_B \\ 1 & 1 & 1 \\ z_R/y_R & z_G/y_G & z_B/y_B \end{pmatrix} \begin{pmatrix} S_R \\ S_G \\ S_B \end{pmatrix} = \begin{pmatrix} x_w/y_w \\ 1 \\ z_w/y_w \end{pmatrix}$$

This later equation formulates the additivity constraints.

⁵ $dist()$: Euclidian distance in each color space.

⁶ $Max(RGB) = 1023$ for a 10 bit dynamic signal

For the non-linear model, standard 3D lattice-based interpolation technique (thin plate splines and tetrahedral) is used. For the characterization, R , G , B values in a regular lattice ($7 \times 7 \times 7$) are measured to obtain corresponding X , Y , Z tristimulus values. Thin plate splines interpolation algorithm is used for the forward mapping (RGB to XYZ) and a tetrahedral interpolation of the oversampled dataset is used for the inverse mapping (XYZ to RGB).

Comparison results are shown in the following tables for two target displays: NEC NC800C digital cinema projector and Sony BVM 32" CRT.

The NEC NC800C projector was used with different settings:

- ITU Rec.709 mode
- Cinema mode 1 – DCI P3
- Cinema mode 2 – P7v2 telecine - Secondaries (Cyan, Magenta and Yellow) reinforced

NEC NC800C Projector ITU Rec. 709 mode			
Error type	3x3 linear model	3D non-linear model	Parametric model
Avg. ΔE_{Lab}	1.39	1.04	0.86
95 th . ΔE_{Lab}	2.97	2.81	1.65
Max. ΔE_{Lab}	3.51	4.58	2.26
$\Delta_{RGB\%}$	1.27%	2.41%	1.84%

NEC NC800C Projector Cinema mode 1 – DCI P3 - (SMPTE 431-2)			
Error type	3x3 linear model	3D non-linear model	Parametric model
Avg. ΔE_{Lab}	2.44	1.27	1.16
95 th . ΔE_{Lab}	5.34	3.26	2.74
Max. ΔE_{Lab}	6.43	5.00	6.14
$\Delta_{RGB\%}$	5.53%	3.24%	3.32%

NEC NC800C Projector Cinema mode 2 – P7v2 telecine - Secondaries reinforced			
Error type	3x3 linear model	3D non-linear model	Parametric model
Avg. ΔE_{Lab}	8.32	1.47	1.00
95 th . ΔE_{Lab}	16.95	3.01	2.50
Max. ΔE_{Lab}	19.18	5.23	3.29
$\Delta_{RGB\%}$	7.29%	3.57%	3.38%

Sony BVM 32" CRT			
Error type	3x3 linear model	3D non-linear model	Parametric model
Avg. ΔE_{Lab}	4.10	0.82	0.68
95 th . ΔE_{Lab}	6.24	2.02	1.40
Max. ΔE_{Lab}	7.16	2.95	2.58
$\Delta_{RGB\%}$	3.69%	0.89%	0.78%

Table 3. Results

Results in the above tables show that the proposed two step parametric model can significantly outperform the simple linear model (3x3 matrix) and perform slightly better over the complex non-linear model (splines and tetrahedral). We see most specifically the case of P7v2 telecine mode where non-additive color variations can be correctly estimated by the 2D step of the proposed parametric model.

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Authors Biography

Laurent Blondé is graduate engineer of the Institut d'Optique (1985). Hired as a research engineer for THOMSON R & D, he participated in more than a few R&D projects including: Infrared Image Synthesis, Special Effects and Virtual Studio, Display processing, Anti-Camcorder and Color Management for Cinema applications. Supporting both of the Technicolor and Grass Valley businesses, Blondé is currently Technical Advisor. His research interests involve all domains of image processing and synthesis for the media industry.

Jürgen Stauder received in 1999 the PhD degree from University of Hannover (Germany) in the field of computer vision. He then stayed with INRIA (France) for two years, before joining Thomson Research Labs in Rennes (France). Jürgen is guest lecturer at University of Rennes for applied color science. His research interests are computer vision, color science and computer graphics with application to video asset management, color management and content production.

Bongsun Lee received a PhD degree in Electrical and Computer Engineering from Purdue University, West Lafayette, IN, in 2005. His research interests included psychophysics and human visual system, printer halftoning, printer defect analysis, and digital device characterization. Since June, 2005, he has been employed at Thomson Corporate Research, Burbank, CA as a research staff working on color management and calibration for a film post production, digital cinema, and professional and consumer level display systems.