

# Visual Contrast Mapping based on Weber's Fraction

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## Abstract

Center/Surround (C/S) response is the first step in human vision, which represents many aspects of visual perception. The C/S model has hinted to sharpness, lightness, and contrast improvements in image processing. Retinex is a root of C/S based vision model which restores an intrinsic reflectance image by removing the spatial non-uniformities of illumination. To recreate a viewer's sensation of real scenes, Tone Mapping Operators (TMO) have been actively developed during the past decade in two ways of Spatially-invariant and Spatially-variant approaches and now the latter is becoming major stream. However, since most of spatially-variant TMO are designed independent of input/output Tone Reproduction Curve (TRC) and based on "pixel-based" C/S process, they meet a difficulty in keeping the output local contrast as same as the input image.

This paper proposes an improved spatially-variant TMO based on an "area-based" C/S process different from "pixel-based" C/S process that maintains local visual Contrast based on Weber Fraction criterion under the given system TRC. The paper describes a basic mathematical formula and introduces experimental results applied to natural images.

## Introduction

The Center/Surround (C/S) response is the first step in human vision, which represents many aspects of visual perception [1], [2]. The C/S model has hinted to sharpness, lightness, and contrast improvements in image processing. Retinex is a C/S based vision model which restores an intrinsic reflectance image just as seen under uniform illumination by removing the spatial non-uniformities.

During the past decade, the visual tone mapping has been common and important topics because the imaging systems have been strikingly evolving to capture HDR scenes, while most available displays are still limited in their LDR. To recreate the viewer's sensation of the captured scene, HDR has to be compressed to LDR of display devices. Tone Mapping Operators (TMO) aiming at this objective are categorized into two types,

- Spatially-invariant global operators
- Spatially-variant local operators

Spatially-invariant mappings apply a global TRC to a whole image, which is determined by the viewing condition and the content of the image. Since the input-to-output mapping can be provided with a LUT, it's realized simply and quickly. However, it's hard to preserve the local visual contrast because the TRC with monotonously increasing characteristics operates as an image-independent point process not to cause the tonal inversion.

On the other hand, Spatially-variant mappings apply an image-dependent spatial TMO to improve the local visual contrast. Since these operators realize flexible mappings, they are becoming main stream in recent years.

This paper proposes an improved spatially-variant TMO based on an "area-based" C/S process different from "pixel-based" C/S process that maintains local visual Contrast based on Weber Fraction criterion under the given system TRC. Our "area-based" TMO manages a local contrast gain defined by Weber Fraction in a different manner from conventional TMO.

## Pixel-based Contrast Mapping

Retinex [5] is a root of *spatially-variant* TMO. Basically it is a model to remove the spatial non-uniformity of illumination based on C/S process, where S reflects a spatial average in the surround luminance. Simply, an image  $I$  captured by camera is equivalent to the product of the reflectance  $R$  and illuminant  $L$ . According to  $R \equiv I/L$ , the reflectance  $R$  is restored from Image  $I$  by inferring the illumination  $L$  [6]-[9]. Our *Adaptive Scale-Gain Multi-Scale Retinex* [10], [11] worked well to improve the image contrast by setting the weights automatically. However, since Retinex is designed *irrelevant* to the input/output TRC, sometimes it causes unnatural gradation in tonal rendition or degradation in local contrast hard. Hence we developed a *spatially-variant* TMO to enhance the local visual contrast *relevant* to the input/output TRC. We have reported two types of *spatially-variant* TMO for local contrast enhancement as follows.

## Local Contrast Range Transform (LCRT)

Recently HDR to LDR TMO has been developed actively. *Spatially-invariant* TRC operates point-wise on the image based on the global adaptation of vision, as reported by Tumblin and Rushmeier [12] or Ward Larson [13]. While, *spatially-variant* TMOs proposed by Chiu [14], Pattanaik [15], Fattal [16], Tumblin [17] or Fairchild [18] try to recreate a real world appearance by local process.

In our previous papers, Monobe et al [19]-[21], introduced a new criterion to a local contrast. We proposed *LCRT (Local Contrast Range Transform)* operator to preserve a local contrast between input and output images *relevant* to the given TRC. The conceptual model of *LCRT* is illustrated in Fig.1.

The condition to preserve the local contrast is given by

$$\frac{g(x,y)}{g_{ave}(x,y)} = \frac{f(x,y)}{f_{ave}(x,y)} \quad (1)$$

Where,  $f(x,y)$  and  $f_{ave}(x,y)$  denote the input pixel luminance and its local average as a surround. As well,  $g(x,y)$  and  $g_{ave}(x,y)$  denote the corresponding output pixel luminance and its local average, respectively.

*LCRT* operator should meet the Eq. (1) under the given TRC as

$$g(x,y) = TRC\{f(x,y)\} \quad (2)$$

Taking the *log* and denoting the variables in capital letters,

$$G(x,y) - G_{ave}(x,y) = F(x,y) - F_{ave}(x,y) \quad (3)$$

Solving the Eq. (3) by the first-order Taylor expansion, the output  $g(x, y)$  in linear space is described by

$$g(x, y) = TRC\{f(x, y)\} \cdot LCRT(x, y) \quad (4)$$

$$LCRT(x, y) = \left( \frac{f(x, y)}{f_{ave}(x, y)} \right)^{\left\{ 1 - \frac{f(x, y)}{TRC(f(x, y))} \frac{dTRC(f(x, y))}{df(x, y)} \right\}} \quad (5)$$

The solution (4) is composed of a product of two terms. The first term corresponds to the system TRC itself and the second term reflects the LCRT operator in Eq. (5).

LCRT operator is described as a function of input C/S ratio with the exponent term of “spatially-variant” contrast gain factor,

$$LCgain(x, y) = 1 - \frac{f(x, y)}{TRC(f(x, y))} \cdot \frac{dTRC(f(x, y))}{df(x, y)} \quad (6)$$

Since  $LCgain(x, y)$  includes a first-order differential operation for TRC, it should be a differentiable smoothed curve in entire range. Fig.2 shows how the LCRT works different from Retinex. It restores the highlight visibility lost by conventional knee-compressed TRC used to commercial digital video camera. As shown in a scan line profile, LCRT keeps smoothed tone under the knee TRC, while Retinex restores a scene reflectance just as seen under uniform illumination. That is to say, LCRT claims the preservation of local contrast different from the preservation of reflectance in Retinex.

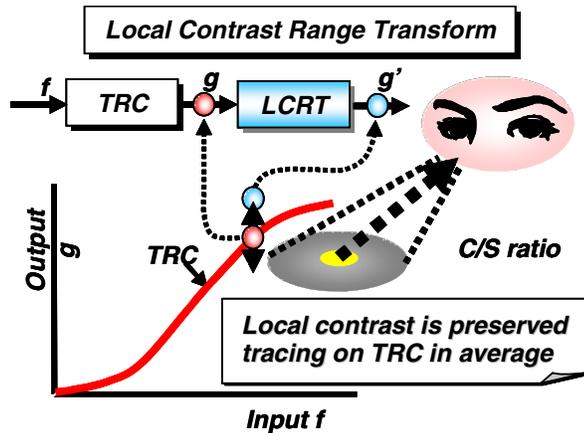


Figure 1. C/S based Local Contrast preservation model LCRT

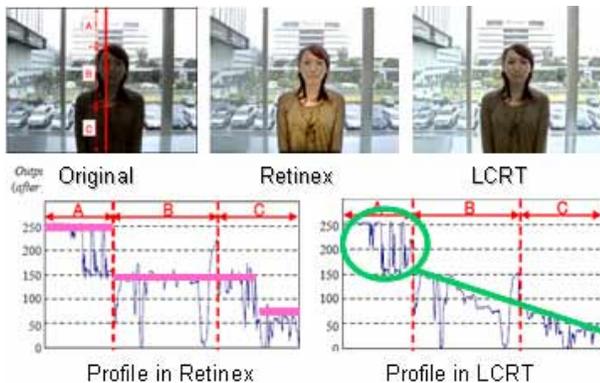


Figure 2. Essential difference between Retinex and LCRT

## Weber Contrast Gain (C-Gain) TMO by Yamashita

Although LCRT preserves the local contrast under a given TRC by matching the input C/S ratio to that of output as described by Eq. (4), it doesn't include contrast gain function.

More recently, in our previous paper [22], Yamashita newly introduced a contrast gain factor called  $Cgain$ .

The definition of contrast is not standardized but defined in different manners such as

- (A) Simple Contrast:  $C_R = L_{max} / L_{min}$
- (B) Michelson Contrast:  $C_M = (L_{max} - L_{min}) / (L_{max} + L_{min})$
- (C) Weber Fraction:  $C_W = \Delta L / L$
- (D) RMS Contrast: root mean square of Weber fraction

Since Weber fraction is a simple but vision-based useful measure for contrast management, we applied it to enhance the local visual contrast as follows.

$Cgain$  is defined by the ratio of Weber fraction  $C_{wg}$  of output  $g(x, y)$  vs.  $C_{wf}$  of input  $f(x, y)$  as,

$$C_{gain}(x, y) = \frac{C_{wg}}{C_{wf}} = \frac{dg(x, y)/g(x, y)}{df(x, y)/f(x, y)} = \frac{dG(x, y)}{dF(x, y)} \quad (7)$$

$F(x, y)$  and  $G(x, y)$  are the logarithm of  $f(x, y)$  and  $g(x, y)$ .

This signifies the slope of input vs. output in log space.

Eq. (7) is written by the following formula.

$$C_{gain}(x, y) = \frac{f(x, y)}{TRC(f(x, y))} \cdot \frac{dTRC(f(x, y))}{df(x, y)} \quad (8)$$

The condition that the average output traces just on a given TRC to maintain the smoothed tonal mapping is given by

$$g_{ave}(x, y) = TRC(f_{ave}(x, y)) \quad (9)$$

Solving Eq. (8) for the most simple linear TRC:  $g_{ave}(x, y) = f_{ave}(x, y)$ ,

$$g(x, y) = TRC(f(x, y)) \cdot \left( \frac{f(x, y)}{f_{ave}(x, y)} \right)^{C_{gain}} \quad (10)$$

Where, the surround  $f_{ave}(x, y)$  is given by a Gaussian average

$$f_{ave}(x, y) = G_m(x, y) \otimes f(x, y); \quad \otimes \text{ denotes convolution} \\ G_m(x, y) = K \exp\left\{ -(x^2 + y^2) / \sigma_m^2 \right\}; \quad \iint G_m(x, y) dx dy = 1 \quad (11)$$

## Area-based Weber's Contrast Mapping

Both LCRT and C-Gain TMO by Yamashita, control a local contrast based on C/S relations under the constraints of given system TRC. Since the center  $C$  corresponds to the “pixel” in attention and  $S$  to its “surround” given by the average luminance, the model works to enhance the image contrast in “pixel-based” processing.

While, more universally, the center  $C$  should be better to be treated not by “pixel-based” but by “area-based” in the C/S visual field, because a tiny pixel as a center  $C$  may not have enough size to contribute to the visual contrast due to the spatial frequency response limit of human vision.

Here we introduce an “area-based” C/S TMO defined by generalized Weber Fraction as shown in Fig. 3.

Letting a small and a large surrounds be  $S_S$  with scale  $m=S$  and  $S_L$  with scale  $m=L$ , the “area-based” Weber Fraction  $C_W$  is simply given by the difference between center and surround as

$$C_w(x,y) = \{S_s(x,y) - S_L(x,y)\} / S_L(x,y) = \beta(x,y) - 1 \quad (12)$$

$$\beta(x,y) = S_s(x,y) / S_L(x,y); \text{ C/S ratio} \quad (13)$$

Now we introduce *Weber Contrast Gain* factor by taking the ratio of output vs. input *Weber Fraction* as

$$WC_{gain}(x,y) = C_w^{out} / C_w^{in} = (\beta^{out} - 1) / (\beta^{in} - 1) \quad (14)$$

Where,  $\beta^{in}$  and  $\beta^{out}$  denote the input and output C/S ratios. If we can keep the *Weber Contrast Gain* at constant

$$WC_{gain}(x,y) \cong \alpha \text{ (constant)} \quad (15)$$

the local contrast in output image will be uniformly enhanced for  $\alpha > 1$  under a given TRC.

The output C/S ratio  $\beta^{out}$  to meet the Eq. (15) is given by

$$\beta^{out}(x,y) = \alpha \{\beta^{in}(x,y) - 1\} + 1 \quad (16)$$

Considering the following relations in a “area-based” small and large surrounds corresponding to C and S for input and output

$$\beta^{in}(x,y) = S_s^{in}(x,y) / S_L^{in}(x,y) \quad (17)$$

$$\beta^{out}(x,y) = S_s^{out}(x,y) / S_L^{out}(x,y)$$

we obtain the small surround for output to meet Eq. (17) as

$$S_s^{out}(x,y) = S_L^{out}(x,y) \left[ \alpha \left\{ \frac{S_s^{in}(x,y)}{S_L^{in}(x,y)} - 1 \right\} + 1 \right] \quad (18)$$

Since the input surrounds are directly calculated from original image  $f(x,y)$  by

$$S_s^{in}(x,y) = G_s(x,y) \otimes f(x,y) \quad (19)$$

$$S_L^{in}(x,y) = G_L(x,y) \otimes f(x,y)$$

we can easily get the term inside the bracket in Eq. (18).

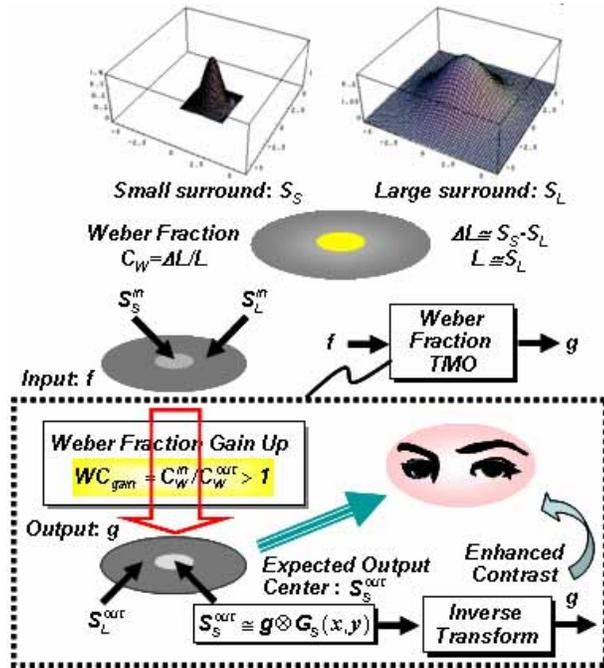


Figure 3. Area-based Contrast Enhancement model by Weber Fraction

However, the output surrounds

$$S_s^{out}(x,y) = G_s(x,y) \otimes \hat{g}(x,y) \quad (20)$$

$$S_L^{out}(x,y) = G_L(x,y) \otimes \hat{g}(x,y)$$

are still unknown.

Where,  $\hat{g}(x,y)$  means the final solution we want to get after *Weber Contrast* enhancement to meet the Eq. (16).

Assuming the large surround for output obeys system TRC as

$$S_L^{out}(x,y) = TRC\{S_L^{in}(x,y)\}, \quad (21)$$

the right side of Eq. (18) is decided.

Hence, we arrive at a conclusion to get an inverse solution for  $\hat{g}(x,y)$  by a de-convolution of

$$G_s(x,y) \otimes \hat{g}(x,y) = TRC\{S_L^{in}(x,y)\} \times \left[ \alpha \left\{ \frac{S_s^{in}(x,y)}{S_L^{in}(x,y)} - 1 \right\} + 1 \right] \quad (22)$$

### [A: Simple Solution for Weber Fraction TMO]

In case of simple linear TRC, assuming

$$S_L^{out}(x,y) \cong TRC\{S_L^{in}(x,y)\} = S_L^{in}(x,y) \quad (23)$$

to maintain the output average luminance as same as input after processing, Eq. (18) is reduced to

$$S_s^{out}(x,y) = G_s(x,y) \otimes \hat{g}(x,y) = \alpha S_s^{in}(x,y) + (1 - \alpha) S_L^{in}(x,y) \quad (24)$$

This means a small surround around the output  $\hat{g}(x,y)$  should be composed of two fractions of small surround  $\alpha S_s^{in}(x,y)$  and large surround  $(1 - \alpha) S_L^{in}(x,y)$  for the input.

2When the center C is handled as “pixel-based” just as *C-Gain TMO* by Yamashita, the small surrounds are replaced by

$$S_s^{in}(x,y) = f(x,y) \text{ and } S_s^{out}(x,y) = \hat{g}(x,y)$$

Thus the “pixel-based” simple linear solution is given by

$$\hat{g}(x,y) = \alpha f(x,y) + (1 - \alpha) S_L^{in}(x,y) \quad (25)$$

### Area-based Solution for Weber Fraction TMO

We tried to get “area-based” solution for Eq. (24) by two types of de-convolution,

- [1] De-Blurring
- [2] Fourier Transform

Fig.4 illustrates the inverse transform procedure to meet the expected *Weber Fraction gain* after contrast enhancement using de-convolution method for the proposed *Weber Fraction TMO*.

### [B: De-Blurring Solution]

Since the left term in Eq. (22) represents a *Gaussian diffusion* process as a image blurring model, the objective output  $\hat{g}(x,y)$  is restored by de-blurring process for the output center  $S = S_s^{out}(x,y)$ , that is, by image sharpening process [3],[4] as follows.

$$\hat{g}(x,y) \cong S_s^{out}(x,y) - \nabla^2 G_s(x,y) \otimes S_s^{out}(x,y) \quad (26)$$

Where,  $\nabla^2 G_s(x,y)$  is well-known *Laplacian* operator as described here by second order *Gaussian derivative* as

$$\nabla^2 G_s(x,y) = \frac{1}{\pi \sigma_s^4} \left( \frac{x^2 + y^2}{2\sigma_s^2} - 1 \right) \exp\left( -\frac{x^2 + y^2}{2\sigma_s^2} \right) \quad (27)$$

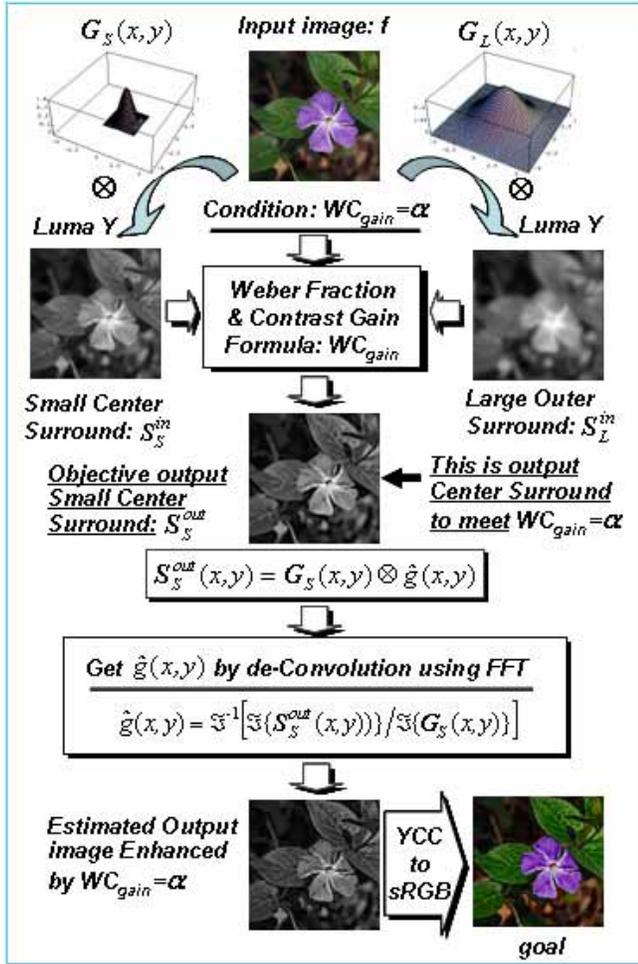


Figure 4. De-convolution Procedure by Fourier Transform for solving Weber Fraction modeled TMO

### [C: Fourier Solution]

Taking the Fourier transform  $\mathfrak{F}\{\bullet\}$  of Eq. (22), we get

$$\mathfrak{F}\{G_S(x,y) \otimes \hat{g}(x,y)\} = \mathfrak{F}\{G_S(x,y)\} \cdot \mathfrak{F}\{\hat{g}(x,y)\} \quad (28)$$

Thus  $\hat{g}(x,y)$  is given by inverse Fourier transform  $\mathfrak{F}^{-1}\{\bullet\}$  as

$$\hat{g}(x,y) = \mathfrak{F}^{-1}\left[\frac{\mathfrak{F}\{h(S_L^{in}(x,y))\}}{\mathfrak{F}\{G_S(x,y)\}}\right] \quad (29)$$

where  $h\{\bullet\}$  represents the right term in Eq. (22), that is,

$$h(S_L^{in}(x,y)) = TRC\left[S_L^{in}(x,y)\right] \left[ \alpha \left\{ \frac{S_S^{in}(x,y)}{S_L^{in}(x,y)} - 1 \right\} + 1 \right] \quad (30)$$

## Experimental Results

We verified the basic contrast enhancement function in the proposed Weber Fraction TMO through experiments.

Fig.5 shows a result for test image “Alhambra” by our model in comparison with C-gain TMO by Yamashita in case of linear TRC. Both models are tested under the same contrast gain of  $C_{gain}=2.0$  and  $\alpha=2.0$ .

All the solutions seem to be much the same in well-enhanced contrasts without changing the average luminance of original. But looking carefully the close-up details, our [A: simple Weber TMO] is almost close to [C-gain TMO by Yamashita], because both are based on “pixel-based” Weber Fraction, that is, a center  $C$  in input image is manipulated as pixel  $f(x,y)$  itself as well as output. Maybe both will come down to the same mathematical description, though we haven’t proved yet.

While, our [B: Weber TMO by De-Blurring] and [C: Weber TMO by Fourier] solutions work in different, because they are modeled not by “pixel-based” but by “area-based”  $C/S$  fields. However in general, [B: De-Blurring] algorithm loses its fine resolution a little bit in spite of nice impression in whole image contrast after enhancement, because a second order Gaussian Derivative isn’t an exact de-convolution but an approximation to the inverse diffusion. Among all, [C: Weber TMO by Fourier] solution resulted in the best that the local contrast in fine details is nicely enhanced and the color rendition is better than others.

Fig.6 shows another result in case of a non-linear TRC. Here we tested a simple gamma compression TRC given by

$$g(x,y) = TRC(f(x,y)) = f(x,y)^\gamma; \quad 0 < \gamma < 1.0 \quad (31)$$

The test image “Genoa” is beforehand corrected with  $\gamma=0.65$  to improve the shadow visibility but reduced in highlight contrast by gamma compression. By applying the same models as above, all the resultant images are well improved in the highlight visibility as seen in the close-up of “watch tower”. Among them, [C-gain TMO by Yamashita] worked better in spite of its simple algorithm and again [C: Weber TMO by Fourier] resulted in the best.

Fig.7 is a picture of “white mausoleum” with delicate texture in the highlight. In comparison with “pixel-based” contrast enhancements by [C-gain TMO by Yamashita] and [A: simple Weber TMO], the proposed “area-based” [C: Weber TMO by Fourier] look to be best in the appearance of whitish dome texture, while [B: Weber TMO by De-Blurring] lacks in the fine textural appearance, maybe due to the imperfect de-convolution filtering.

## Conclusions

The paper presented a visual contrast enhancement model based on well-known Weber Fraction. We proposed a new “area-based” Center/Surround contrast mapping model and verified its basic function through experiments. Weber fraction model derived from “area-based”  $C/S$  ratio claims that the visual contrast would be better enhanced than “pixel-based” model, because the resultant center field  $C$  with appropriate size is more clearly receptive to human vision.

However we haven’t clarified the optimal size of  $C/S$  fields yet. It may be determined depending on the image contents. At present, the de-convolution solution by FFT resulted in the best, but takes high computation costs. A simplification and proper selection of  $C/S$  field’s sizes through much more image tests under different TRC are left for our future works.



Original



Input with Gamma =0.65 Compression TRC:



C-gain TMO by Yamashita (pixel-based)



C-gain TMO by Yamashita (pixel-based)



A: Simple Weber TMO (pixel-based)



A: Simple Weber TMO (pixel-based)



B: Weber TMO by De-Blurring (area-based)



B: Weber TMO by De-Blurring (area-based)



C: Weber TMO by Fourier Solution (area-based)

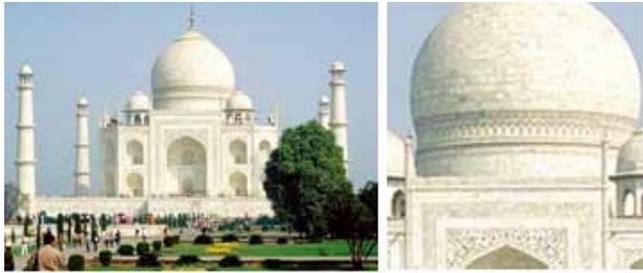


C: Weber TMO by Fourier Solution (area-based)



**Figure 5.** Contrast enhancement by Weber Fraction TMO  
(Test image "Alhambra with linear TRC")

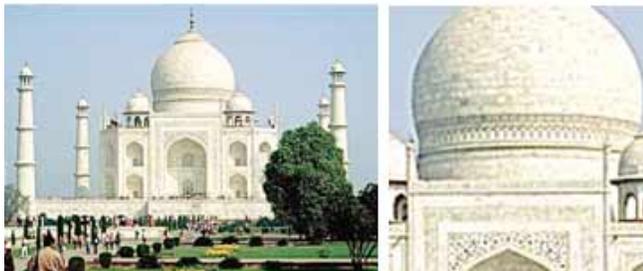
**Figure 6.** Contrast enhancement by Weber Fraction TMO  
(Test image "Genoa" with gamma compression TRC)



Original



C-gain TMO by Yamashita (pixel-based)



A: Simple Weber TMO (pixel-based)



B: Weber TMO by De-Blurring Solution (area-based)



C: Weber TMO by Fourier Solution (area-based)

**Figure 7.** Contrast enhancement by Weber Fraction TMO  
(Test image "White mausoleum" with linear TRC)

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## Author Biography

Hiroaki Kotera received his B.S degree from Nagoya Inst. Tech. and Doctorate from Univ. of Tokyo. In 1963, he joined Matsushita Electric Industrial Co. Since 1973, he worked in image processing at Matsushita Res. Inst. Tokyo. In 1996, he moved to Chiba University. He was a professor at Dept. Information and Image Sciences until his retirement in this spring, 2006. He received Johann Gutenberg prize from SID in 1995 and journal awards from IS&T in 1993, from IIEEJ in 1990 and 2000, and from ISJ in 2003. He is a Fellow of IS&T.