## **Two Invariance Properties on Object Color Changes**

### under Daylights

Johji Tajima, Eiichi Niinomi; Nagoya City University; Nagoya, Japan

#### Abstract

Two invariance properties concerning object color changes under daylights are discussed. The first is the invariance over object variety. Color changes under daylights with various correlated color temperatures were simulated using the Standard Object Colour Spectra Database (SOCS). It was found that the color changes do not depend on object categories. When we changed the illumination from D50 to the 9,300K CIE daylight, the tristimulus values of possible metamers under D50 were found to be very similar under the 9,300K daylight. It means that the metameric relation is very stable under daylights. The second is the invariance over the correlated color temperatures of daylight. The color change trajectories in the tristimulus space are almost stable even when the daylight color temperature is different. In addition, the degree of color changes is almost constant when the color temperature is measured in 'mired'. This result quantitatively justifies the conventional use of color conversion filters in the photographic industry. These invariance properties should be useful for simple realization of color constancy ability found in human vision.

#### Introduction

One of the most important purposes of the human vision is to recognize objects under daylight. It is necessary for humans to recognize an object under various conditions. If we limit the discussion to color perception, the perception of object colors that is independent of illumination color change is desirable. For millions of years, our ancestors lived under earthly daylight. The daylight, however, varies over time and so does the color appearance of objects. Thus we have obtained the ability to automatically ignore the color difference caused by the daylight variations, though it is not perfect. We call the ability 'color constancy' in this paper, taking no notice of its mechanism, though it might be realized by chromatic adaption.

Human color vision is based on the three cone inputs to the retina. The output is the tristimulus values. However, there are metamers whose spectral reflectance is different from each other while looking the same under certain illumination. This metameric relation between object colors should be preserved under illumination change for the purpose of object recognition, meaning that objects that look similar under one illumination should do so under another, even if their spectral reflectance is not similar.

Because the spectral reflectance of most objects were not known, so far it has been difficult to investigate the metameric relation. However, now that we have the 'Standard Object Colour Spectra Database for Colour Reproduction Evaluation (SOCS)' [1][2], which includes spectral reflectance/transmittance data for most object colors in the world, we can investigate whether the relation is kept under illumination change.

In this paper, we discuss the color-change invariance among metamers, which underlies the human ability of color constancy. The SOCS database includes object color spectra in tens of object categories, which may be composed of various materials. Since the human being has to cope with only the metamers that really exist in the world, it is not necessary to investigate all metamers that can be mathematically generated. As the SOCS database includes the spectra of most object colors in the world, it is very suitable for the evaluation of human color constancy.

In addition, we discuss the color-change invariance under color temperature variation of daylight. This might be empirically known in the photographic world, but here it is numerically verified by the use of spectral reflectance estimation.

#### **Spectral Reflectance Approximation**

Although the SOCS database includes spectral reflectance data for many object categories, it is impossible to find perfect metamers in the measured samples. In a typical category, the data base has the spectral reflectance of hundreds of samples. For example, it includes the spectral reflectance of 288 color samples of photographic material, and 928 samples of ink for offset printing. These numbers are sufficient for most color reproduction evaluations. However, they are not enough for the metamer evaluation. By definition, metamers must look the same color, or at least the difference should be less than 1 under an illumination in some uniform color space, CIELAB or CIELUV for example. As the volume of the color gamut of NTSC television is about two million in the CIELUV space [3], we need about 2,000,000 color samples in each object category in order to be always able to find a metamer in a category whose color is almost the same as given sample in another category. To simulate the metamers that look the same but have the spectral reflectance appropriate for each object category, we must simulate the spectral reflectance from the existing samples.

For the evaluation of color difference, we estimate the spectral reflectance from a given set of tristimulus values in an object category under a given daylight, and then calculate the new set of tristimulus values under another daylight from the spectral reflectance. The accuracy of approximation can be confirmed using the existing SOCS data. For the estimation of spectral reflectance from a given tristimulus values, several methods are known [4][5][6]. To our case that the object category is given, the methods proposed in [4][6] that use the statistics of the spectral reflectances included in the object category are appropriate and applied. According to the references, the accuracy of the spectral reflectance estimation is not high. However, in this study, it is

essential that the spectral reflectances which are similar to the existing metamers are estimated. High accuracy is not necessarily important.

# Principal component analysis with an average (PCAA)

The first method is 'PCAA' (Principal Component Analysis with Average). If the object spectral reflectance set A includes N samples, let the i-th sample be  $\beta_{Ai}$  (i=1,...,N), where

$$\boldsymbol{\beta}_{Ai} = \begin{pmatrix} \boldsymbol{\beta}_{Ai1} & \boldsymbol{\beta}_{Ai2} & \cdots & \boldsymbol{\beta}_{AiM} \end{pmatrix}^{t}$$
(1).

Each component is the reflectance at the j-th (j=1,2,...,M) wavelength. Wavelengths are sampled between 400nm to 700nm with an interval of 10nm in this study. PCA is applied to the N vectors, and a mean value  $\overline{\beta}_A$  and three principal components  $(\xi_{A1} \quad \xi_{A2} \quad \xi_{A3})$  with the largest eigenvalues are obtained.

When the tristimulus values in CIE 1931 XYZ system under  $I_1(\lambda)$ ,  $X_{I_1A,i}$ ,  $Y_{I_1A,i}$  and  $Z_{I_1A,i}$ , are known, the estimated spectrum is expressed by:

$$\hat{\boldsymbol{\beta}}_{I_1Ai} = \overline{\boldsymbol{\beta}}_A + a_{A1i}\boldsymbol{\xi}_{A1} + a_{A2i}\boldsymbol{\xi}_{A2} + a_{A3i}\boldsymbol{\xi}_{A3}$$
(2)

The coefficients  $a_{A1i}, a_{A2i}, a_{A3i}$  are solved as:

$$\begin{pmatrix} a_{A1i} \\ a_{A2i} \\ a_{A3i} \end{pmatrix} = \begin{pmatrix} \overline{\mathbf{x}}_{1}^{\prime} \xi_{A1} & \overline{\mathbf{x}}_{1}^{\prime} \xi_{A2} & \overline{\mathbf{x}}_{1}^{\prime} \xi_{A3} \\ \overline{\mathbf{y}}_{1}^{\prime} \xi_{A1} & \overline{\mathbf{y}}_{1}^{\prime} \xi_{A2} & \overline{\mathbf{y}}_{1}^{\prime} \xi_{A3} \\ \overline{\mathbf{z}}_{1}^{\prime} \xi_{A1} & \overline{\mathbf{z}}_{1}^{\prime} \xi_{A2} & \overline{\mathbf{z}}_{1}^{\prime} \xi_{A3} \end{pmatrix}^{-1} \begin{pmatrix} X_{I_{1}A,i} - \overline{X}_{I_{1}A} \\ Y_{I_{1}A,i} - \overline{Y}_{I_{1}A} \\ Z_{I_{1}A,i} - \overline{Z}_{I_{1}A} \end{pmatrix},$$
(3)

where

$$(\overline{\mathbf{x}}_{1}, \overline{\mathbf{y}}_{1}, \overline{\mathbf{z}}_{1}) = \begin{pmatrix} I_{1,1}\overline{x}_{1} \\ I_{1,2}\overline{x}_{2} \\ \vdots \\ I_{1,M}\overline{x}_{M} \end{pmatrix}, \begin{pmatrix} I_{1,1}\overline{y}_{1} \\ I_{1,2}\overline{y}_{2} \\ \vdots \\ I_{1,M}\overline{y}_{M} \end{pmatrix}, \begin{pmatrix} I_{1,1}\overline{z}_{1} \\ I_{1,2}\overline{z}_{2} \\ \vdots \\ I_{1,M}\overline{z}_{M} \end{pmatrix} \end{pmatrix};$$

 $(X_{I_1A,i} \quad Y_{I_1A,i} \quad Z_{I_1A,i})$  are the tristimulus values of the color of the i-th sample;  $(\overline{X}_{I_1A} \quad \overline{Y}_{I_1A} \quad \overline{Z}_{I_1A})$  are the tristimulus values of the mean reflectance under  $I_1(\lambda)$ ; and  $I_{1,j}$  and  $\overline{x}_j, \overline{y}_j, \overline{z}_j$  are the components of  $I_1(\lambda)$  and the color matching functions defined in the CIE1931 XYZ system, respectively.

#### Least square method (LSQ)

Let us represent the N spectral reflectance values in Set A as a matrix  $\mathbf{B}_{A} = (\boldsymbol{\beta}_{A1} \quad \boldsymbol{\beta}_{A2} \quad \cdots \quad \boldsymbol{\beta}_{AN})$  and their tristimulus values under the illumination  $I_{1}(\lambda)$  as

$$\boldsymbol{\chi}_{I_{1}A} \equiv \begin{pmatrix} X_{I_{1}A,1} & X_{I_{1}A,2} & \cdots & X_{I_{1}A,N} \\ Y_{I_{1}A,1} & Y_{I_{1}A,2} & \cdots & Y_{I_{1}A,N} \\ Z_{I_{1}A,1} & Z_{I_{1}A,2} & \cdots & Z_{I_{1}A,N} \end{pmatrix} = \begin{pmatrix} \overline{\mathbf{x}}_{1}^{r} \\ \overline{\mathbf{y}}_{1}^{r} \\ \overline{\mathbf{z}}_{1}^{r} \end{pmatrix} \mathbf{B}_{A} .$$
(4)

If we assume that each spectral reflectance  $\beta_{A_i}$  is estimated from the tristimulus values  $\begin{pmatrix} X_{I_iA,i} & Y_{I_iA,i} & Z_{I_iA,i} \end{pmatrix}$  by

$$\hat{\boldsymbol{\beta}}'_{I_{1}Ai} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \vdots & \vdots & \vdots \\ a_{M1} & a_{M2} & a_{M3} \end{pmatrix} \begin{pmatrix} \boldsymbol{X}_{I_{1}A,i} \\ \boldsymbol{Y}_{I_{1}A,i} \\ \boldsymbol{Z}_{I_{1}A,i} \end{pmatrix},$$
(5)



Figure 1 Spectral reflectance estimation example.

then the sum of the square of spectral reflectance estimation errors at j-th wavelength is expressed as:

$$E_{j} = \sum_{i=1}^{N} \left( \beta_{Aij} - \hat{\beta}^{*}_{I_{i}Aij} \right)^{2} = \sum_{i=1}^{N} \left( \beta_{Aij} - a_{j1} X_{I_{1}A,i} - a_{j2} Y_{I_{i}A,i} - a_{j3} Z_{I_{i}A,j} \right)^{2}$$
(6)

Solving this least square problem, we obtain the least square estimate of the spectral reflectance as:

$$\hat{\boldsymbol{\beta}}'_{I_{l}Ai} = \boldsymbol{B}_{A} \boldsymbol{\chi}'_{I_{l}A} \left( \boldsymbol{\chi}_{I_{l}A} \boldsymbol{\chi}'_{I_{l}A} \right)^{-1} \begin{pmatrix} \boldsymbol{X}_{I_{l}Ai} \\ \boldsymbol{Y}_{I_{l}A,i} \\ \boldsymbol{Z}_{I_{l}A,i} \end{pmatrix}$$
(7)

#### Accuracy of the estimated spectral reflectance and colors

From the original reflectance, the tristimulus values under the illuminant D50 were calculated. The two estimation methods (PCAA and LSQ) were applied to the tristimulus values. Figure 1 shows an example of the estimated reflectance. For the 64-th sample in the pr\_ds\_4 file in SOCS, the original reflectance, the estimation results by PCAA, and those by LSQ are shown. The calculations were carried out, using 512 color samples by a dye sublimation printer. The estimated relectance is similar to the original, though it deviates by a few percent at some wavelengths. Using this estimated spectral reflectance, we can estimate the color of the samples under another illumination.

Next, we verify the accuracy of the tristimulus values calculated from the estimated spectral reflectance  $\hat{\beta}_{I_1Ai}$  and  $\hat{\beta}'_{I_1Ai}$ . Let the illumination  $I_1(\lambda)$  be D50, and illumination  $I_2(\lambda)$  be the CIE daylight for 9300K, since the former illumination is important to observe hardcopies, and the latter is often the white point of off-the-shelf displays. The spectral power distribution of the CIE daylight is calculated from the following formula:

$$I(\lambda) = S_0(\lambda) + M_1 S_1(\lambda) + M_2 S_2(\lambda)$$
(8)

where  $S_i(\lambda)$  (i=0,1,2) are the mean and the principal components of measured daylight spectra, and  $M_i$  (i=1,2) are coefficients that are fixed by the correlated color temperature of the daylight.

Let A be the reflectance set of 512 samples in the pr\_ds\_4 file. First, for these samples, the tristimulus values

 $(X_{D50A,i} \quad Y_{D50A,i} \quad Z_{D50A,i})^{t}$  (i=1,2,...512) are calculated. Second,

from the tristimulus values, the spectral reflectance  $\hat{\beta}_{D50Ai}$  and

 $\beta'_{D50Ai}$  are estimated using both methods. Third, applying the CIE

in CIE-19	1.00	
	PCAA	LSQ
Maximum difference	1.54	1.45
rms difference	0.48	0.48

Table 1 Difference of estimated color from original in CIE-1931 XYZ space.

daylight for 9300K to these  $\hat{\beta}_{D50Ai}$  and  $\hat{\beta}'_{D50Ai}$ , the tristimulus values for this illumination are obtained as

 $(\hat{X}_{D9300A,i} \quad \hat{Y}_{D9300A,i} \quad \hat{Z}_{D9300A,i})$ . Table 1 shows the root mean

square difference and the maximum difference from the true values. The true values are calculated applying the CIE daylight for 9300K to each original spectral reflectance. The differences are evaluated in CIE-1931 XYZ space. Although such color differences should be evaluated in one of the uniform color spaces (e.g. CIELAB), they are not necessarily appropriate in this case since they are defined for standard C or D65 illuminant. The rms difference is 0.48 and the maximum difference is about 1.5. This result shows that the two spectrum estimation methods are accurate enough in estimating the colors under the daylight for 9300K, and that the two methods do not have large difference in their estimation ability.

#### Color change invariance among metamers under various daylights

When we estimate the spectral reflectance based on the statistics of a sample set other than its original, the estimated result may be dissimilar to the original. Figure 2 shows an example, where the spectral reflectance of the 64-th sample in the  $pr_ds_4$  file is estimated from the tristimulus values by the LSQ method, using the statistics on 512 color sample by another dye sublimation printer ( $pr_ds_3$ ). The result is compared with the estimation result based on the statistics of  $pr_ds_4$  and the original spectral reflectance.

The difference is large at several wavelengths, though these three are metamers under D50. It is anticipated that their color appearances are not the same under different daylight. To verify this anticipation, the following numerical experiment was carried out.

 From the tristimulus values of the above 512 samples in 'pr\_ds\_4' under D50, spectral reflectance was estimated using eleven object categories in SOCS. The categories are as follows (The names in the parentheses are the file names in SOCS. As the two spectral reflectance estimation methods generate almost the same results, only LSQ method was used



Figure 2 Spectral reflectance estimated based on statistics of two sample sets.

for this evaluation.):

- a) Graphic prints (of05\_g)
- b) Natural flowers, leaves and skin colors (flower + leaf +  $t2fbchc1 \rightarrow fllfskin$ )
- c) Paint not for art (paint)
- d) Oil paints (pa\_o + pa\_s  $\rightarrow$  oil)
- e) Photographic transparency (ph01\_t)
- f) Photographic reflection prints (ph01\_r)
- g) Dye sublimation printer (pr\_ds\_3)
- h) Dye sublimation printer (pr\_ds\_4)
- i) Electro-static printer (pr\_es\_3)
- j) Inkjet printer (pr\_ij\_1)
- k) Polyester textiles (poly)
- 2) The tristimulus values (X,Y,Z) corresponding to the estimated spectral reflectance under the daylight for 9300K were calculated and compared with the true values.

The simulation result is summarized in Table 2. The color differences are unexpectedly small in the spectral estimation not only with their own sample set (pr\_ds\_4) but also with other sample sets. The sample sets, except for the 'fllfskin' color samples, are uniformly distributed in their color gamut. In these cases, the rms color differences are less than 1 in the CIE-1931 XYZ space with the statistics of those sample sets. Though the set 'fllfskin' is a collection of natural object colors (flowers, leaves and human skin) and thus its color distribution is biased, the estimated tristimulus values are surprisingly close to the true values.

From this numerical experiment, it may be said that the color change under two daylight illumination conditions is almost invariant over the variety of existing objects. Humans could learn

and the true values under the daylight of 9500K in CIE-1951 X1Z space.											
	of05_g	fllfskin	paint	oil	ph01_t	ph01_r	pr_ds_3	pr_ds_4	pr_es_3	pr_ij_1	poly
maximum difference	1.86	2.47	1.60	1.95	1.94	1.37	1.71	1.45	1.83	1.41	1.71
rms difference	0.98	1.12	0.83	0.92	0.80	0.63	0.77	0.48	0.85	0.60	0.62

Table 2 Color difference between the estimated X, Y and Z values and the true values under the daylight of 9300K in CIE-1031 XXZ space



(a) Trajectory is dependent on *T*. (b) Trajectory is independent of *T*. Fig.3 Color change trajectories from *T*<sub>1</sub>(●) to *T*<sub>2</sub>(×), and from *T*<sub>3</sub>(\*) to *T*<sub>4</sub>(+).

the color correspondence between typical daylight conditions in spite of the existence of metamers.

## Color change invariance with correlated color temperatures

The above experiment shows that the human can identify objects by estimating the color under given daylight  $I_2(\lambda)$  from the color under another daylight  $I_1(\lambda)$ , if he knows the color correspondence. This is the chromatic adaptation ability. This correspondence is depicted in Fig.3. Let us assume that the space is the tristimulus space. Let the symbol • indicate the sample colors under a daylight  $I_1(\lambda)$  and the symbol  $\times$  those under another daylight  $I_2(\lambda)$ . As daylights can be defined by their correlated temperatures, we can say that  $\bullet$  are for temperature  $T_1$  and  $\times$  are for temperature  $T_2 = T_1 + \Delta T$ . However, colors with the same tristimulus values indicated by • may exist also for the 3rd temperature. Let these samples be depicted by \*. If the color change trajectory from  $T_3$  to the fourth temperature  $T_4 = T_3 + \Delta T'$ is very different from the trajectory from  $T_1$  to  $T_2$ , as depicted by + in Fig.3(a), human chromatic adaptation ability must be a complicated system. On the other hand, if the trajectories of + and  $\times$  lie near as in Fig.3(b), color conversions between various daylights may be described in a simple manner. In the photographic industry, it is known that color temperature conversion filters can convert color images from those for a color temperature to another temperature and, that if the color temperatures are measured in the unit 'mired (micro reciprocal degree)', which is defined by

$$M = 100,000/T , (9)$$

the degree of color change is independent of the color temperature. For example, if  $T_1$  is 5,000K,  $M_1$  is defined to be 200 *mired*. If the second temperature is defined as  $M_2 = M_1 - 60 = 140$ , it is  $T_2=7,143$ K in Kelvin. If the above statement about color conversion filters is true, a color change from 200 *mired* to 140 *mired* is the same as the one from 250 *mired* (4,000K) to 190 *mired* (5,263K). We investigated how accurately this relation holds.

We employed the 512 colors that we used in the above experiments. However, because the tristimulus values were obtained by applying D50 illuminant to the spectral reflectance in 'pr\_ds\_4', it is not guaranteed that the values may exist under other illuminations. We used the correlated color temperature

Table 3 Color change difference for various temperatures  $(\Delta M = -60)$ .

	(	).		
$M_1 \rightarrow M_2$	200→140	167→107	143→83	
250→190	0.792	1.755	2.559	
200→140		1.000	2.823	
167→107			0.824	

Table 4 Color change difference for various temperatures  $(\Delta M = \pm 60)$ 

$(\Delta m = 100)$ .						
$M_1 \rightarrow M_2$	140→200	107→167	83→143			
190→250	0.785	1.590	2.269			
140→200		0.810	1.518			
107→167			0.744			

range 250 mired > M > 83 mired (4,000K < T < 12,048K) in this experiment. The spectral reflectance for each tristimulus values was estimated by the PCAA method. Any color whose estimated spectral reflectance takes values larger than 100% or smaller than 0% at any wavelength was regarded as an impossible color and was not used. Hence, 320 colors out of 512 remained for this experiment. The experimental procedure was as follows.

- 1) Spectral reflectance is estimated from the tristimulus values for the 320 colors under illumination  $M = M_1$ .
- 2) Tristimulus values  $(X_2, Y_2, Z_2)$  under  $M = M_2 = M_1 + \Delta M$  are calculated for each estimated spectral reflectance.

This procedure was repeated, changing the  $M_1$  value. We employed  $M_1$ =250, 200, 167, 143,  $\Delta M = -60$ , and  $M_2$ =190, 140, 107, 83. These *mired* value changes correspond to the changes 4,000K $\rightarrow$ 5,263K, 5,000K $\rightarrow$ 7,143K, 5,988K $\rightarrow$ 9,346K, and 6,993K $\rightarrow$ 12,048K in Kelvin. The difference of  $(X_2, Y_2, Z_2)$  values between each change was evaluated and the root mean square (rms) of the difference values was obtained as  $\Delta E$  by Eq.(10), where *i* is the index for the color. The  $\Delta E$  values are summarized in Table 3.

$$\Delta E = \sqrt{\frac{\sum_{i=1}^{320} (X_{2i} - X'_{2i})^2 + (Y_{2i} - Y'_{2i})^2 + (Z_{2i} - Z'_{2i})^2}{320}}$$
(10)

Color changes in the reverse direction, i.e.,  $\Delta M = +60$ , were also investigated. The result is summarized in Table 4. The rms color difference between various  $M_1$ 's is less than 3. It is sufficiently small in many color management cases, and the color change can be regarded very stable. This means that the knowledge about color conversion filters is true. Color change due to daylight change does not depend on the correlated color temperature. If the color temperature is measured by '*mired*', the color change is about the same, independent of color temperatures. This is a simple relation, and the human color vision does not need a complicated adaptation mechanism for daylight variations.

#### Conclusions

In this paper, we discussed two kinds of color change invariance under various daylights. The first was the invariance over object variety. We verified that metamers under one daylight remain metamers under daylights with any correlated color temperatures. This verification was made possible by the development of the Standard Object Colour Spectra Database (SOCS), and the spectral reflectance estimation methods. Because of this fact, human color vision only has to adapt to color changes of daylight color temperatures, oblivious of object variety. This invariance shows that the human color constancy ability is supported by the relation between the sensitivities of the cones in the retina and the existing object reflectance varieties. It suggests that the cone sensitivities may have adapted to the existing object reflectance in the world.

The second was the invariance over the correlated color temperatures of daylight. The color change trajectories in the tristimulus space are almost stable and do not depend on the correlated color temperature. Because of this, human color vision has to memorize only one set of color change trajectories. In addition, the degree of color change is almost constant when the color temperature is measured in 'mired'. Though this was known qualitatively in the photography industry, quantitative verification was first carried out in this study.

#### References

[1] ISO/TR16066, Graphic technology -- Standard object colour spectra database for colour reproduction evaluation (SOCS) (2003)

- [2] J. Tajima, H. Haneishi, N. Ojima and M. Tsukada, Representative Data Selection for Standard Object Colour Spectra Database (SOCS), 10th Color Imaging Conference, pages 155-160, 2002
- J. Tajima: "Uniform Color Scale Applications to Computer Graphics", Computer vision, graphics, and image processing, Vol.21, pp.305-325 (1983)
- [4] F. H. Imai and R. S. Berns, Spectral Estimation Using Trichromatic Digital Cameras, International Symposium on Multispectral Imaging and Color Reproduction for Digital Archives, 1999, pp.42-49
- [5] C. Li and M. R. Luo, The Estimation of Spectral Reflectances Using the Smoothness Constraint Condition, Ninth Color Imaging Conference, pages 62-67, 2001
- [6] J. Tajima, Consideration and Experiments on Object Spectral Reflectance for Color Sensor Evaluation/Calibration, 15th Conference on Pattern Recognition, Vol.3, pp.592-595, 2000

### **Author Biography**

Johji Tajima received his BS (1971) and a doctorate (1990) from the Faculty of Science, the University of Tokyo. From 1971 to 2003, he was a research member of NEC Corporation. Since 2003 he is a professor of Graduate School of Natural Sciences, Nagoya City University. His work has focused on image processing and pattern analysis, especially color image processing and 3D vision. Prof. Tajima is a fellow of the International Association for Pattern Recognition