Colour Constancy Based on Reflectance Functions

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Abstract

Reconstructing a colour constant reflectance function under various practical light sources is important in some of practical applications. Linear programming method was proposed by Takahama and Nayatani (1973), and Berns et al (1985) respectively. Their methods were derived based on minimisation in the tristimulus values colour space in order to avoid nonlinear optimisation. In this paper a least-square method was derived for reconstructing reflectance functions based upon the concept of colour constancy. The main contributions of this method are a) directly using colour difference as a measure of colour constancy; b) avoiding highly nonlinear optimisation; c) generating much smooth reflectance with very good colour inconstancy index.

Introduction

When a colour object is given in terms of tristimulus values vector p ($p^t = (X, Y, Z)$, the transpose of the column vector p) under a particular light source and a CIE standard colorimetric observer, to reconstruct the reflectance function $r^t = (r_1, r_2, \cdots, r_n)$ of the object is equivalent to solve the inverse problem:

$$p = M^t r \tag{1}$$

Here the $n \times 3$ ($n > 3$) matrix M is the weighting table computed from the spectral power distribution of a light source and a CIE standard colormetric observer. It is clear that there is no unique solution with solving equation (1) for r. The problem can be overcome by adding the smoothest condition [1,2,3], or using the basis vectors [4]. Unfortunately, reconstructed reflectance function r using either smoothest condition or basis vectors is not necessary to have a colour constancy property, which refers to the invariance of the perceived colour of surfaces under changes in illuminations. This characteristic of colour constancy is important in practical applications such as digital printing, textile and paint industries.

Notations

In order to describe the new method and methods related, some notations are needed to define clearly first. All vectors used in this paper are column vectors. A row vector is the transpose of a column vector. For example, r^t is a row vector. As defined early, p is a vector of tristimulus values. Subscripts R and T relate to the reference and test illuminants respectively. Superscripts C and W in bracket relate to tristimulus values of the corresponding colour and the relevant illuminant respectively. Thus, p_R and p_T indicate the tristimulus values are computed under the reference and test illuminant respectively. $p_R^{(C)}$ is a vector of corresponding colour's tristimulus values transformed from p_T using a chromatic adaptation transform (CAT). $p_R^{(W)}$ is a vector of tristimulus values of reference illuminant. Finally, M_R and M_T stand for the weighting tables under the reference and test illuminants (combined with CIE 1931 standard observer) respectively.

The Takahama and Nayatani Method

In 1973, Takahama and Nayatani [5] gave a linear programming algorithm for reconstructing reflectance function with colour constancy property. Their method can be described using the following flowing chart:

Given tristimulus values p_R

⇩

To find reflectance r via solving the following:

$$\frac{Min}{r} d(p_R, p_R^{(C)})$$

Subject to: $p_R = (M_R)^t r$ and $0 \le r \le 1$

Figure 1: *The flowing chart of the Takahama and Nayatani method*

Note that the $d(p_R, p_R^{(C)})$ in the above chart is a distance measure between the two vectors of tristimulus values. Ideally, it should be a colour difference measure, such as CIELAB colour difference $\Delta E = \Delta E(p_R, p_R^{(C)})$. In order to reduce the computational complexity, they used the following distance measure:

$$d(p_R, p_R^{(C)}) = | X_R - X_R^{(C)} | + | Y_R - Y_R^{(C)} | + | Z_R - Z_R^{(C)} | \tag{2}$$

Secondly, we note that, several test illuminants can be used, see N of them, thus, the above minimisation is to find r that minimises:

$$\frac{1}{N} \sum_{i=1}^{N} d(p_R, p_{R,i}^{(C)}) \tag{3}$$

Here, the second subscript i relates to the i-th test illuminant. For example, $p_{R,2}^{(C)}$ is the vector of tristimulus values (or corresponding colour under the reference illuminant) transformed from $p_{T,2}$, the tristimulus values calculated under the second test illuminant, via a CAT.

Thirdly, we note that the smooth constraint was incorporated and defined by the following:

$$|r_j - r_{j+1}| \le S_j, \quad j = 1,2,\cdots,n-1 \tag{4}$$

Here, S_j, $j = 1,2,\cdots,n-1$, are some pre-specified constants. Finally, they transformed the above problem to a linear programming problem.

It is found that the method can generate reflectance with lower colour inconstancy index, through the minimization is in the tristimulus value space. However, the reflectance is too rugged, or step-like as shown in Figure 4, where the thick dot curves were generated using this method. This phenomena was illustrated in their own example [5], and was noted in references [1,2] as well.

The Berns, Billmeyer, and Sacher Method

In 1985, Berns, Billmeyer, and Sacher [6] also gave a linear programming method to reconstruct the reflectance functions for the Munsell colour order system with colour constancy property. The flowing chart is given in Figure 2. The reference illuminant they used is illuminant C since Munsell colour order system was rigorously defined under the illuminant and CIE 1931 standard observer. The 10 chosen test illimunants were continuous spectrum filtered incandescent type daylight plus illuminant A. The nonlinear chromatic adaptation transform developed by Nayatani et al [7] was used. Like Takahama and Nayatani method, the minimisation was made is still in the tristimulus space. They also introduced hue vector h_T (the difference between $p_T^{(W)}$ and $p_T^{(C)}$) and allow the solution to move off and along the hue vector. Thus, the solution is constrained by the following equation:

$$p_T = p_T^{(C)} + b_T h_T + \sum_{j=1}^{8} a_{T,j} v_{T,j} \quad \text{with } a_{T,j} \ge 0 \tag{5}$$

where $v_{T,j}$'s are unit vectors and orthogonal to h_T, and angle between successive $v_{T,j}$'s is 45 degree. Therefore, minimization measure they used is defined by the following:

$$d(p_T, p_T^{(C)}) = |b_T| + \sum_{j=1}^{8} a_{T,j} \tag{6}$$

Besides, the smoothness for the reconstructed reflectance was achieved by using

$$-0.03 \le 0.5 r_{j-1} - r_j + 0.5 r_{j+1} \le 0.03, \quad j = 2,\cdots,n-1 \tag{7}$$

which was proposed by Ohta [8].

As noted to the Takahama and Nayatani Method, when more test illuminants are used, the averaged $d(p_T, p_T^{(C)})$ of equation (6) is used. It is found that generated reflectance using this method is 'smoother' than that generated by Takahama and Nayatani Method, but it is oscillated too much, as shown in Figure 4. This phenomenon was also illustrated by their own example (Fig. 3, reference [6]).

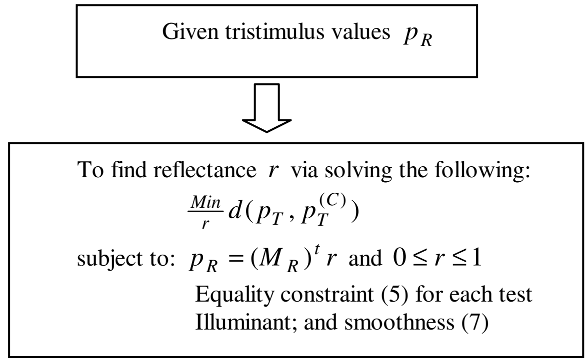

Figure 2: *The flowing chart of the Berns, Billmeyer and Sacher Method*

The Estimate CIELAB Colour Difference

In order to describe the new method, we need to find an approximation to the CIELAB colour difference. Let $q^t = (L, a, b)$ and $d^t = (16,0,0)$, then transformation from tristimulus space to CIELAB space can be done using

$$q = AF(p, p^{(W)}) - d \tag{8}$$

Here, $p^{(W)}$ is the tristimulus vector of the illuminant, and

$$A = \begin{pmatrix} 0 & 116 & 0 \\ 500 & -500 & 0 \\ 0 & 200 & -200 \end{pmatrix}, \quad F(p, p_w) = \begin{pmatrix} f(X/X_W) \\ f(Y/Y_W) \\ f(Z/Z_W) \end{pmatrix} \tag{9}$$

where

$$f(h) = \begin{cases} h^{1/3}, & \text{if } h \ge 0.008856 \\ 7.787t + 16/116 & \text{if } h < 0.008856 \end{cases} \tag{10}$$

Let p_1 and p_2 be tristimulus values of the two colour samples and q_1 and q_2 be the corresponding L, a, b vectors, then

$$\begin{aligned} q_1 - q_2 &= A[F(p_1, p^{(W)}) - F(p_2, p^{(W)})] \\ &= AF'(g, p^{(W)})(p_1 - p_2) \end{aligned} \tag{11}$$

Here, g ($g^t = (X, Y, Z)$) is a tristimulus vector with X between X_1 and X_2, Y between Y_1 and Y_2, and Z between Z_1 and Z_2. The prime "'" represents derivative and

$$F'(g, p^{(W)}) = \begin{pmatrix} F_1 & 0 & 0 \\ 0 & F_2 & 0 \\ 0 & 0 & F_3 \end{pmatrix} \tag{12}$$

with $F_1 = f'(X / X_W) / X_W$,
$F_2 = f'(Y / Y_W) / Y_W, F_3 = f'(Z / Z_W) / Z_W$

Thus, if we define a vector $g_0 = (X_0, Y_0, Z_0)^t$ with

$$X_0 = \begin{cases} X_1 & \text{if} \quad X_1 / X_w \geq 0.1 \\ 0.0241 X_W & \text{otherwise} \end{cases} \tag{13}$$

and similarly with Y_0 and Z_0, we have

$$F'(g, p^{(W)}) \approx F'(g_0, p^{(W)}) \tag{14}$$

Therefore, we have

$$\begin{aligned} [\Delta E(p_1 - p_2)]^2 &= \| q_1 - q_2 \|^2 \\ &= \| AF'(g, p^{(W)})(p_1 - p_2) \|^2 \\ &\approx \| AF'(g_0, p^{(W)})(p_1 - p_2) \|^2 \end{aligned} \tag{15}$$

Here $\| p \|$ denotes the 2-norm of the vector p, i.e., the square root of the sum of the squares of each of the components of the vector p.

The New Method

We still let p_R and $p_R^{(W)}$ be the tristimulus vectors of the colour sample and the reference illuminant, and p_R and the wanted reflectance function r satisfy:

$$p_R = (M_R)^t r \tag{16}$$

Now we choose CAT02 [9] to transform $P_{T,j}$ to the corresponding colours $p_{R,j}^{(C)}$ in the reference tristimulus values space. Then we have

$$p_{R,i}^{(C)} = U_i p_{T,i} = U_i (M_{T,i})^t r \tag{17}$$

where U_i is a 3×3 matrix depending on the reference and i-th test illuminant and the matrix of the CAT02. Besides, we use the following colour inconstancy index (CII)

$$\text{CII} = \frac{1}{N} \sum_{i=1}^{N} (\Delta E_i)^2 \tag{18}$$

Now it can be shown that

$$\begin{aligned} \text{CII} &= \frac{1}{N} \sum_{i=1}^{N} (\Delta E_i)^2 = \frac{1}{N} \sum_{i=1}^{N} \| q_R - q_{R,i}^{(C)} \|^2 \\ &= \frac{1}{N} \sum_{i=1}^{N} \| AF'(g_i, p_R^{(W)})(p_R - p_{R,i}^{(C)}) \|^2 \\ &\approx \frac{1}{N} \sum_{i=1}^{N} \| AF'(g_0, p_R^{(W)})(p_R - p_{R,i}^{(C)}) \|^2 \\ &= \frac{1}{N} \sum_{i=1}^{N} \| AF'(g_0, p_R^{(W)})[(M_R)^t - U_i (M_{T,i})^t] r \|^2 \end{aligned}$$

Note g_0 is chosen according to p_R and $p_R^{(W)}$ using equation (13). If we let

$$B = \frac{1}{\sqrt{N}} \begin{pmatrix} AF'(g_0, p_R^{(W)})[(M_R)^t - U_1 (M_{T,1})^t] \\ AF'(g_0, p_R^{(W)})[(M_R)^t - U_2 (M_{T,2})^t] \\ \vdots \\ AF'(g_0, p_R^{(W)})[(M_R)^t - U_N (M_{T,N})^t] \end{pmatrix} \tag{19}$$

$$\text{CII} \approx \| Br \|^2 \tag{20}$$

The smoothness of the reconstructed reflectance can be achieved by introducing a smooth operator matrix G [3], and the smoothness can be modeled by

$$\text{Minimise} \| Gr \|^2 \tag{21}$$

The above discussion leads to the following constraint least squares problem:

$$\text{Minimise} \quad \| Br \|^2 + s \| Gr \|^2$$

$$\text{Subject to: } 0 \leq r \leq 1 \text{, and } p_R = (M_R)^t r$$

The variable s is the weight, which balances the important contributions from CII and smoothness.

Performance Comparison

1560 measured reflectance functions between 400nm and 700nm at 10nm interval from Munsell colour book were used for generating tristimulus values. CIE illuminant C was used as reference illuminant. The test illuminants were D65, A, D50, F2, F7, and F11. CIE 1931 standard observer was used for generating tristimulus values. Thus, for each reflectance, a tristimulus value vector can be computed under illuminant C and CIE 1931 standard observer. Hence a new reflectance can be generated using each of the above methods.

For measuring the reflectance generated, the colour inconstancy index CMCCON02 [9] will be used. Note that when generating reflectance, the reference illuminant is the CIE illuminant C. However, for the performance measure, the CIE illuminant D65 is used for the reference illuminant, and illuminant C, D50, A, F2, F7 and F11 were used as test illuminants. Thus, for each reflectance (original or generated), its colour inconstancy index is measured using the following:

$$\text{CII} = \frac{1}{6} \sum_{i=1}^{6} \Delta E(p_R, p_{R,i}^{(C)}) \tag{22}$$

where p_R is the tristimulus values computed under the reference illuminant D65, $p_{R,i}^{(C)}$ is the corresponding tristimulus values transformed from $p_{T,i}$, tristimulus values computed under each of the test illuminants: C, D50, A, F2, F7 and F11, using the chromatic adaptation transform CAT02.

The CII of generated reflectance functions on the vertical axis versus the CII of the original reflectance functions on the horizontal axis were drawn in Figure 3. The 45 degree line was also drawn on each of the diagrams in Figure 3. If all points are on or around the 45 degree line, there is no improvement according to this measure. While, all the points are located below the 45 degree line, the generated reflectance functions have a better colour inconsyancy index compared with its original. The left, middle and right diagrams of Figure 3 show the performances of the proposed, the Berns, Billmeyer, and Sacher, and Takahama and Nayatani methods respectively. According to this measure, the proposed and Takahama and Nayatani methods have a similar performance and they are much better than the Berns, Billmeyer, and Sacher method.

Figure 4 shows the original and generated reflectance functions using each of the three methods. The thin full curves are the original reflectance functions; the thick full curves were generated using the proposed method; the thin dot curves were generated using the Berns, Billmeyer, and Sacher method; and the thick dot curves were generated using the Takahama and Nayatani method. It can be seen that the reflectance functions generated by Takahama and Nayatani method are stair-like, while the reflectance functions generated by the Berns, Billmeyer, and Sacher method are 'smoother' than those generated by by Takahama and Nayatani method, but they are oscillated too much. In general, the reflectance functions generated by the proposed method are much smooth.

Finally, we note that all the computations were done using MATLAB. It is found that the proposed method took the least CPU time compared with the other two methods.

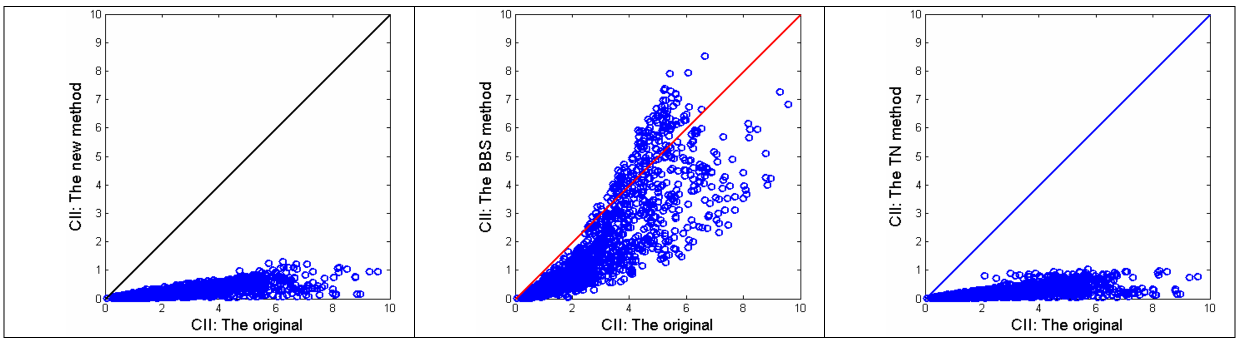

Figure 3: The CII of generated reflectance (vertical axis) generated by the new method (Left), Berns, Billmeyer, and Sacher (Middle), and Takahama and Nayatani method (Right) versus the CII of the original (horizontal axis).

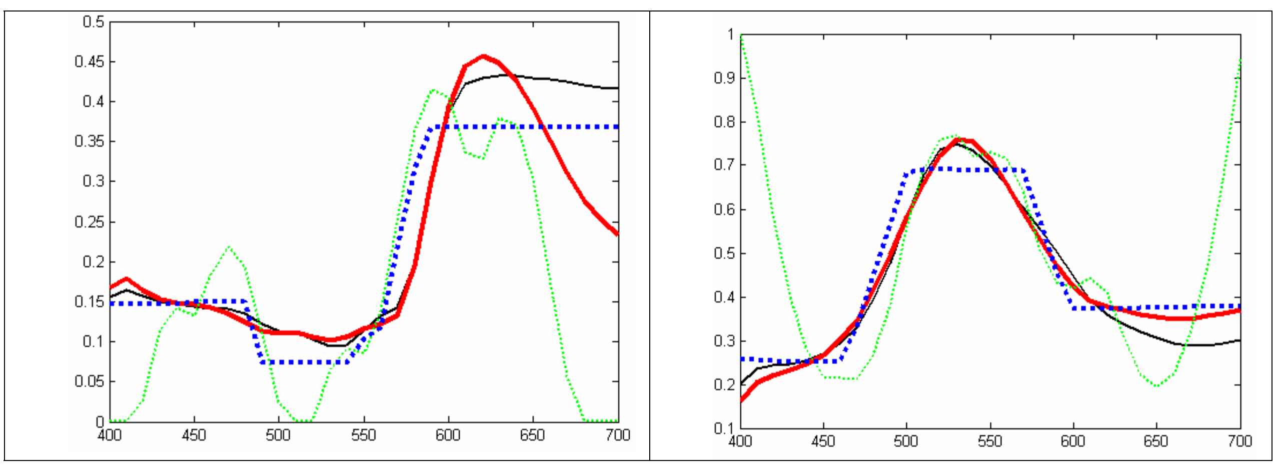

Figure 4: Reflectance functions: the original input (full thin curve), generated by the New method (full thick curve), generated by Berns, Billmeyer and Sacher method, and generated by Takahama and Nayatani method (thick dot curve).

Conclusions

In this paper, a new method for generating reflectance with a better colour inconstancy index is developed. The proposed method is simple and leads to a constrained least squares problem. The main contributions are: a) directly using colour difference as measure of colour inconstancy; b) avoiding highly nonlinear optimisation; c) generating much smooth reflectance with very good colour inconstancy index.

References

[1]. C. van Trigt, Smoothest reflectance functions. I: Definition and main results, J. Opt. Soc. Am. A, 7, 1891-1904(1990).

[2]. C. van Trigt, Smoothest reflectance functions. II: Complete results, J. Opt. Soc. Am. A. 7, 2208-2222(1990).
[3]. Changjun Li and M.R.Luo, The estimation of spectral reflectances using smoothness constraint condition, in the proceedings of TS&T/SID Colour Imaging Conference'2001, Arizona, USA, pages 62-67.
[4]. Changjun Li and M Ronnier Luo, How Many Basis Vectors Can Be Used For Generating Reflectance, 10th Congress of the International Colour Association, 8-13 May 2005, Granada, Spain, pages 1637-1640.
[5]. K Takahama and Y Nyatani, Derivation of spectral reflectance functions of object colours with good colour constancy, ACTA CHROMATICA, 2(4), 187-191(1973).
[6]. RS Berns, FW Billmeyer (Jr.), and RS Sacher, Methods for generating spectral reflectance functions leading to colour constancy properties, Color Research and Application, 10(2), 73-83(1985).
[7]. Y Nayatani, K Takahama and H Sobagaki, Formulation of a nonlinear model of chromatic adaptation, Color Research and Application, 6, 161-171(1981).
[8]. N Ohta, A simplified method for formulating pseudo object colours, Color Research and Application, 7, 78-81(1982).
[9]. MR Luo, CJ Li, RWG Hunt, B Rigg, and KJ Smith, CMC 2002 Colour Inconstancy Index: CMCCON02, Journal of Coloration Technology, 119, 280-285(2003).