Estimating reflectances from multi-spectral video responses

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Abstract

Common, tri-chromatic, RGB colour-acquisition devices capture spectral signals by a coarse sampling through three broad colour filters. Due to metamerism, a single response from a RGB device corresponds to an infinite set of possible surface spectral reflectances. In order to acquire higher quality surface colour descriptors that reduce metamerism in the acquisition process, multi-spectral imaging devices are used. These sample the spectral signals more finely through > 3 colour filters, enabling better estimation of surface spectral reflectance and consequently mitigating the problem of metamerism.

In this paper the performance of a 6-channel digital video camera is evaluated in terms of it's accuracy to capture surface spectral reflectances. A number of known techniques to estimate reflectance from device response are examined, such as linear least squares, the Wiener estimation technique, Tikhonov regularised estimation as well as the Metamer Set Maximum Likelihood method.

The performance of each algorithm is compared under different training and testing conditions. The experiments show that there estimation accuracy is significantly increased by using multi-spectral acquisition and furthermore that there is benefit in using more advanced estimation techniques still.

Introduction

In conventional tri-chromatic colour-acquisition devices, colour rendering properties of surfaces in a scene are recorded as a vector of three values, commonly known as the RGB. These compactly express the interaction of light, surface and device characteristics. However, the RGB is also a compressed representation of surface colour. It is both illuminant and device dependent and significantly under–samples the higher dimensional colour signal by representing it in 3 dimensions. Surface spectral reflectances and colour signals on the other hand, have been found to be of much higher dimensionality[19]. This discrepancy gives rise to the phenomenon of metamerism, whereby an infinite set of surfaces induce the same RGB response. Both the compressed, ambiguous nature of RGBs and it's dependence on device and illuminant conditions renders this colour descriptor inaccurate. Consequently, it is necessary to perform device characterisation (mapping RGBs to a device-independent colour space such as CIE XYZ tristimulus space) or reflectance estimation (estimating the spectral reflectance of a surface from the recorded RGB response). With the present colour rendering quality requirements, it is increasingly desirable to opt for the latter approach, as it yields not only device and illuminant independent descriptors, but also reduces the effect of metamerism[1]. Given the difference in dimension between tri-chromatic device responses and surface spectral signals, a multi–spectral approach is more suitable.

In a multi–spectral (or hyper–spectral) colour–acquisition device the surface colour properties of surfaces in a scene are filtered more finely than in tri-chromatic devices. Instead of employing three broad colour filters, effectively more filters are used to measure a higher dimensional representation of surface colour. While the majority of devices do not measure spectral reflectance directly without error, it is possible to achieve better estimation accuracy compared to tri-chromatic devices. Note however that even if a multi-spectral device has more linearly independent colour filters[2], representing surface colour with a higher dimensional response vector compared to the dimensionality of surface spectral reflectance, it does not mean that the device is necessarily error–free. Furthermore, while the phenomenon of metamerism is greatly reduced, it is still present in multi–spectral acquisition.

An acquisition device can be thought of as a linear transform, mapping surface spectral reflectances to representations within the space of the device. Unless the linear space spanned by the device's spectral sensitivities and the space spanned by surface spectral reflectances are within a linear transform of each other, metamerism will theoretically exist. So, even for devices with more sensors it is necessary to perform some form of estimation algorithm from response to surface spectral reflectance. In this paper a number of such techniques are examined and evaluated.

It has been argued that techniques based on statistical knowledge of the space of surface spectral reflectances, such as Principal Component Analysis (PCA or Characteristic Vector Analysis (CVA) are superior to other methods[3, 4]. All but one method considered in this paper indeed make use of PCA, in order to represent surface spectral reflectances in a lower dimensional linear model. The methods considered in this paper can be further classified in two classes: approaches based on a least squares minimisation (unconstrained linear least squares, constrained linear least squares using a Wiener filter[24] or regularising the solution using Tikhonov regularisation[27, 6]) and the Metamer Set Maximum Likelihood method[21].

In order to evaluate reflectance estimation methods for multi-spectral colour-acquisition devices, the 6 band multi-spectral (6B–HDTV) video camera spectral sensitivities, developed as part of the Akasaka Natural Vision project (NVP)[22, 32], have been used. There are several other approaches to mutli-spectral

[1]Reflectance estimation is a more general form of the problem of device characterisation. The former solves the latter as well, since from an accurately estimated reflectance, accurate CIE XYZ values can be had.

[2]Such that none of the filters can be substituted as a combination of the remaining ones.

imaging, such as tunable liquid-crystal filters, solid-state filter wheels[3], interferometry based devices[7], and others. However, the majority is used for digital still images instead of video. Multi-spectral digital video is an emerging technology used at present mainly in tele-medicine and high definition video. High accuracy of spectral estimation is particularly important in the former field, as diagnoses are often made based primarily on colour[32].

Results are presented on a set of 2086 reflectances including natural as well as man-made surfaces, showing statistically significant benefit in estimation accuracy by using a metamer set based technique. However, results using regularised, constrained least-squares techniques also offer high estimation accuracy from multi-spectral device responses, with the advantage of being faster to compute and easier to implement.

The article is organised as follows. First the mathematical background for reflectance estimation is established, introducing colour image formation as well as the linear model representation of surface spectral reflectances. Then, each of the considered estimation techniques is briefly introduced as well as a measure to evaluate the methods is presented. Experimental set-up, results and discussion are given afterwards. Finally some conclusions are drawn from the experiments.

Background

Let $S_i(\lambda)$ be the i-th sensor spectral sensitivity ($i = 1,\dots,N$), $E(\lambda)$ the illuminant spectral power distribution, $R(\lambda)$ the surface spectral reflectance of a surface in a scene, ρ_i the i-th response to the surface and ω the interval of visible wavelengths. Colour image formation is then defined through the integral equation:

$$\rho_i = \int_\omega S_i(\lambda)E(\lambda)R_j(\lambda)d\lambda \tag{1}$$

Assuming each quantity is represented as a discrete vector sampled at q equi-distant sampling points (e. g. $q = 31$ sampling points for $\omega = [400,700]nm$ at a $d\lambda = 10nm$ sampling frequency), then $\mathbf{S}$ can be denoted as the $q \times N$ matrix of N device spectral sensitivities, $\mathbf{e}$ the $q \times 1$ vector of illuminant spectral power distribution and $\mathbf{r}$ the $q \times 1$ vector of surface spectral reflectance and $\underline{\rho}$ the $N \times 1$ response vector (the RGB for tri-chromatic devices). Colour image formation can be re-written in matrix algebra form as:

$$\underline{\rho} = \mathbf{S}^T D(\mathbf{e})\mathbf{r} \tag{2}$$

where ^{T} is the vector transpose operator and $D()$ transforms a column vector into a diagonal matrix, with the vector elements along the diagonal of the matrix.

Surface spectral reflectance estimation amounts to inverting Eq. 2, i.e. solving for $\mathbf{r}$ given $\underline{\rho}$ and assuming linear response of a device with known spectral sensitivities $\mathbf{S}$ captured under a known light source with illuminant spectral power distribution $\mathbf{e}$.

Surface spectral reflectances can be well described by a small number of basis vectors within a linear model representation[17]. The number of basis vectors needed to represent reflectances varies depending on the data, but as a rule of thumb between 5 and 10 bases suffice. Let $\mathbf{B}$ be the $q \times D$ matrix of D basis vectors, such that $3 \leq D \leq q$. A reflectance $\mathbf{r}$ can then be approximated as:

$$\mathbf{r} \approx \mathbf{B}\underline{\sigma} \tag{3}$$

where σ is the the $D \times 1$ vector of linear model weights, representing surface spectral reflectance within the basis $\mathbf{B}$. Depending on how well matrix $\mathbf{B}$ spans the space of surface spectral reflectances it represents, the approximation in Eq. (3) is more or less accurate. Assuming the knowledge of a set of reflectances that are representative of the surfaces that might be observed, statistical tools such as Principal Component Analysis[19] serve to find the smallest number of bases that best represent the given data.

Within this representation, colour image formation can be re-written as:

$$\underline{\rho} = \mathbf{S}^T D(\mathbf{e})\mathbf{B}\underline{\sigma} \tag{4}$$

Under this assumption, estimating the surface spectral reflectance amounts to solving for the $\underline{\sigma}$ given $\underline{\rho}$, since $\underline{\sigma}$ corresponds to a reflectance $\mathbf{r}$ through matrix $\mathbf{B}$.

Let $\mathbf{L} = \mathbf{S}^T D(\mathbf{e})\mathbf{B}$, also referred to as the *lighting matrix*[18] relating linear model weights and sensor responses, then colour image formation becomes:

$$\underline{\rho} = \mathbf{L}\underline{\sigma} \tag{5}$$

This is a system of N linear equations (one per sensor) of D unknowns (one per linear model dimension). Since in general $N \ll D$, the system is referred to as *under–determined* and has no unique solution.

Unconstrained Linear Least Squares (LSQ)

A straight–forward way to solve under–determined linear systems of equations is to find a linear mapping matrix that minimises the error in the sum-of-squares sense. Let $\mathbf{X}$ be a $M \times N$ matrix of responses $\underline{\rho}$ to M surfaces, and $\mathbf{Y}$ a $M \times D$ matrix of linear model weights $\underline{\sigma}$ corresponding to the same M surfaces. Then the linear least squares minimisation can be found as:

$$\min_T \parallel \mathbf{XT} - \mathbf{Y} \parallel \tag{6}$$

where $\parallel \cdot \parallel$ is the L2 Euclidean norm.

The solution to this minimisation is defined as $\mathbf{T} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$ and writing $\mathbf{X}^+ = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$ where $^+$ denotes the Moore-Penrose (Pseudo) inverse, Eq. (6) is solved as:

$$\mathbf{T} = \mathbf{X}^+\mathbf{Y} \tag{7}$$

An alternative way of looking at the problem from the same, error-minimisation perspective is to assume knowledge of sensor spectral sensitivities, illuminant and a linear model basis, combined in the $N \times D$ matrix $\mathbf{L}$, and solve for a data-independent linear mapping, again in the least-squares sense:

$$\mathbf{T} = (\mathbf{L}^T\mathbf{L})^{-1}\mathbf{L}^T \tag{8}$$

In general however, this approach performs less well than the data-driven linear least squares matrix in Eq. (7), hence we only consider the former method.

Constrained Least Squares - Tikhonov Regularisation (LSQT)

A simple least squares matrix need not be a good fit when applied to data other than that used to derive itself (i.e. the *training set*). A consequence is that the *condition number*[3] of $\mathbf{T}$ might

[3] The ratio of the largest and smallest singular value in the singular value decomposition of a matrix.

be very large, meaning it's inverse could be difficult to compute due to computational precision. It is therefore desirable to regularise this solution. Instead of solving for the simple least squares matrix, a new optimisation can be formulated as follows:

$$\min_T \parallel \mathbf{XT} - \mathbf{Y} \parallel + \alpha \parallel \mathbf{T} \parallel \tag{9}$$

where α is a weighting factor of the norm of the sought transform. In effect this amounts to adding a penalty term associated with the norm of the transformation matrix. This approach is known as the Tikhonov regularisation[6]. The solution to Eq. (9) can be derived directly as:

$$\mathbf{T} = \mathbf{Z}^{+}\mathbf{X}^{T}\mathbf{Y} \tag{10}$$

$$\mathbf{Z} = \mathbf{X}^{T}\mathbf{X} + \alpha\mathbf{I} \tag{11}$$

where $\mathbf{I}$ is the $N \times N$ identity matrix.

Finding the weighting factor α is non-trivial. For the purpose of this experiment a simple routine based on Newton's iterative bisection technique was implemented to find the α that results in minimal error for the training set.

Constrained Least Squares - Wiener (LSQW)

Another way to regularise the solution of the least squares problem is to assume more knowledge about the estimated quantities, the suraface reflectances. Let the $D \times D$ matrix $\mathbf{C}$ be the covariance matrix of a set of representative surface spectral reflectances (the training set), then the matrix $\mathbf{T}$ mapping $\underline{\rho}$ to $\mathbf{r}$ can be solved for as[24]:

$$\mathbf{T} = \mathbf{CL}^{T}(\mathbf{LCL}^{T})^{-1} \tag{12}$$

Note that this method does not employ the linear model representation and neither does it build on PCA. However, including the covariance matrix in the solution does make use of statistical knowledge about the estimated surface spectral reflectances and has been extensively used in reflectance estimation (e.g [11]).

Metamer Set Based Estimation

In metamer set based estimation[21] first the entire set of plausible surface reflectance estimates, the *metamer set* is solved for and then, at a second stage, one reflectance from within this set is chosen as the estimate.

The existence of the metamer set follows from the under–determined nature of Eq. (5), the colour image formation equations. Since Eq. (5) is a set of N linear equations of D unknowns, and $N < D$, it follows that there are at least $D - N$ degrees of freedom. In other words, matrix $\mathbf{L}$ spans at most N dimensions assuming linearly independent spectral sensitivities of the device. The remaining $D - N$ dimensions are the so-called *metameric black space* (or null-space or kernel) of the system, as they span the space that results in responses of zeros.

The metamer set is defined as:

$$\mathscr{M}(\underline{\rho}) = \{\underline{\sigma}_i | \underline{\rho} = \mathbf{L}\underline{\sigma}_i\} \tag{13}$$

that is, a set of linear model weights $\underline{\sigma}_i$ that induce identical response $\underline{\rho}$, such that all $\underline{\sigma}_i$ correspond to surface spectral reflectances that are "natural".

A reflectance is natural if and only if it can be written as a convex combination[4] of a set of representative, real reflectances from a training set. In other words, all reflectances in the metamer sets are within a convex polyhedron described by the extreme vertices of the set of representative reflectances[8, 10].

In order to chose a single representative metamer we assume a probability distribution of reflectances within the metamer set and choose the most likely reflectance. Let us denote the probability of $\underline{\sigma}$ as $P(\underline{\sigma})$ and conditional probability, that is the probability of $\underline{\sigma}$ given $\underline{\rho}$, as $P(\underline{\sigma}|\underline{\rho})$ then the maximum likelihood estimate is found by solving:

$$\max_{\forall \underline{\sigma}_i \in \mathscr{M}(\underline{\rho})} P(\underline{\sigma}_i | \underline{\rho}) \tag{14}$$

that is, maximise the probability of $\underline{\sigma}$ given the known response vector $\underline{\rho}$. We assume a truncated normal distribution, which has been successfully employed before[9, 33].

Other Methods

The methods described above are by no means a complete and exhaustive list. There are other known approaches to estimating reflectances from device responses. In what follows a brief reference is given to some. However, many of these approaches start from different premises, solving more specific or more general versions of the present problem, or have different objectives such as colour correction.

van Trigt's "smoothest reflectance" estimates[28] (and their discrete optimisation solution by Li and Luo[16]) are an elegant way to analytically solve for the reflectance that is the smoothest possible function. While reflectances are smoothly varying functions, they need not necessarily be the smoothest possible, hence this method has been found to give high estimation error[21]. The Hawkyard method [12, 2] can estimate reflectances to a high accuracy in terms of CIE L*a*b* ΔE for a single or small number of illuminants, however results in high estimation error for illuminants not used to derive the reflectances[5]. Shi and Healey[26] presented a method that, similarly to the Metamer Set based methods described above, employs higher dimensional linear models for mapping device responses to device independent CIE XYZs, while estimating surface spectral reflectance as an intermediate step. This method performs very well if training and testing on the same data set however gives considerably worse results otherwise. Dupont[5] presents and compares other numerical optimisation techniques such as simulated annealing, linear programming and genetic algorithms, from the point of view of achieving accurate colorimetric matches and minimise metamerism.

Error Measures

Let $\hat{\mathbf{r}}$ denote the estimated surface spectral reflectance and $\mathbf{r}$ the original, sought spectral reflectance. The estimation accuracy is evaluated in terms of spectral error.

Spectral error, denoted henceforth Δ, is the L2 Euclidean distance between $\hat{\mathbf{r}}$ and $\mathbf{r}$:

$$\Delta = \parallel \hat{\mathbf{r}} - \mathbf{r} \parallel = \sqrt{\sum_i (\hat{\mathbf{r}}_i - \mathbf{r}_i)^2} \tag{15}$$

[4] A convex combination is a weighted linear combination with weights summing to 1 while being non-negative and less than 1.

There are several other ways to express the accuracy of spectral matches. The aim in this investigation however is general accuracy, irrespective of viewing conditions such as observers or illuminants. Hence the dominant measure considered here is the spectral error Δ. This has been found to be a good choice given the objective [29].

In addition however, CIE L*a*b* ΔE colour differences are also reported, as an approximate, informative measure. Note however that the correlation between the two error measures is weak (the correlation coefficient between Δ and CIE ΔE for 16688 pairs of estimate and original reflectances is $R = 0.7250$), due to the non-linear nature of the transform from reflectance to CIE L*a*b*.

Finally, feasibility, denoted $\mathscr{F}$, is also examined. This is a simple test as to whether the estimated reflectance can be considered indeed a feasible reflectance and thus has no negatives, nor values above 1. In the results section, $\mathscr{F}$ is given as a %–value, referring to the proportion of estimates that are feasible reflectances.

Experimental Set-up

The spectral estimation algorithms are tested on synthetic data. The set of surface spectral reflectances used as the training and testing sets are made up of the following data sets: the Macbeth Colorchecker Chart (24)[20], Munsell paper chips (1269)[23], the Vrhel and Gershon's Object set (170)[30], Krinov's Natural set (219)[15] and Westland et al.'s set (404)[31]. In total, the surface spectral reflectances used in the experiments amount to a set of 2086 samples.

Three training and testing strategies are reported. In the first experiment training is performed on the 24 Macbeth ColorChecker Chart surfaces, while testing on all 2086 samples. This training set is a small and not necessarily representative set that, however, is a common standard chart used for calibration and characterisation. In the second experiment training is performed on a random half (1043) of all reflectances from the set and tested on the remaining 1043. Finally, we also report results using the entire set of 2086 reflectances for training and testing. While this is the least robust test, it shows ideal performance of the algorithms.

The training sets were used for determining the linear model basis vector matrix **B**, the linear least squares (constrained and unconstrained) mapping matrices **T** for methods LSQ, LSQT and LSQW, and the covariance matrix for LSQW.

Each algorithm under each of the training and testing conditions was evaluated using the 6B–HDTV sensor set as well as a set of spectral sensitivities of a standard 3 channel RGB digital still camera for comparison (see Fig. 1). An equi–energy illuminant was used as the scene illuminant.

To establish the correct linear model dimension the 2086 surface spectral reflectances were examined in terms of the error arising from a representation using a linear model of a particular dimension. The error was evaluated using both the Δ and the ΔE measures. A linear model dimension of 8 was found to be sufficiently high for the given purposes as it resulted in a negligible average and acceptable maximum estimation error in terms of both spectral error Δ and perceptual error ΔE.

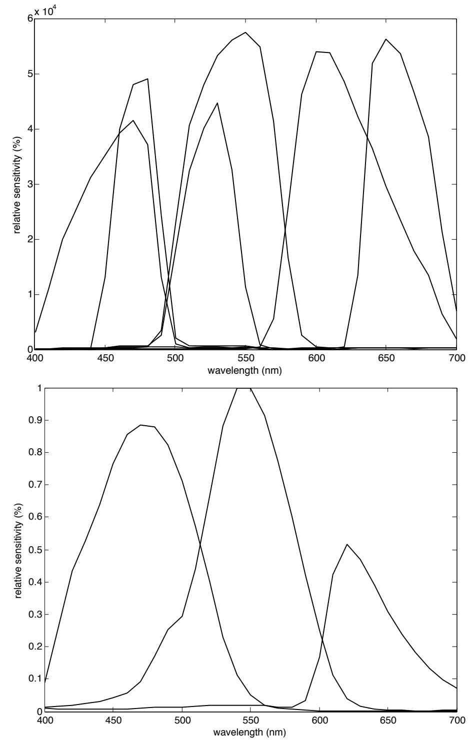

Figure 1. HDTV sensitivities (top) and SONY RGB sensitivities (bottom).

Results and Discussion

In Tables 1, 2 and 3 the results of the reflectance estimation experiments, as described above, in terms of all three error measures is reported. The smallest errors are shown in bold–face. In Table 1, all algorithms used the 24 Macbeth ColorChecker Chart reflectances as their training set, while testing on the entire set of 2086 samples. In Table 2, algorithms used a random half of the 2086 samples while testing on the other half and in Table 3 the entire set was used to train and test the algorithms.

The mean and maximum estimation error summary statistics are commonly reported in reflectance estimation literature and hence are provided here as well. However, performing the Jarque–Bera test[1] for goodness–of–fit to a normal distribution on the error distributions from all tested algorithms shows that on a 99% significance level ($\alpha = 0.01$) we can reject the hypothesis of a normal distribution. Therefore we also conducted another statistical test of the error distributions in order to establish with whether there is significance in the differences in performance of the algorithms.

We employed the two–sample Kolmogorov–Smirnov (K–S) test [14, 13] to compare distributions of two sets of estimation errors and establish whether or not they are significantly different. The K–S test is a non-parametric test making no assumption about the nature of the underlying error distribution and we test for alternative hypotheses determining whether the cumulative distribution functions likely to correspond to the samples from the error distributions of different algorithms are *smaller* or *larger* or simply *unequal*.

In Table 4 we show the results of the K–S test for $\alpha = 0.01$

	6B–HDTV				
	$\mu\Delta$	max Δ	$\mu\Delta E$	max ΔE	$\mathscr{F}$ (%)
LSQ	0.48	1.65	3.25	24.71	33.4
LSQW	0.13	0.88	0.75	9.15	95.0
LSQT	**0.10**	**0.80**	**0.57**	6.16	98.4
MSML	**0.10**	0.82	0.58	**5.96**	**100.0**
	RGB				
	$\mu\Delta$	max Δ	$\mu\Delta E$	max ΔE	$\mathscr{F}$ (%)
LSQ	0.67	2.06	7.94	79.71	88.6
LSQW	0.21	0.97	4.52	62.08	87.3
LSQT	0.18	0.94	4.43	51.96	95.3
MSML	**0.18**	**0.91**	**4.33**	**25.46**	**100.0**

Training: 24 Macbeths; Testing: all 2086 reflectances; Sensors: 6B–HDTV device (top part), 3 channel RGB device (bottom part).

	6B–HDTV				
	$\mu\Delta$	max Δ	$\mu\Delta E$	max ΔE	$\mathscr{F}$ (%)
LSQ	0.45	1.57	1.36	5.58	0.0
LSQW	0.11	0.58	0.55	5.21	96.9
LSQT	**0.09**	**0.52**	0.48	4.67	97.9
MSML	**0.09**	**0.52**	**0.46**	**3.48**	**100.0**
	RGB				
	$\mu\Delta$	max Δ	$\mu\Delta E$	max ΔE	$\mathscr{F}$ (%)
LSQ	0.66	2.05	8.15	90.02	38.6
LSQW	0.19	0.97	4.40	41.70	94.4
LSQT	0.18	0.92	4.36	43.31	95.1
MSML	**0.16**	**0.82**	**3.90**	**25.42**	**100.0**

Training: 1043 reflectances; Testing: remaining 1043 reflectances; Sensors: 6B–HDTV device (top part), 3 channel RGB device (bottom part).

	6B–HDTV				
	$\mu\Delta$	max Δ	$\mu\Delta E$	max ΔE	$\mathscr{F}$ (%)
LSQ	0.44	1.6	1.29	5.15	0.0
LSQW	0.10	0.78	0.56	10.29	96.8
LSQT	**0.09**	**0.72**	0.49	**6.9**	97.6
MSML	**0.09**	0.74	**0.45**	9.63	**100.0**
	RGB				
	$\mu\Delta$	max Δ	$\mu\Delta E$	max ΔE	$\mathscr{F}$ (%)
LSQ	0.66	2.05	8.16	91.24	43.2
LSQW	0.19	0.97	4.43	43.45	94.9
LSQT	0.17	0.92	4.39	45.19	95.5
MSML	**0.16**	**0.87**	**3.96**	**25.45**	**100.0**

Training: all 2086 reflectances; Testing: all 2086 reflectances; Sensors: 6B–HDTV device (top part), 3 channel RGB device (bottom part).

	LSQ	LSQW	LSQT	MSML
LSQ		-	-	-
LSQW	+		-	-
LSQT	+	+		-
MSML	+	+	+	

Kolmogorov–Smirnov two sample test of reflectance estimation algorithms. A $-/+$ in row i and column j means that algorithm i is significantly worse/better, in terms of the Δ metric compared to algorithm j.

(i.e. at a 99% significance level) for the case of training on the 24 patches of the Macbeth ColorChecker chart against testing on the entire set of 2086 reflectances – corresponding to the results shown in Table 1.

The K–S test shows that the Metamer Set Maximum Likelihood method is best, followed by Tikhonov Regularised LSQ, Wiener LSQ and finally simple LSQ. The results in terms of mean and maximum Δ (and ΔE) in Table 1 for MSML and LSQT are very similar, however the K–S test shows statistically significant superiority of the Metamer Set method. The K–S test for the other two training–testing set-ups is also similar, although for the ideal case, when training and testing is performed on the same data, LSQT gives better results. This can be explained by the fact that the α parameter in this case is optimised such that it results in smallest possible error for the testing set.

Our results are in line with previous findings, that simple, unconstrained linear least squares does not work well. This approach gives a high maximum error and on average is the worst performing method. Constrained least squares methods give significantly better results - both the Wiener and the Tikhonov regularisation approach reduce maximum error and on average work very well, with Tikhonov resulting in smaller estimation error and approaching the performance of the Metamer Set Maximum Likelihood approach.

A Δ of 0.1 and a ΔE of 0.5, common performances for both the constrained least–squares methods as well as the metamer set maximum likelihood method, are very small errors and mean very high accuracy. The mean errors for the 3 channel RGB camera are, in comparison, almost twice higher in the reflectance domain and significantly above the just–noticeable 1 ΔE threshold. Thus, given a 6 channel device and using an appropriate estimation technique we are able to estimate reflectance very well. What also needs to be considered however, is the feasibility of the estimates – whether the estimated reflectances are indeed physically possible surface spectral reflectances.

None of the constrained or unconstrained least squares methods guarantee physical realisability – it is possible that the resulting estimate has negatives or values above 1. The feasibility measure $\mathscr{F}$ in some cases is below 40%, while constrained leas squares methods LSQW and LSQT have a feasibility of over 90%. The Metamer Set method instead is by design such that 100% of estimates are physically possible reflectances.

Other factors will influence the choice of reflectance estimation algorithm for multi–spectral imaging, such as computational expense and accuracy requirements. While for fine–art, high–quality reproductions the Metamer Set based method seems to be of most interest, it may be less appropriate in the case of live

High Definition Video or tele–medicine where speed is important. There is a significant difference between the two methods with highest accuracy. Metamer Set computation involves the calculation of convex hulls[25], and the subsequent Maximum Likelihood choice requires the solution of a Quadratic Programming problem. Compared to the simple matrix multiplication by which LSQT can be implemented, this is a considerable overhead. Thus depending on the application and the requirement for accuracy, either of the two methods can be appropriate.

A final and important aspect not considered here is noise. Good results have been achieved with the Metamer Set Maximum Likelihood method for 3 channel RGB cameras[21]. However, given the overall higher accuracy of estimation in the multi–spectral case, this needs further investigation.

Conclusions

In this paper the problem of estimating surface spectral reflectances from multi-spectral device responses was addressed. Four approaches have been compared in terms of spectral reconstruction accuracy: unconstrained linear least squares, Wiener linear least squares and Tikhonov regularised linear least squares, as well as the Metmer Set based Maximum Likelihood method.

The experiments conducted in this paper have shown that there is significant benefit in introducing regularisation constraints to the simple least squares approach and that there is further benefit in accuracy and feasibility in using the Metamer Set based estimation technique.

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