Can Highly Chromatic Stimuli Have A Low Color Inconstancy Index?

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Abstract
Both surface color and imaging industries strive for minimum color inconstancy in the materials they produce, while trying to match as many colors as possible. In this context the aim of this paper is to look at the relationship between color inconstancy properties of surface colors and their relationship to their chromas - specifically with the intention of understanding whether the two are opposed. Following the generation of spectra that match a sampling of the gamut of all possible surface colors it is shown clearly that the two properties are indeed inter-related and that highly chromatic colors cannot be color constant. Finally, the results of this work are proposed as a basis for setting color inconstancy targets and for having realistic expectations in terms of this property.

Introduction
An important aim in many color and imaging industries is to produce materials or images that maintain their appearances regardless of the viewing conditions under which they are viewed. In the surface color (e.g. textile, plastics) industries the produced materials have carefully chosen colors and any change to their appearance can significantly change their desirability or utility. In imaging, the value of a printed or displayed output's constant appearance is equally great as images are typically tuned under one set of viewing conditions, but viewed also under a myriad of others. The appearance that has been achieved by a creative professional under the viewing conditions they used is what needs to be transmitted and changes to it are not desirable. In fact there is existing research that aims to minimize the color inconstancy of a color separation[1,2] as well as a hypothesis that it cannot be kept at zero for high–chroma colors postulated by Berns *et al.*[2]

Another important aim in developing color imaging systems is to allow them to reproduce as wide a range of colors as possible. In other words, it is important to give access to the largest possible color gamut, which in turn entails using primaries (e.g. inks, phosphors, filters) that are as chromatic as possible. Hence, the ideal imaging system would be one that, among many other features, would allow for the generation of colors ranging from neutral to highly chromatic and looking the same under a wide range of viewing conditions.

Put more specifically, the aim is to be able to generate highly chromatic colors that have a low Color Inconstancy Index (CII). The CII of a surface can be expressed, for example using the CMCCON02[4] metric, where a sample's tristimulus values are obtained for a test illuminant (or light source), transformed using a chromatic adaptation transform to a reference illuminant and color difference is computed between these tristimulus values and the ones obtained directly for the sample under the reference illuminant (Figure 1). These color difference values express the degree to which the sample's color changes between the pair of test and reference illuminants.

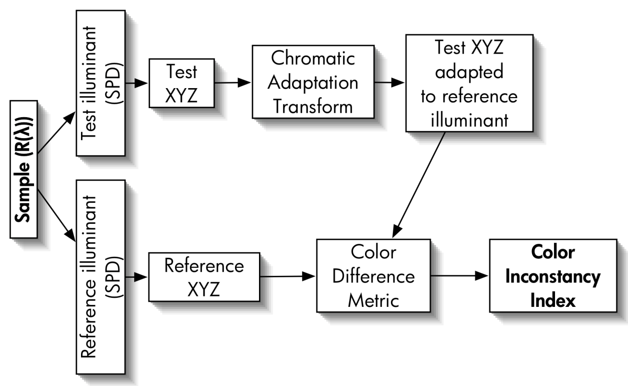

Figure 1. Color Inconstancy Index computation workflow

What will be presented in the remainder of this paper is an analysis of surface spectra in terms of the relationship between their chromas and color inconstancy indices. The aim of the analysis is to make explicit the link between these two properties and the constraints it places on what outputs imaging systems can have.

Experimental Framework
To look at the relationship of a sample's chroma under reference conditions and the CII values it could have if viewed under a test illuminant, the concept of CII potential will be introduced here.

CII potential will refer to the range of CII values that are obtained for a sampling of the spectra that under a reference illuminant have a given set of tristimulus values. In other words, for a set of tristimulus values (that also correspond to certain lightness, chroma and hue angle values) under a reference illuminant, there is set of metamers that correspond to it. For a test illuminant, each of the metameric spectra has a specific CII and their minimum and maximum values are then the CII potential of the given XYZ values for the pair of reference and test illuminants (Figure 2).

What this concept allows is to take any possible surface XYZ and express the minimum and maximum CIIs possible at its location. In other words, it is possible to express the greatest and least degree of color inconstancy that can be had at a given location in color space. If then CII potential varies with chroma it will be possible to see what impact the choice of a more chromatic primary is likely to have on the color inconstancy of an imaging system that uses it.

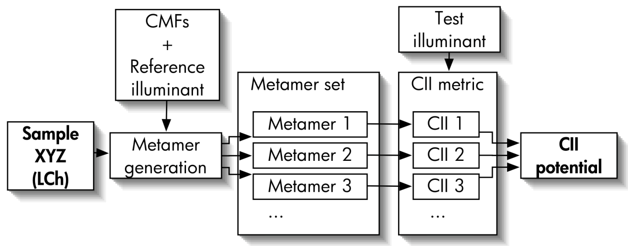

Figure 2. CII potential computation.

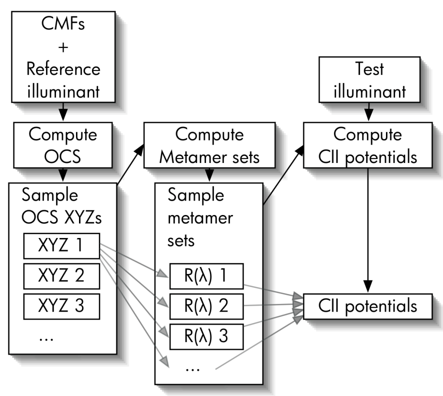

Figure 3. CII versus chroma computational framework.

To compute CII potential for a given test illuminant throughout the gamut of surface XYZs possible under a given reference illuminant, the following stages are required (Figure 3):

1. A means of determining the gamut of all possible surface XYZs (i.e. the Object Color Solid – OCS[5]).
2. A sampling of the OCS.
3. For each XYZ sampled from the OCS, a means of delimiting the set of all metameric spectra that have the given XYZ.
4. A sampling of the spectra from each set of the metamer sets.
5. The computation of CII potential for each metamer set (and therefore for each sampled XYZ).

Next, details will be provided of the individual stages:

Computing And Sampling The OCS

The set of all possible surface reflectances – the Object Color Solid – is bounded and determined by reflectance spectra that are bounded themselves in each wavelength interval: in terms of their minima by zero and in terms of their maxima by 100% (or some other finite value if fluorescence were to be considered). To compute the gamut boundary of the colors corresponding to all possible surface spectra, an approach outlined previously[6] will be used. In summary the approach is to first compute the gamut of an exhaustive sampling of spectra that at each wavelength interval have either zero or 100% reflectance and then to scale the XYZs of the resulting gamut boundary points arbitrarily between zero and 100%. The result of this computation in CIELAB and for D65 as the reference illuminant is shown in Figure 4.

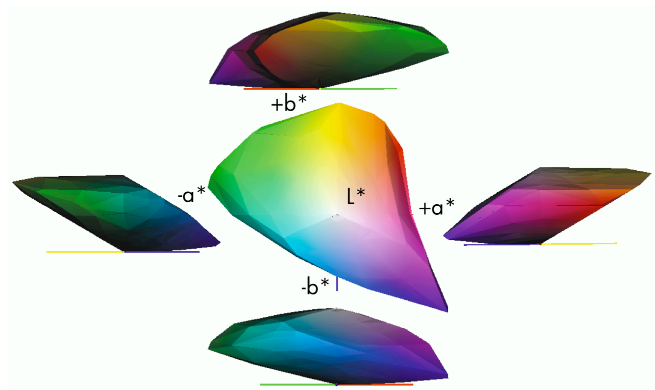

Figure 4. The Object Color Solid in CIELAB (D65, 2° observer – (half)axes are 100 units long).

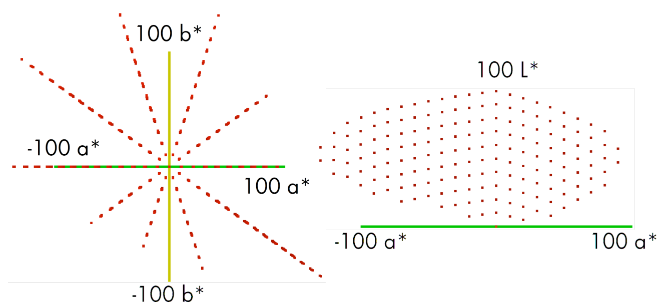

Figure 5. Sampling of OCS.

Given the OCS' gamut boundary, CIELAB samples will be generated by first evenly dividing hue angle and at each hue angle then evenly dividing the available chroma range. At each resulting hue and chroma combination, the available lightness range is then divided so as to provide samples that along constant hue and chroma lines have the same distances throughout the space (Figure 5). The reason for arranging the sampling in this way is to then allow for a direct evaluation of CII potential as a function of chroma.

Computing And Sampling Metamer Sets

The computation of metamer sets, given an LAB or XYZ vector has also been introduced previously.[7,6] In summary the approach consists of the following:

Estimating a spectral reflectance $R(\lambda)$ that corresponds to a given set of XYZ values requires inverting the color formation equations (i.e. the equations that show how XYZs are computed from reflectance, illuminant SPD and observer color matching functions). However, given that reflectance is a continuous function, while the set of XYZs consists of three values only, the inverse of color formation is in general an underdetermined set of linear equations and as such has potentially an infinite set of solutions, which also implies the phenomenon of metamerism.

The first step to computing the set of spectral reflectances that result in a given XYZ is to represent them in terms of a linear basis obtained by applying characteristic vector analysis[8] to measured spectra. R(λ) can then be written as a weighted linear sum $R(\lambda) = B_1(\lambda)w_1 + B_2(\lambda)w_2 + \ldots + B_d(\lambda)w_d$, where d is the dimension of the linear model and is significantly smaller then the number of spectral samples used for R(λ). Hence it is no longer necessary to solve for the n dimensional R(λ) values but only for the d linear coefficients $[w_1, w_2, \ldots w_d]$ that uniquely define the reflectance within the known linear basis $[B_1(\lambda), B_2(\lambda), \ldots, B_d(\lambda)]$. In general, solving such under-determined linear systems results in an convex, infinite and unbounded set of solutions. However this set needs to be constrained to contain only solutions corresponding to meaningful spectral reflectances.

In the context of matte surfaces uniformly illuminated with a diffuse light source, spectral reflectances are at all wavelengths less than or equal to 100% (no more than all light is reflected) and more than or equal to 0% (no less than no light is reflected). This restriction, also referred to as physical realizability, can be formulated as a linear inequality in terms of spectral reflectance: $0 \leq R(\lambda) \leq 1$ and bounds the set of solutions to an infinite, closed and convex set made up only of physically possible, metameric spectral reflectances. Furthermore we constrain the linear model weights to lie between the minimum and maximum of the weights corresponding to some real, measured surface reflectance spectra. Physical realizability coupled with this "box" constraint define what we consider meaningful surface reflectances.

Once the metamer set corresponding to a given XYZ is computed, it is sampled in terms of the linear coefficients of the metameric blacks. Here the vertices of the convex hull in the linear coefficient space were used as samples from the metamer set. The reason for making this choice is that these vertices represent the spectra that have extreme amounts of contribution from the metameric blacks used (i.e. both minimal and maximal per dimension and their combinations) while giving spectra that match the desired LAB values.

Note also that while the above procedure will be used for the vast majority of OCS samples, the cusps (where the cusp at a given hue is the most chromatic colors there) cannot be matched using the above linear model. Instead spectra obtained during the OCS gamut computation will be used for them. Also note that for OCS cusp spectra there is no room for metamerism since they have either 100% or 0% reflectance at each wavelength and adding a metameric black to them would mean that this would always need to be scaled by zero – to satisfy physical realizability. Hence OCS cusp LABs will have CII potentials equal to the single spectra's CII that can match them.

Computing CII Potential

Finally, the CIIs of samples from each of the metamer sets computed for the OCS samples are computed using the CMCCON02[4] metric and the CIEDE2000 color difference equation[9] and the CII potential of the set (and therefore corresponding OCS sample) is determined as the minimum and maximum values of the CII distribution.

Experimental Setup

Table 1, shows details of the experiment whose results will be presented in the following section.

Table 1. Details of experimental setup.

Parameter	Value
Spectral sampling	400 to 700 nm at 10 nm intervals
Reference illuminant	D65
Test illuminant	A
Observer	2º
Luminance of adapting field	L_A=60 cd/m^2
Degree of adaptation	complete (D=1)
Surround conditions	average (F=1)
OCS samples	749 (10 hues, L and C spacing – 10 CIELAB units)
Metamer computation[7]	12D basis with physical realizability and "box" constraints
Metamer set samples per set	Between 2 and 36410 with a median of 363 (i.e. number of metamer set vertices in linear coefficient space)
CII metric	CMCCON02[4] (using CIEDE2000 color difference equations)

Results

Taking the 749 member sampling of the OCS in CIELAB, taking the vertices of the metamer sets of each of these samples that match it under D65 and computing the CII potential of each set of spectra resulted in the data that will be looked at here. In Figure 6 we see a summary of the CII potentials for the ten hues considered here, plotted as a function of CIELAB C*. As can be seen there is an important relationship between the CII potential (i.e. the minimum and maximum possible CIIs) of a color and its chroma. Namely as chroma increases the minimum possible CII increases, confirming the hypothesis of Berns *et al.*[3] Looking at maximum CII as function of chroma presents a less clear relationship and one that could be studied more closely in the future. However, as it is CII minima that are of more importance here, the above results do lead to clear conclusions.

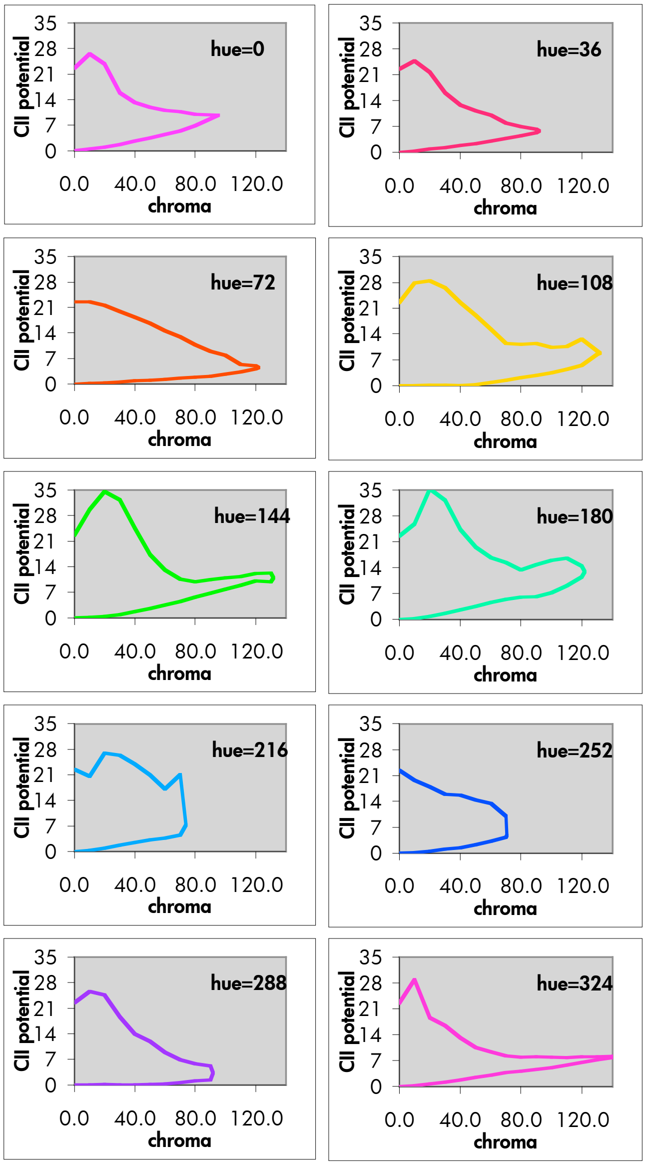

Figure 6. CII potential as function of CIELAB C*.

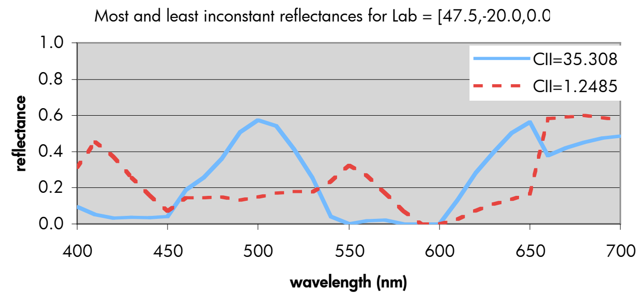

Figure 7: Spectra with smallest and largest CIIs matching the same LABs under D65.

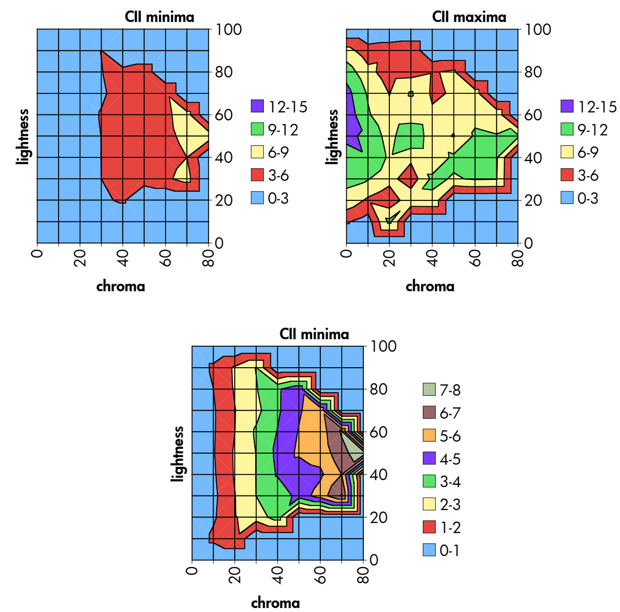

Figure 8: CII potential across h^*_{ab}=0 hue page. Top: minima and maxima on 0–15 scale, bottom: higher resolution view of minima.

To illustrate the above results, Figure 7 shows the most and least color inconstant spectra for the LAB sample that lead to the largest CII value in this data set.

To take a closer look at the above results it is also of value to see the CII potentials of an entire hue plane rather than summarized as a function of chroma. Figure 8 Therefore shows minimum and maximum CIIs at the locations in the h^*_{ab}=0 hue plane where nonempty metamer sets were computed. As can be seen, the minimum values are pretty much a function only of chroma, whereas the maximum ones also vary significantly with lightness. In particular, maximum CII values are greater around the middle of the lightness range and drop off as the OCS gamut boundary is approached.

Conclusions

As has become apparent, the simple answer to the question posed in the title of this paper: "Can highly chromatic stimuli have a low color inconstancy index?" is a clear "No." The implications especially of CII minima increasing as a function of chroma are important in that they set theoretical lower limits to the CIIs that can be aimed for, given a color of certain chroma. For example, it is not possible to aim for a CII of zero when formulating a colorant with a chroma of C*=70. In that case, the best that can be hoped for are values of around one for purples and around seven for cyans, depending on the color's hue angle. Looking at CII potentials for the 10 hues considered here we can also see, for example, that one is more likely to succeed getting a blue colorant with lower color inconstancy than a green one.

In summary CII potentials, such as those shown in Figure 6, can be used both for setting achievable targets and having realistic expectations when formulating new colorants and when evaluating the color inconstancy properties of entire imaging systems, by relating CII targets to CII potentials in different parts of color space.

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Author Biographies

Ján Morovič received a Ph.D. in color science at the Colour & Imaging Institute (CII) of the University of Derby (UK) in 1998, where the title of his thesis was To Develop a Universal Gamut Mapping Algorithm. He then worked as a lecturer in digital color reproduction at the CII and is currently a senior color scientist at Hewlett-Packard in Barcelona, Spain. He is also the chairman of the CIE's Technical Committee 8–03 on Gamut Mapping.

Peter Morovič received a Ph.D. in computing sciences at the School of Computing Sciences, University of East Anglia in Norwich (UK) in 2002, where the title of his thesis was Metamer Sets. He is currently a senior research associate at the School of Computing Sciences, University of East Anglia in Norwich (UK)..