Vectorial Computation of the Optical Flow in Color Image Sequences

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Abstract

Actually in most applications, optical flow is computed from one luminance Y-plane and only a few methods refer to color optical flow. In fact, a brief analysis shows that these methods are either marginal approaches, whether dramatically time consuming techniques. Here, we propose a vectorial approach based upon a joint analysis of a structure tensor and a so called flow tensor, both computed from image derivatives.

Introduction

In this paper we address a still open problem, an accurate recovery of the optical flow from color image sequences. Currently in most applications, optical flow is computed from one luminance Y-plane and only a few methods refer to color optical flow, explicitly taking into account the color spaces. In fact, most works on color optical flow estimation are based on the extension of the Brightness Change Constraint Equation (BCCE)[1]:

$$\partial_1 I\, u + \partial_2 I\, v + \partial_3 I = 0 \tag{1}$$

to multi-band images. A color image sequence is denoted $\mathbf{I} : x = (x_i)_{i=1..3} \rightarrow (I_j(x))_{j=1..3}$ where x_1 and x_2 are the spatial coordinates, x_3 the temporal one and ∂_i stands for $\partial/\partial x_i$. So, the extension consists in applying the previous equation (1) to each I_j band[2] and the color optical flow is a vector $\mathrm{v}^T = (u\ v)$ such as:

$$\begin{cases} \partial_1 I_1\, u + \partial_2 I_1\, v + \partial_3 I_1 = 0 \\ \partial_1 I_2\, u + \partial_2 I_2\, v + \partial_3 I_2 = 0 \\ \partial_1 I_3\, u + \partial_2 I_3\, v + \partial_3 I_3 = 0 \end{cases} \tag{2}$$

Obviously, this system is over-determinate and three main strategies appear:

- Selecting two independent equations from the system in order to obtain a direct estimation of v using Gaussian elimination.[2,3] In Ref. [4], the authors assume that the quantities representing color under constant illumination are invariant and can be used for motion estimation. Consequently, two equations can be selected from the two chromatic bands of the HSV,[4,5] YUV, UCS, YCbCr... color spaces.
- Solving the system as it is, using either least squares or pseudo-inverse methods.[2,4-6] This is in fact a direct extension of the Lucas and Kanade work.[7]
- Separately computing the optical flow on each band using traditional grayscale techniques and then fuse the results to recover one vector field. For example, in Ref. [8] Andrews and Lovell simply select the estimated vector with the smallest intrinsic error at each point.

A brief analysis of system (2) shows that any color optical flow vector v must also be a potential optical flow vector for each scalar band. In that way, we can say that the proposed methods are quasi-marginal approaches. Here, we will compute a color optical flow where the $\mathbf{v}^T = (u\ v\ 1)$ vector have only to satisfy:

$$\mathbf{v}^T\, \nabla \mathbf{I} = 0 \tag{3}$$

where $\nabla\mathbf{I}$ is the multi-band color gradient direction of the vectorial image $\mathbf{I}$. This vectorial approach is based upon a joint analysis of a structure tensor and a so called flow tensor, both computed from image derivatives.

The paper will be organized as follows: in the first part, we briefly present the methods using a structure tensor for retrieving the optical flow in gray-level images; in the second part, we give some simple but useful properties concerning tensorial calculus; in the third part, we first point out the main drawback for a direct extension to color spaces, and then we present our solution using the properties given in part two; finally, in the fourth part, we discuss about results and performances.

Optical Flow and Structure Tensor for Scalar Images

As seen in the introduction, we consider a gray-level image sequence I and we have a first order differential characteristic given by the gradient vector $\nabla I^T = (\partial I_1\ \partial I_2\ \partial I_3)$. This vector can generate[9] a structure tensor $\boldsymbol{S}$ given by:

$$\boldsymbol{S} = \nabla I\ \nabla I^T = \begin{pmatrix} \partial_1 I^2 & \partial_1 I\,\partial_2 I & \partial_1 I\,\partial_3 I \\ \partial_1 I\,\partial_2 I & \partial_2 I^2 & \partial_2 I\,\partial_3 I \\ \partial_1 I\,\partial_3 I & \partial_2 I\,\partial_3 I & \partial_3 I^2 \end{pmatrix} \tag{4}$$

The spectral elements of $\boldsymbol{S}$ are for the eigenvalues:

$$\beta_1^{(\mathbf{S})} = \partial_1 I^2 + \partial_2 I^2 + \partial_3 I^2, \quad \beta_2^{(\mathbf{S})} = \beta_3^{(\mathbf{S})} = 0 \tag{5}$$

Concerning the eigenvectors, the one associated to $\beta_1^{(\mathbf{S})}$ is the gradient vector $\mathbf{V}_1^{(\mathbf{S})} = \nabla I$. Furthermore, the subspace generated by the two others eigenvectors $\mathbf{V}_2^{(\mathbf{S})}$ and $\mathbf{V}_3^{(\mathbf{S})}$, associated to the null eigenvalues, is orthogonal to ∇I. So, any vector belonging to the kernel of $\boldsymbol{S}$ is a possible solution of the BCCE equation (1). In fact, $\mathbf{V}_2^{(\mathbf{S})}$ and $\mathbf{V}_3^{(\mathbf{S})}$ can be chosen such as the three eigenvectors form a direct orthogonal basis:

$$\mathbf{V}_1^{(\boldsymbol{S})} = \begin{pmatrix} \partial_1 I \\ \partial_2 I \\ \partial_3 I \end{pmatrix}, \quad \mathbf{V}_2^{(\boldsymbol{S})} = \begin{pmatrix} \partial_2 I \\ -\partial_1 I \\ 0 \end{pmatrix}, \quad \mathbf{V}_3^{(\boldsymbol{S})} = \begin{pmatrix} \partial_1 I\, \partial_3 I \\ \partial_2 I\, \partial_3 I \\ -(\partial_1 I^2 + \partial_2 I^2) \end{pmatrix} \tag{6}$$

As shown in Ref. [10], $\mathbf{V}_3^{(\boldsymbol{S})}$ can then be taken as a local representative of the optical flow. Of course, in this purely local approach, the optical flow does not solve the aperture problem meets with the motion estimation. That is why Lauze and al.[11] propose to combine tensorial optical flow and diffusion processes when they want to estimate the apparent motion from image sequences. However we have to say that, in the present article, we will only focus on the optical flow extraction, and reserve the motion estimation question to the future.

Some Properties of Tensors

To simplify the presentation as a whole and in order to directly use them in the following, we now give a general spectral property and reminder about the quadratic forms.

Spectral Directions Stability

Let us consider a vector $\mathbf{U}^T = (a\ b\ c)$ and the tensor $\boldsymbol{T} = \mathbf{U}\,\mathbf{U}^T$. The eigenvalues of $\boldsymbol{T}$ are $\beta_1 = a^2 + b^2 + c^2$, $\beta_2 = \beta_3 = 0$ with respectively the eigenvectors

$$\mathbf{V}_1 = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \quad \mathbf{V}_2 = \begin{pmatrix} b \\ -a \\ 0 \end{pmatrix}, \quad \mathbf{V}_3 = \begin{pmatrix} ac \\ bc \\ -(a^2+b^2) \end{pmatrix}$$

forming a direct orthogonal basis, cf (6).

Obviously, we remark that $\boldsymbol{T}_1 = \mathbf{V}_1\,\mathbf{V}_1^T$ is nothing but the original tensor $\boldsymbol{T}$, so the eigenvectors of $\boldsymbol{T}_1$ will be $\mathbf{V}_1^{(\boldsymbol{T}1)} = \mathrm{U} = \mathbf{V}_1$, $\mathbf{V}_2^{(\boldsymbol{T}1)} = \mathbf{V}_2$ and $\mathbf{V}_3^{(\boldsymbol{T}1)} = \mathbf{V}_3$. In a same way, we have $\boldsymbol{T}_3 = \mathbf{V}_3\,\mathbf{V}_3^T$, whose eigenvalues are $\beta_1^{(\boldsymbol{T}3)} = (a^2 + b^2)(a^2 + b^2 + c^2)$ and $\beta_2^{(\boldsymbol{T}3)} = \beta_3^{(\boldsymbol{T}3)} = 0$ with the eigenvectors

$$\mathbf{V}_1^{(\boldsymbol{T}3)} = \begin{pmatrix} ac \\ bc \\ -(a^2+b^2) \end{pmatrix}, \quad \mathbf{V}_2^{(\boldsymbol{T}3)} = \begin{pmatrix} b \\ -a \\ 0 \end{pmatrix}, \quad \mathbf{V}_3^{(\boldsymbol{T}3)} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

As we have

$$\mathbf{V}_1^{(\boldsymbol{T}1)} = \mathbf{V}_3^{(\boldsymbol{T}3)}, \quad \mathbf{V}_2^{(\boldsymbol{T}1)} = \mathbf{V}_2^{(\boldsymbol{T}3)} =, \quad \mathbf{V}_3^{(\boldsymbol{T}1)} = \mathbf{V}_1^{(\boldsymbol{T}3)} \tag{7}$$

we call this property the spectral direction stability of the $\boldsymbol{T}$ tensor.

Vector Projection and Quadratic Form

Let us consider a vector $\mathbf{U}_j^T = (a_j\ b_j\ c_j)$ and the tensor $\boldsymbol{Q}_j = \mathbf{U}_j\,\mathbf{U}_j^T$. Otherwise, we consider the matrix element $\mathbf{q} = (\mathbf{U}_1 \ldots \mathbf{U}_n)$ which we associate the $\boldsymbol{Q} = \mathbf{q}\,\mathbf{q}^T$ tensor verifying $\boldsymbol{Q} = \sum_j \mathbf{U}_j\,\mathbf{U}_j^T$ or $\boldsymbol{Q} = \sum_j \boldsymbol{Q}_j$ where $\boldsymbol{Q}_j$ is the tensor associated to the $\mathbf{U}_j$ vector. Now, let $\mathbf{V}$ be some vector and if we evaluate the product $\boldsymbol{Q}\mathbf{V}$, we find the relation

$$\boldsymbol{Q}\mathbf{V} = \sum_{j=1}^{n} (\mathbf{U}_j^T\,\mathbf{V})\,\mathbf{U}_j \tag{8}$$

that is the right projection of a vector V relative to the matrix element $\mathbf{q}$, or to the set of vectors $(\mathbf{U}_1 \ldots \mathbf{U}_n)$. In a similar way, we can form $\mathbf{V}^T\boldsymbol{Q}\mathbf{V}$ that gives the expression $\sum_j (\mathbf{U}_j^T\mathbf{V})\mathbf{V}^T\mathbf{U}_j$. This leads to a new relation

$$\mathbf{V}^T\boldsymbol{Q}\mathbf{V} = \sum_{j=1}^{n} (\mathbf{U}_j^T\,\mathbf{V})^2 \tag{9}$$

that is the quadratic form applied to a vector $\mathbf{V}$ and defined by the matrix element $\mathbf{q}$, or to the set of vectors $(\mathbf{U}_1 \ldots \mathbf{U}_n)$.

Vectorial Computation of the Color Optical Flow

Our aim is the extraction of a vectorial field representative of the apparent displacement in a color image sequence. We want to explicitly take into account the intrinsic characteristics of the color spaces and that for, we avoid any kind of marginal approach in the computation of the optical flow. So, we consider that a color sequence is a vectorial function denoted $\mathbf{I} : x = (x_i)_{i=1..3} \rightarrow \mathbf{I}(x) = (I_j(x))_{j=1..3}$ with the I_j functions that can be associated to any of the three different channels of most standard color spaces. We define the color optical flow, by extension of the BCCE equation (1) as a vector $\boldsymbol{\nu}$ which satisfy equation (3), i.e. $\boldsymbol{\nu}^T\,\nabla\mathbf{I} = 0$. In this last relation, $\nabla\mathbf{I}$ is obviously not exactly a gradient but its traditional extension, the multi-band color gradient direction of the vectorial image $\mathbf{I}$. By the way of the definition, $\boldsymbol{\nu}$ will be a vectorial color optical flow, no peculiar relation being pointed out with any peculiar color channel.

From Structure Tensor to Flow Tensor

For every band we have a first order differential element, the gradient vector ∇I_j. Thus we obtain a global first order differential characteristic given by the matrix element $\mathbf{g} = (\nabla I_1\ \nabla I_2\ \nabla I_3)$. A natural extension of the scalar case leads to build a structure tensor and to search for its spectral components. So we obtain a structure tensor given by $\boldsymbol{G} = \mathbf{g}\,\mathbf{g}^T$ and equal to

$$\boldsymbol{G} = \sum_{j=1}^{3} \nabla I_j \nabla I_j^T, \quad \boldsymbol{G} = \sum_{j=1}^{3} \boldsymbol{S}_j \tag{10}$$

where $\boldsymbol{S}_j$ is the structure tensor of the j channel, cf (4). This structure tensor is generally a full rank matrix, which means that $\boldsymbol{G}$ have three real eigenvalues generally distinct and positive. Anyway, $\beta_1^{(\boldsymbol{G})}$ the dominant eigenvalue and $\mathbf{V}_1^{(\boldsymbol{G})}$ the associated eigenvector are significant of the value and the direction of the maximal variation. These spectral elements are so considered as the natural extension of the gradient in the case of color image sequences,[9] and we have

$$\nabla\mathbf{I} \equiv \mathbf{V}_1^{(\boldsymbol{G})} \tag{11}$$

Moreover, we can consider that $\boldsymbol{\nu}$ must be in the orthogonal subspace generated by the two others eigenvectors $\mathbf{V}_2^{(\boldsymbol{G})}$ and $\mathbf{V}_3^{(\boldsymbol{G})}$. Unfortunately, some difficulties arise. In fact, the interpretation of the two last spectral directions and consequently the construction of a combination giving the flow as a function of these directions is quite tricky. For example, nothing allows to assert that the flow will be linked to $\mathbf{V}_3^{(\boldsymbol{G})}$ the direction associated to the smallest eigenvalue, because each $\boldsymbol{S}_j$ has two null eigenvalues and that forbids any extrapolation.

To overcome these drawbacks we use the spectral directions stability property (7). So, and for any channel, we define $\boldsymbol{F}_j$ a flow tensor

$$\boldsymbol{F}_j = \mathbf{V}_3^{(\boldsymbol{S}j)}\,\mathbf{V}_3^{(\boldsymbol{S}j)T} \tag{12}$$

where $\mathbf{V}_3^{(\boldsymbol{S}j)} = \nu_j$ is the optical flow computed from the structure tensor of the j channel. From equations (6) and (7) we know that the eigenvalues of $\boldsymbol{F}_j$ are

$$\beta_1^{(\boldsymbol{F}j)} = (\partial_1 I_j^2 + \partial_2 I_j^2)(\partial_1 I_j^2 + \partial_2 I_j^2 + \partial_3 I_j^2), \quad \beta_2^{(\boldsymbol{F}j)} = \beta_3^{(\boldsymbol{F}j)} = 0$$

and the eigenvectors are

$$\begin{gathered}\mathbf{V}_1^{(\boldsymbol{F}j)} = \begin{pmatrix} \partial_1 I_j \partial_3 I_j \\ \partial_2 I_j \partial_3 I_j \\ -(\partial_1 I_j^2 + \partial_2 I_j^2) \end{pmatrix} = \nu_j, \\ \mathbf{V}_2^{(\boldsymbol{F}j)} = \begin{pmatrix} \partial_2 I_j \\ -\partial_1 I_j \\ 0 \end{pmatrix} = \mathbf{V}_2^{(\boldsymbol{S}j)}, \quad \mathbf{V}_3^{(\boldsymbol{F}j)} = \begin{pmatrix} \partial_1 I_j \\ \partial_2 I_j \\ \partial_3 I_j \end{pmatrix} = \nabla \mathbf{I}\end{gathered} \tag{13}$$

Color Optical Flow

Now and with the per-band flow tensors we build a global flow tensor

$$\boldsymbol{F} = \sum_{j=1}^{3} \boldsymbol{F}_j, \quad \boldsymbol{F} = \sum_{j=1}^{3} \nu_j \nu_j^T \tag{14}$$

and we define ν the color optical flow as $\mathbf{V}_1^{(\boldsymbol{F})}$ the direction given by the dominant eigenvalue, this assertion being a consequence of the properties previously enounced (7), (8) and (9) . A first solution to find $\beta_1^{(\boldsymbol{F})}$ and $\mathbf{V}_1^{(\boldsymbol{F})}$ goes through finding the roots of the $\boldsymbol{F}$ characteristic polynomial and through resolving a linear system. In fact, we can use ν in conjunction with (8) and (9), or more directly from (14), to obtain

$$\boldsymbol{F}\nu = \sum_{j=1}^{3} (\nu_j^T \nu)\nu_j \quad \text{and} \quad \nu^T \boldsymbol{F} \nu = \sum_{j=1}^{3} (\nu_j^T \nu)^2$$

As ν is the dominant spectral element of $\boldsymbol{F}$, we have $\boldsymbol{F}\nu = \beta_1^{(\boldsymbol{F})}\nu$, and that gives the two relations

$$\beta_1^{(\boldsymbol{F})} = \sum_{j=1}^{3} \left(\frac{\nu_j^T \nu}{\|\nu\|} \right)^2 \qquad \nu = \frac{1}{\beta_1^{(\boldsymbol{F})}} \sum_{j=1}^{3} (\nu_j^T \nu)\nu_j \tag{15}$$

which define a peculiar version of the well known iterated power method, using the ν_j optical flow vectors directly computed from the structure tensors of the I_j band. We can notice that the spectral direction stability assures that we will effectively satisfy (3). So, these two relations simply describe the core of our algorithm for the computation of the ν vectorial color optical flow.

Experimental Results

To illustrate the accuracy of the color optical flow such as defined in (15) we present the results obtained with a mpeg sequence provided by the CAVIAR project (Context Aware Vision using Image-based Active Recognition). In Fig. 1 we have two consecutive images of a corridor shopping center sequence.

Figure 1. Two images from the corridor sequence

The corresponding color optical flow are shown using a color convention for representing direction and intensity of the vector fields. We use three different algorithms to compare with our method: in Fig. 2, a flow computed from the Y-plane with Horn and Schunk algorithm[1] and, in Fig. 3, a flow computed from the UV planes such as described by Barron[5]; in Fig. 4, a least square method[2] applied to YUV space and, in Fig. 5, the same algorithm applied to RGB space. With our method, results were obtained with the implementation of the iterative process (15), the initial ν vectors being systematically the normalized unit vector (1 1 1) and the maximal iterations number being set to 20 which is generally large enough, the average convergence rate being around 5 iterations. We use both YUV and RGB spaces to show the efficiency of our algorithm, as we retrieve very similar color optical flow in both spaces as shown in Fig. 6 for the YUV space and in Fig. 7 for the RGB space.

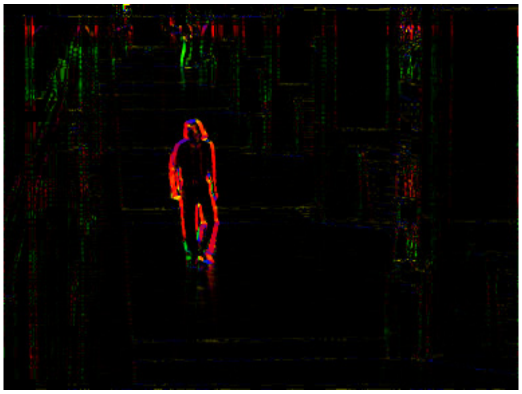

Figure 2. Horn and Schunk: optical flow from Y-plane

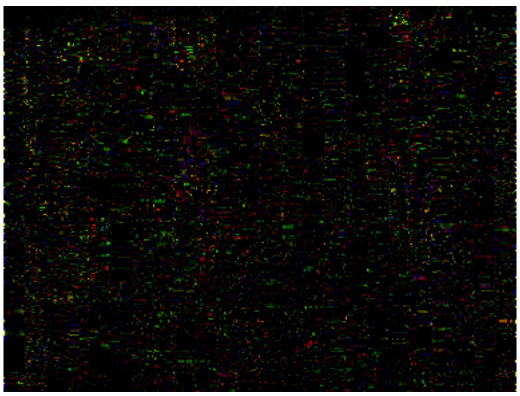

Figure 3. Barron: color optical flow from UV-planes

Figure 4. Least square: color optical flow from YUV space

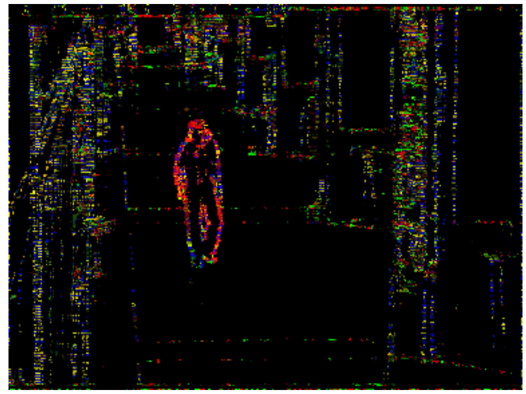

Figure 5. Least square: color optical flow from RGB space

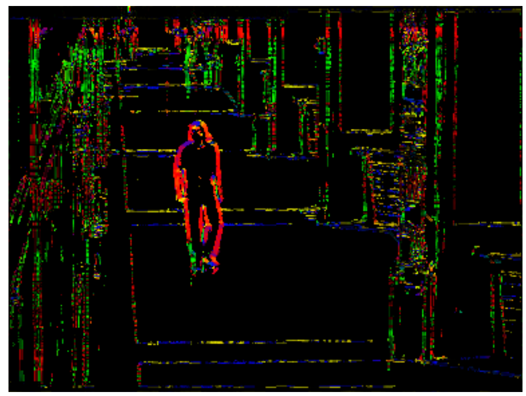

Figure 6. Our algorithm: color optical flow from YUV space

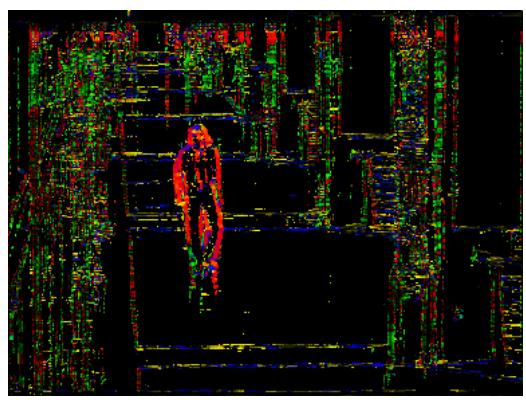

Figure 7. Our algorithm: color optical flow from RGB space

We have to notice that our method effectively produce a significant color optical flow, combining results of both luminance or chromatic dedicated methods. Our results are comparable to those obtained with the least square method which is known as one of the more reliable. Furthermore, we observe that the motion fields obtained with our method seem to be more regular, i.e. more independent of the space color representation, than the ones obtained with the least square method. We also can notice that motion coming from the instability of the mpeg color artifacts is very well perceived and that demonstrates, a contrario, the color accuracy of our method.

Conclusion

In this paper we present an original and efficient method of computation of the color optical flow. This method uses a flow tensor coming from the usual structure tensor and provides a vectorial solution to the color optical flow question. Moreover, the computational cost of the algorithm is quite reasonable, due to the direct computation of the tensorial per-band optical flows and to the fast convergence (about 5 iterations) of the iterative process.

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