Calculating Metamer Sets Without Spectral Calibration

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Abstract
The set of metamers for a given device response can be calculated given the device's spectral sensitivities. Knowledge of the metamer set has been useful in practical applications such as colour correction and reflectance recovery; and also in furthering our understanding of aspects of colour science such as the tendency of metamers to cross at certain preferred wavelengths. Unfortunately, the device sensitivities of a camera or scanner are not known and they are difficult to estimate reliably outside the lab.

The main contribution of this paper is to show how metamer sets can be calculated when a device's spectral sensitivities are not known. The result is built on two observations: first, the set of all reflectance spectra are convex combinations of certain basic colours which tend to be very bright (or dark) and have high chroma. Second, the convex combinations which describe reflectance spectra result in convex combinations of RGBs. Thus, given an rgb it is possible to find the set of convex combination of the RGBs of the basic colours which generate the same rgb. The corresponding set of convex combinations of the basic spectra is precisely the metamers set.

The practical import of this result is that the theory of metamer sets can be used without the need for difficult and tedious calibrations. Experiments validate our approach.

1. Introduction
Colorimetric device calibration is the problem of estimating the responses of a device **A** in the space of a second device **B**. As an example, if device **A** is an rgb camera and its responses to a given scene are contained in an rgb image; then colorimetric calibration is the problem of estimating a mathematical transformation of the image data to the space of device **B**. The latter might be the human's XYZ space or a monitor's RGB space such as sRGB.

The process of integrating the continuous light signal reflected of an object to a device response is linear. This linearity has for a long time been the motivation for employing linear transforms to the problem of colorimetric device calibration. It is, however, known that; for a linear transform to perfectly map the responses of device **A** to the space of **B** the sensor curves of **A** must span the same space as those of **B**. A camera whose spectral sensitivities are a linear combination of the colour matching functions[1] is said to satisfy the Luther condition[2] and is colorimetric.

Although colour scanner and camera manufacturers strive to achieve colorimetric colour reproduction, they have to take into account other design factors such as noise amplification and manufacturing limitations.[3] The result of the design compromise is that cameras and scanners are not perfectly colorimetric and linear transforms fall short from being the ideal solution for the problem of colour correction.

Several methods have been exploited to improve upon the performance of linear transforms. These methods include: constrained linear transforms,[4] polynomial fitting,[5] look up tables[5] and local linear transforms. Unfortunately, all these techniques have their own limitations; some of which are practical, i.e. relate to technique's performance while others are theoretical. As an example of the practical limitations, polynomial transformations are known to perform well for the calibration target but may not generalize adequately on other data. But, our main criticism is that these methods represent (albeit useful) heuristics and they fail to shed light on the underlying problem.

Finlayson and Morovic[6,7] developed a theory of metamer sets which provided a strong fundamental basis for color correction. They showed that given a single rgb there is a corresponding metamer set of reflectances that might have induced the rgb (where we assume the spectral sensitivities of the device is known as is the prevailing lighting conditions). This *metamer set* when projected onto the target sensitivities (e.g. XYZ colour matching functions) is usually non singular: a single rgb maps to a convex set (a convex polygon in XYZ space) of xyzs. Algorithms were presented for recovering metamer sets and also for selecting the best metamer to represent the set. It is this single reflectance which, when projected onto the target colour space, is the endpoint of colour correction. Experiments validated their method with significantly improved correction being reported with marked improvement with saturated colours.

Unfortunately, the calculation of metamers requires knowledge about the camera's spectral sensitivity curves. These are typically not known. Moreover, they are very difficult to estimate. In certain restricted cases a monochrometer might be used to measure the system response to different monochromatic spectra (in effect we run a colour matching function with a camera). However, monochromaters are expensive and the experiment is tedious to carry out. Alternatively, given a reference target (such as a Macbeth Colour Checker) one can infer the spectral sensitivities. While this method can work the approach is subject to significant numerical errors and in some cases the estimated sensitivities can be quite different than the true sensor curves. This problem can be mitigated in part by solving for the set of all plausible sensors,[8] But, ultimately a single set must be chosen and this may still be the wrong choice.

The main contribution of this paper is to show how metamer sets can be calculated when the device's spectral sensitivities are not known. The result is built on two observations. First, the set of all reflectance spectra are convex combinations of certain basic colours which tend to be very bright (or dark) and have high chroma. Second, the convex combinations which describe reflectance spectra result in convex combinations of RGBs. Thus, given an rgb if we can find the set of convex combination of the RGBs for the basic colours which generate the same rgb then the corresponding set of convex combinations of the basic spectra is precisely the metamers set.

2. Background

The response of a linear sensor to a spectral stimulus can be modelled as:

$$p^i = (Es)^T r^i, \quad i = 1, 2, 3 \tag{1}$$

where E is a diagonal matrix whose diagonal elements are the intensity of the scene's illumination at each discrete wavelength i.e. $E = diag(e)$, s is the surface reflectance, r^i is the camera sensitivity vector at channel i, and T is the matrix transpose operator. Spectral functions are adequately represented by sampling at 10 nanometer intervals across the visible spectrum: 400 to 700 Namometres.[9] Hence e, s and r are 31×1 vectors. Writing,

$$c = Es, \tag{2}$$

we can write the sensor response to a spectral stimuli as:

$$p = c^T R \tag{3}$$

where R is a (31×3) matrix whose columns are the red, green and blue sensitivities of the camera respectively and p is an 1×3 camera response vector.

In Equation (3) the colour signal c is an 1×31 dimensional vector while the response of the device is 1×3. This property means that the sensors of a camera collapse the information in the colour signal from a 31dimensional space onto a much lower dimensional space, normally 3-*d*. As a result of this projection, it is impossible to exactly recover the spectral information of a surface based on the 3dimensional camera response, as many spectrally different signals can integrate to a single response triplet when projected down to the 3-dimensional space.

Surfaces which integrate to the same camera response are said to be metameric to each other.[10] Further, as Horn pointed out[11] the metamers of one device are different to those of another unless *both devices are within an exact linear combination of each other*.

Let us consider a camera's response to a single surface, c^T ,such as that in Equation (2). It is possible to decompose c^T into two components, one in the range of R (it integrates to a non zero response), and another in its null space (it integrates to 0), i.e.:

$$c^T = c^T_{ra} + c^T_{nu} \tag{4}$$

Mathematically, for a $m \times n$ matrix A, the range[12] is defined as:

$$ran(A) = \{y \in R^m : y = Ax \text{ for some } x \in R^n\} \tag{5}$$

and the null space[12] of A is defined as:

$$null(A) = \{x \in R^n : Ax = 0\} \tag{6}$$

From the definitions in Equations (5) and (6), we are able to state that for a vector c^T_{ra} in the range of R we have:

$$c^T_{ra} R = p, \tag{7}$$

while for c^T_{nu} in the null space of R we have:

$$c^T_{nu} R = 0 \tag{8}$$

In Equations (7) and (8) we have decomposed a surface c^T into c^T_{ra} + c^T_{nu} into c. The vector c^T_{ra} is sometimes called the **fundamental metamer**, while c^T_{nu} is known as a **metameric black**.[13] The fundamental metamer must be unique since any vector in the range of A will project to a unique point. And, because metameric blacks project to a unique point we can add them arbitrarily to create new metamers.

Suppose now we wish to solve for the set of all metamers that induce a given response. We write this set as $Q(p)$ and we can describe the members of this set in the following way:

$$\begin{aligned} &Q(p) = \{c^T_{ra} \text{ and } c^T_{null} \in R^n \text{ s.t.} \\ &c^T_{ra} R = p, \\ &c^T_{nu} R = 0, \\ &\quad c^T_{ra} \in ran(R), \\ &\quad c^T_{nu} \in null(R)\} \end{aligned} \tag{9}$$

In Refs. [7] and [14], it was proved that this set of metameric surfaces, is convex and closed.[15]

As is evident from the set defined in Equation (9) the calculation of the metamers requires knowledge of the sensor curves.

In the remaining parts of this paper we demonstrate that it is possible to solve for the set of all metamers associated with a given *rgb* camera response without having to know the device's spectral sensitivities.

3. Finding Metamer Sets Without Sensor Curves

Our result is based on two insights. First that the set of all reflectance spectra can be written as convex combinations of a set of basic reflectances (e.g. such as those found on a reference colour chart). Second, the convex combinations that model spectral interactions map to the same combinations in the RGB domain: 0.5 * red + 0.5* yellow (in the spectral domain) results in an RGB which is 0.5 * RGB (for red) + 0.5 *RGB (for yellow).

We next give the required background concerning convexity. This is then used to develop a method to solve for metamer sets when the spectral sensitivities of the device are not known.

3.1. Convexity Results

Definition: A set Q in R^n is said to be convex if for every x and y in Q, the line segment joining x and y also lies in Q.

A line segment going from point x to y can be defined as:

$$[x, y] = \{\lambda y + (1-\lambda)\, x \; : \; 0 \leq \lambda \leq 1\} \tag{10}$$

Therefore, a set Q in R^n is convex *if and only if* for every x and y in Q and every λ with $0 \leq \lambda \leq 1$ the vector $\lambda y + (1 - \lambda)x$ is also in Q.[16]

Let us consider a colour signal, c, which is defined as a convex combination of two signals, namely, c_1 and c_2 to which we have measured camera responses p_1 and p_2. We can write c as:

$$c = \lambda c_1 + (1-\lambda)\, c_2 \tag{11}$$

From Equation (3) we know the camera response to c_1 can be written as:

$$p_1 = c_1^T R \tag{12}$$

If we scale c_1 by λ then the response is a scalar multiple of p_1, i.e.,

$$\lambda p_1 = \lambda c_1^T R \tag{13}$$

If the second colour signal, c_2, was multiplied by $1 - \lambda$ then its corresponding response can be written as:

$$(1-\lambda)\, p_2 = (1-\lambda)\, c_2^T R \tag{14}$$

By making use of the second fundamental property of a linear system, i.e. additivity[17] we can group Equations (13) and (14) as:

$$\lambda p_1 + (1-\lambda)\, p_2 = \left(\lambda c_1^T + (1-\lambda)\, c_2^T\right) R \tag{15}$$

By substituting c from Equation (11) in Equation (16) we get:

$$\lambda p_1 + (1-\lambda)\, p_2 = c^T R \tag{16}$$

The response:

$$p = \lambda p_1 + (1-\lambda)\, p_2, \tag{17}$$

is a point in the three dimensional *rgb* space which lies on the line connecting p_1 and p_2. Further, in the case of Equations (11) and (17), it appears that the weights λ and $(1 - \lambda)$ which relate c to c_1 and c_2 are identical to those which relate p to p_1 and p_2.

Finally, for completeness we remark that $\lambda \in [0, 1]$. If say $\lambda = 0.5$ then we can simulate the formation of a new colour signal by painting half of a canvas c_1 and the other half c_2. Viewed from a far enough distance this canvas will map to a single measurement point and it would be as if the scene contained $0.5c_1 + 0.5c_2$. Of course we cannot have less than 0% or more than 100% of something present.

3.2. Solving for the Metamer Set

In the last section we showed that a convex combination of spectra results in a convex combination of responses. While the result was written for 2 spectra it generalizes to n spectra. We can write

$$c = \sum_{i=1}^{n} \lambda_i c_i \; , \lambda_i > 0 \; \& \; \sum_{i=1}^{n} \lambda_i = 1 \; \Rightarrow p = \sum_{i=1}^{n} \lambda_i p_i$$

where p_i is the rgb response of the ith colour signal. Of such that: course the colour signal results from a light multiplying a surface and so the above equation really is informing us that the underlying reflectances are written as convex combinations. Since these will not change with illumination it is useful to think of modelling reflectance as a convex combination of basic reflectances:

$$s = \sum_{i=1}^{n} \lambda_i s_i \; , \lambda_i > 0 \; \& \; \sum_{i=1}^{n} \lambda_i = 1$$

The reflectances s_i might be reflectances from a colour chart. More usefully however the reflectances s_i will lie on the convex hull of measured reflectances. Since all points inside a convex body can be written as a convex combination of points on the boundary (i.e. vertices). Experiments suggest that all reflectances can be written as a linear sum of 8 basis functions. However, in an 8 dimensional space the convex closure of all reflectances requires > 8 vertices. In our experience, we can represent the set of all spectra as a set of a few 10s of basic vertex spectra.

Let us use the 24 reflectances on a Macbeth colour checker as our basic reflectances. The corresponding RGBs are shown in Figure (1). For a new point, such as the one shown in the figure, we would like to calculate all the possible convex weights. From the forgoing discussion this set corresponds to the set of all reflectances that induce the RGB; and, is in fact the Metamer set.

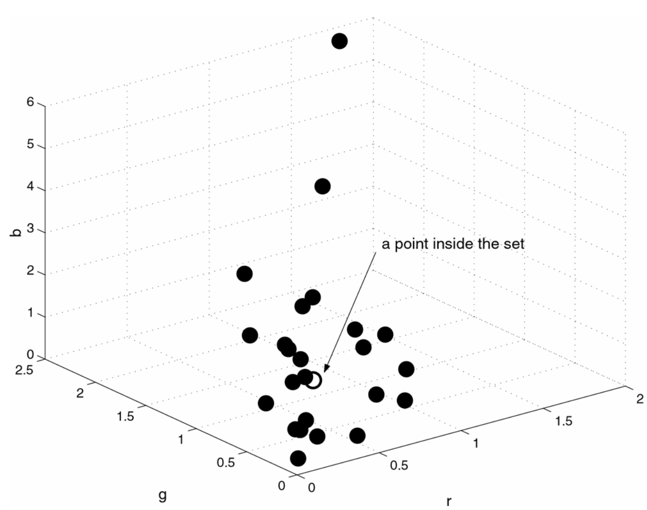

Figure 1. A set of 24 rgb calculated based on the responses of the Sony DX camera to the surfaces of the Macbeth Color Checker are shown as the solid black discs. Further, a point inside is shown as a black ring.

Mathematically, we wish to solve for all the weights λ_i such that:

$$p = \lambda_1 p_1 + \lambda_2 p_2 + ... + \lambda_m p_m \text{ subject to} \quad (18)$$

$$\sum_{i=1}^{m} \lambda_i = 1,$$
$$\forall_i \lambda_i \geq 0, \quad (19)$$

where m is 24 in our example. In matrix format, it is possible to write the system in Eq. (18) as:

$$p = P\lambda \text{ subject to}$$
$$\mathbf{1}^{\mathbf{T}}\lambda = 1,$$
$$I\lambda \geq \mathbf{0},$$
$$I\lambda \leq \mathbf{1} \quad (20)$$

where p is a 3×1 vector, P is a $3 \times m$ matrix, λ is a $m \times 1$ vector, **1** and **0** are $m \times 1$ vectors.

To solve for the set of all vectors λ which would satisfy the system in Eq. (20) we note that each inequality in Eq. (20) defines a hyperplane. A hyperplane, defined by an inequality of the form $ax \leq b$, divides the space into three parts, the first, contains the vectors x which satisfy the inequality, i.e., $ax < b$, the second is the space of all the weights which violates the inequality, i.e. $ax > b$ and the third, satisfies the equality, i.e. $ax = b$. For a linear system of equalities and inequalities, as the one defined in Eq. (20) intersecting all the hyperplanes results in a closed and convex region, which is the space of all feasible solutions to the system. Using computational algorithms such as quickhull[18]; it is possible to solve for the region of all feasible solutions to a system such as the one described in Equation (20). Having done that we need only apply the weights to the set of surfaces available from the calibration data.

We note that in the traditional metamer set formulation,[7,14] additional constraints need to be imposed on the feasible space these are: nonnegativity, smoothness as well as a constraint to restrict the maximum value of a colour signals to be less than one. In our proposed formulation no additional constraints are needed. The proposed formulation constrains a colour signal to be defined inside the space of the available calibration signals. Thus the recovered signal is guaranteed to satisfy the previously mentioned constraints.

4. Results

To test our method we generated a set of 24 responses based on the spectral reflectances of the 24 patches Macbeth Color Checker and the spectral sensitivities of the MegaVision camera. For the calculations equienergy illumination was assumed.

As a test data we used the camera's responses to the 264 surfaces of the Esser calibration chart.[19] In this section we include the results for the reflectance shown in Figure 2. Based on the formulation presented in the previous section we recovered the set of all weights λ which results in exactly the same *rgb* response. When those weights were applied in the spectral space, we recovered the metamer set. In Figure (3) we plotted the maximum and minimum values of that set as well as the actual reflectance. The corresponding colorimetric values were calculated for each reflectance from Figure (3), we arrived at the *xyz* metamer cloud shown in Figure (4). Finally, the metamer cloud in chromaticity space is shown in Figure (5).

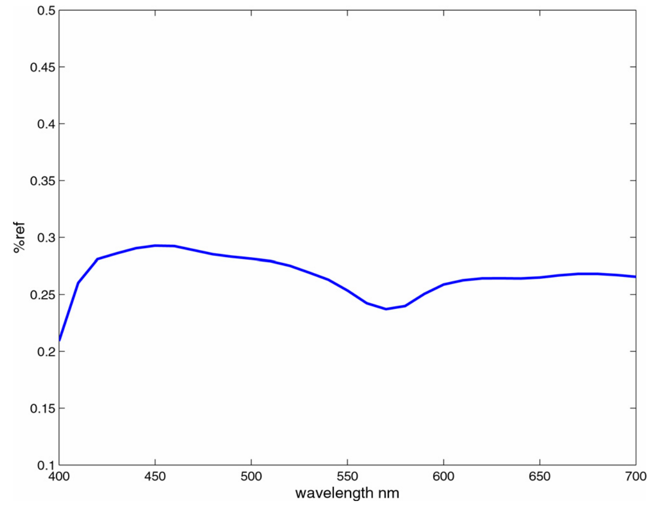

Figure 2. A test reflectance to which we wish to calculate all the metamers

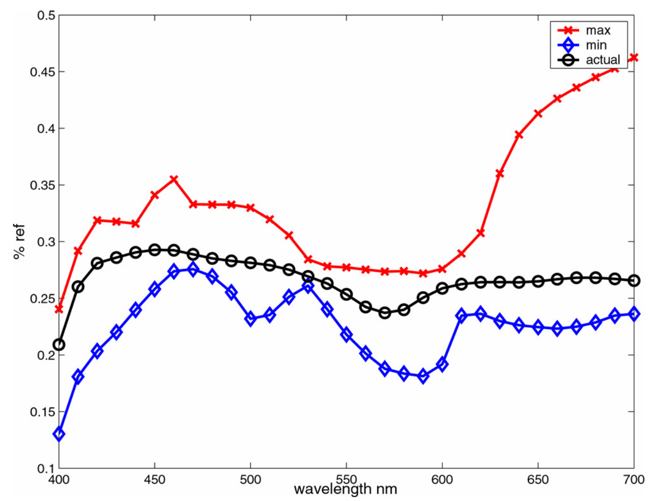

Figure 3. The metamer set in the spectral space.

5. Conclusions

In this paper we presented a novel method to calculate the metamer set with the advantage that; knowledge of device spectral sensitivities is not required.

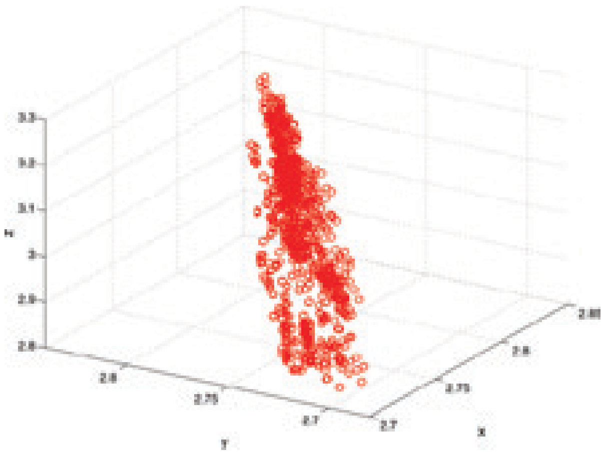

Figure 4. The metamer cloud in the xyz space.

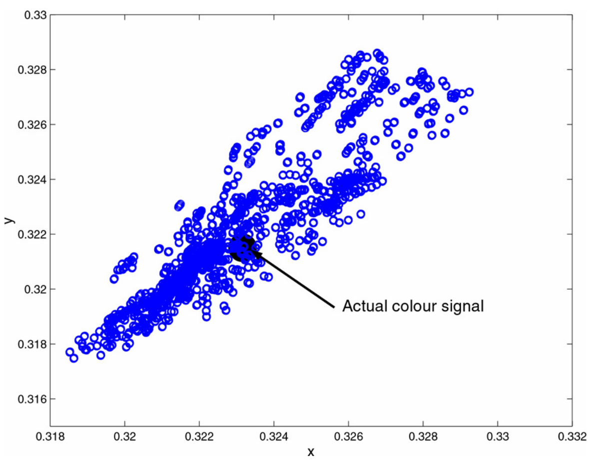

Figure 5.The metamer could in the xyz space.

The result is built on two observations: first, the set of all reflectance spectra are convex combinations of certain basic colours which tend to be very bright (or dark) and have high chroma. Second, the convex combinations which describe reflectance spectra result in convex combinations of RGBs. Thus, given an rgb it is possible to find the set of convex combination of the RGBs of the basic colours which generate the same rgb. The corresponding set of convex combinations of the basic spectra is precisely the metamers set.

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