

# A Novel Approach for Generating Object Spectral Reflectance Functions from Digital Cameras

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## Abstract

*In this paper, a method for generating reflectance spectra from camera signals is proposed. It consists of two main steps. The first step is to characterize the camera by estimating the mapping matrix from colour signal directly to reflectance using a training data set including camera signals and their reflectance functions. The second step is to construct a new reflectance function based on the one generated in the first step via a constrained least squares method. The great advantage of this method over conventional methods is without knowing the information of camera sensors and illuminant. The method together with the conventional methods such as those developed by Wiener, and Hasegawa and Fairchild were tested using two types of data sets: the simulated and real camera data. The results showed that the new method outperformed the conventional methods, especially for the real camera data*

## Introduction

With large progress in recent development of digital colour cameras, they are now being applied for measuring colours of the surface objects. Some systems have already achieved satisfactory results.<sup>1,2</sup> They have advantages over the conventional colour measurement instruments by capturing the total appearance of the object including colour, texture, glossy and other factors. They also have little restriction on the size and shape of objects and the uniformity of the surfaces.

The major requirement for a colour measurement instrument is its high repeatability and accuracy. However, when capturing the same scene or object, the output RGB signals from different cameras or from the same camera with different settings such as shutter speed and exposure are not the same. The way to overcome the problem is known as *device characterisation*, which converts a camera's RGB signals to the CIE tristimulus values. The overall performance of a camera characterisation is dictated by the mathematical model.

Most methods are based on a simple 3 by 3 linear transform,<sup>3</sup> i.e.,

$$f = Mc \quad (1)$$

where  $c^T = (R \ G \ B)$  is the camera response and  $f^T = (X \ Y \ Z)$  is the vector consisted of the tristimulus values, and  $M$  is a 3 by 3 matrix. Here the superscript T stands for the transpose of a vector or matrix. The matrix  $M$  can be determined through some training data sets. More accurate results can be obtained by using a higher order of the camera response.<sup>4,5</sup> When a higher order is used, the matrix  $M$  in Eq. (1) will be 3 by  $m$ . It was found that linearization to the camera response improves the accuracy when  $M$  is 3 by 3.

However, the linearization does not result in accuracy improvement when  $m$  is increased to 11 (second order) or 20 (third order) or 35 (fourth order). This is because the nonlinear relationship is already taken into account by building the mapping matrix  $M$ . The spectral reflectance function of the object can be estimated using methods such as the Hasegawa and Fairchild,<sup>6</sup> denoted as **HF** method, and Wiener<sup>7,8</sup> method based on the computed tristimulus values using Eq. (1). Another approach<sup>6</sup> for the colour correction is based on the assumption that camera response  $c$  and the reflectance function  $r$  of the object have a linear relationship:

$$c = Qr \quad (2)$$

where the column reflectance vector  $r$  has  $n$  components and  $Q$  is a 3 by  $n$  matrix which is dependant on the spectral power distribution of the light for illuminating the object and three CCD sensors of the camera. Normally, the sensors are not available, and can be estimated.<sup>9-11</sup> Thus when  $Q$  is available, the spectral reflectance function  $r$  can be estimated using methods such as those of Wiener and HF.

Now it is clear that both approaches for generating the spectral reflectance function have two steps. The first step includes estimate  $M$  or  $Q$ , and the second step reconstructs the spectral reflectance function. In reality, each step will inevitably introduce error. The method in the second step is more sensitive to the error made in the first step. Thus, the reconstructed spectral reflectance function may largely deviate from the real reflectance function measured by a spectrophotometer. By observing Eq. (2), we have  $r \approx Q^+c$ , where  $Q^+$  is the generalized inverse of the matrix  $Q$ . This motivates us to consider directly to build a matrix  $W$  based on the training data set so that:

$$r = Wv(c) \quad (3)$$

where  $v(c)$  is a column vector and is a function of the camera response  $c$ . For example, the simplest format for the vector function is  $v(c) = c$ . It is believed that this approach for generating spectral reflectance function will be more accurate than the two earlier approaches. In the next section, methods will be introduced to obtain the vector function  $v(c)$ , and the matrix  $W$ . Finally, newly developed method will be compared with the other two methods.

## The New Method

The proposed method includes three steps. Step one defines the vector function  $v(c)$ . Step two derives the matrix  $W$ . Step three calculates the spectral reflectance function.

The vector function  $v(c)$  is defined by the polynomial equation.<sup>5</sup> In order to relate the order  $k$  of the polynomial, the vector function  $v(c)$  is denoted by  $v_k(c)$ , and is defined below.

Definition for  $v_k(c)$

$$v_0(c) = c, v_1(c) = \begin{pmatrix} 1 \\ c \end{pmatrix}, v_k(c) = \begin{pmatrix} v_{k-1}(c) \\ u_k \end{pmatrix}, \quad (4)$$

$u_k$  is a column vector and each element of it has the form of  $R^{j_1}G^{j_2}B^{j_3}$  with  $j_1 + j_2 + j_3 = k$ , and all  $j_1, j_2, j_3$  being non negative integers.

Note that camera response  $R, G, B$  signals must be scaled within the range of zero and one before the calculation of the vector function  $v_k(c)$ .

Now we consider deriving the mapping matrix  $W$  based upon some training data. Suppose that there are  $p$  colour patches and their reflectance vectors  $r^{(j)}$  and camera signals  $c^{(j)}$  are available. Let

$$S = [r^{(1)}, r^{(2)}, \Lambda, r^{(p)}], V = [v_k(c^{(1)}), v_k(c^{(2)}), \Lambda, v_k(c^{(p)})] \quad (5)$$

Then, the matrix  $W$  should satisfy

$$S = WV \quad (6)$$

Now a vector operator:  $vec$  is introduced, which is operated on a matrix, for example  $S$ , giving a column vector  $vec(S)$  defined by

$$[vec(S)]^T = \left( (r^{(1)})^T, (r^{(2)})^T, \Lambda, (r^{(p)})^T \right) \quad (7)$$

Thus, if we let

$$s = vec(S), w = vec(W), A = V^T \otimes I_n \quad (8)$$

then it can be shown from Eq. (6) that

$$Aw = s \quad (9)$$

Here, operator  $\otimes$  is the Kronecker product operator<sup>12</sup>;  $I_n$  is the identity matrix of size  $n$  and  $n$  is the number of elements of a reflectance vector. The linear system of equation (9) may have no solution in normal sense, but it always has a least squares solution. Besides, some constraints can be added. For example, it is possible to map  $C_b$  (the black) and  $C_w$  (the white) camera signals to their exact spectral reflectance vectors  $r_b$  and  $r_w$  respectively. Let

$$S_E = [r_b, r_w], V_E = [v_k(c_b), v_k(c_w)], \quad (10)$$

$$s_e = vec(S_E), A_E = V_E^T \otimes I_n$$

then we have the equality constraint:

$$A_E w = s_e \quad (11)$$

Furthermore, the solution in a certain range can be assumed, for example,  $W$  must not be less than  $b_L$ , and must not be greater than  $b_U$ . Here  $b_L$  and  $b_U$  are pre-specified lower and upper bound vectors. Thus, for finding the matrix  $W$ , the following constrained least squares problem is to be solved.

**The Constrained Least Squares Problem for Finding Matrix  $W$**

$$\frac{Min}{w} \|Aw - s\|^2$$

Subject to:  $A_E w = s_e, b_L \leq w \leq b_U$

Note  $\|x - y\|$  is the Euclidian distance of the two vectors  $x$  and  $y$ .

Finally, the reconstruction of the spectral reflectance function for any given scaled camera signal vector  $c$  can be carried out. It seems that Eq. (3) can be directly used for obtaining the spectral reflectance vector  $r$ . However, the resultant spectral reflectance function using Eq. (3) may include some values outside the range of 0 and 1. In order to overcome this problem, the following constrained least squares problem is proposed again:

**The Constrained Least Squares Problem for Reconstructing Spectral Reflectance Function**

$$\frac{Min}{v} \|v - v_k(c)\|^2$$

Subject to:  $0 \leq Wv \leq 1$

Note that 0 and 1 here represent vectors with all elements being zero and one respectively.

## Testing Methods' Performance

The performance of the above proposed method is compared with the Wiener, and **HF** methods using the simulated and real camera data in this section.

### The Simulated Data

Firstly, camera sensitivity and three sensors shown in Figures 1 and 2 respectively, and the spectral power distribution of the D65 were used to calculate the camera weight  $Q$ . Three sets of reflectance data were used. They were measured from the Munsell colour book (**Munsell**, 1560 samples), a set of textile samples (**Textile**, 705 samples) and a GretagMacbeth ColorChecker Digital Chart (**DC**, 228 from 240 samples) at 10 nm interval between 400 and 700 nm. Thus, the camera signal  $c$  can be generated using Eq. (2). Finally, the signal  $c$  was corrupted by adding quantisation errors (256 grey levels), and a multiplicative Gaussian noise  $err_{noise}$  defined:

$$[err_{noise}]^T = \varepsilon(\xi_1 R, \xi_2 G, \xi_3 B), \text{ with } \varepsilon = 0.01 \quad (12)$$

where random variable  $\xi_i$  has a normal distribution with expectation being zero and standard deviation being one. For the Munsell data set, 10% of them, one every 10 samples was selected as the training set, the whole set (including the training set) was used as the testing test. For the Textile set, one-third, one from every three samples was selected as the training set, and the whole set was used as the testing set. For the DC data set, one-half, selecting one every two samples, was used as the training set and the whole set was used for testing.

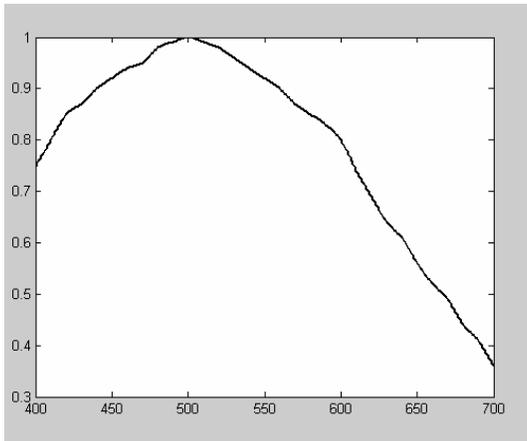


Figure 1. Sensitivity Function of the Camera

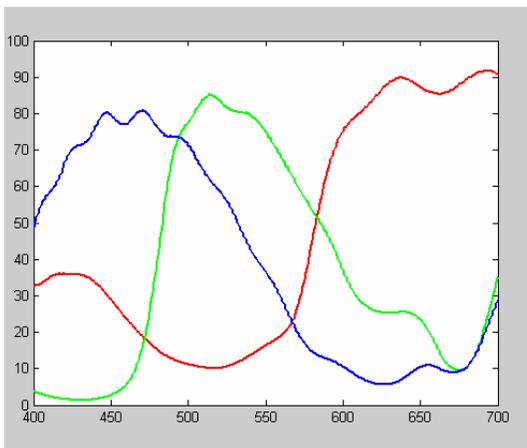


Figure 2. Sensors of the Camera

### The Real Camera Data

The GretagMacbeth ColorChecker DC was also captured by a real camera. Thus, for the DC data set, its real colour signals ( $c$ ) and reflectance functions ( $r$ ) are both available. The data, the one-half defined above, was used as training and the whole set was used as testing. For the Wiener, and HF methods, the camera weight matrix  $Q$  was estimated using the whole DC data (colour signals and reflectance functions), which is shown in Figure 3. Note that the linearization for the camera signals was applied while estimating

the weight matrix  $Q$ , i.e., we found weight matrix  $Q$ , and  $f, g, h$  at the same time when minimizing:

$$\left\| \begin{pmatrix} R^f \\ G^g \\ B^h \end{pmatrix} - Qr \right\|^2$$

over all the training data set plus some constraints.

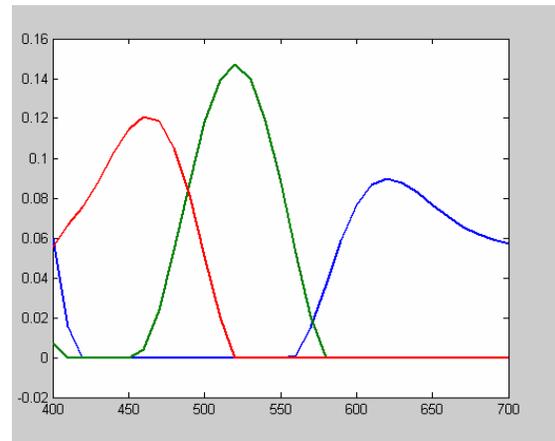


Figure 3. The Estimated Camera Weight

### The Measure of Performance

The spectral reflectance error ( $err_r$ ) and CIELAB colour difference ( $\Delta E$ ) were used for measuring the performance of each method. For the former it is defined by:

$$err_r = \sqrt{\frac{1}{n} \|r_j - \tilde{r}\|^2} = \sqrt{\frac{1}{n} \sum_{j=1}^n (r_j - \tilde{r}_j)^2} \quad (13)$$

where,  $r$  is the original spectral reflectance function and  $\tilde{r}$  is the reconstructed spectral reflectance function.

The tristimulus values computed from  $r$  and  $\tilde{r}$  were used for computing the colour difference using the CIE illuminant D65 and CIE 1964 standard colorimetric observer.

### Results and Discussions

The results using the simulated and real camera data are listed in Tables 1-3 respectively. Two statistical values were computed. One is the median and the other is the maximum. Since both colour and spectra differences are not normally distributed, therefore, the median can reflect the overall performance better than the arithmetic mean. Tables 1 and 2 list the spectral and colour difference between the original and reconstructed colours respectively based on the simulated data sets. The values in bold are the smallest values among the three methods used. From Table 1 it can be seen that for the Textile and DC data sets, the proposed method is the best in terms of median and maximum spectral differences. For the Munsell data set, the proposed method is still the best in terms of median spectral difference, while the HF

method is the best in terms of maximum spectral difference. As for the colorimetric accuracy, it can be seen from Table 2, that the proposed method is still the best in terms of median colour difference. However, the HF method is the best according to the maximum colour difference.

The results using the real camera data set DC are listed in Table 3. It clearly showed that the proposed method outperformed the Wiener and HF methods in terms of all measurers by a large margin. The Wiener and HF methods perform roughly the same. An example is shown in Figure 4. The curve without marking is the

original reflectance. The curves with marking “\*”, “o”, and “+” are the reflectance functions generated by the proposed, Wiener, and HF methods respectively. It is clearly show the curve with marking “\*” is closest to the curve without marking, and the curves with marking “o” and “+” similar in the left and middle parts, but are clearly difference in the right end. But in general they are similar.

However, we have to note that the differences between the proposed method and either Wiener or HF method is much larger for the real camera data than those of the simulated data. This implies the followings.

**Table 1: Performance of each method based on the testing data sets in terms of median (Med) and maximum (Max) of spectra difference using the simulated data**

	Proposed Method		Wiener Method		HF Method	
	Med	Max	Med	Max	Med	Max
Munsell	<b>0.015</b>	0.116	0.020	0.136	0.031	<b>0.089</b>
Textile	<b>0.021</b>	<b>0.099</b>	0.035	0.115	0.032	0.119
DC	<b>0.011</b>	<b>0.083</b>	0.018	0.140	0.026	0.086

**Table 2: Performance of each method based on the testing data sets in terms of median (Med) and maximum (Max) of CIELAB colour difference using the simulated data**

	Proposed Method		Wiener Method		HF Method	
	Med	Max	Med	Max	Med	Max
Munsell	<b>2.30</b>	15.14	2.60	28.77	3.27	<b>10.25</b>
Textile	<b>2.38</b>	12.01	3.46	33.78	2.80	<b>11.39</b>
DC	<b>1.72</b>	10.96	2.61	10.97	2.77	<b>7.65</b>

**Table 3: Performance of each method based on the testing data set (DC) in terms of median (Med) and maximum (Max) of spectra difference and colour difference respectively using the real camera data**

	Proposed Method		Wiener Method		HF Method	
	Med	Max	Med	Max	Med	Max
$err_r$	<b>0.011</b>	<b>0.120</b>	0.051	0.161	0.052	0.165
$\Delta E$	<b>1.38</b>	<b>9.82</b>	13.07	52.05	13.52	41.19

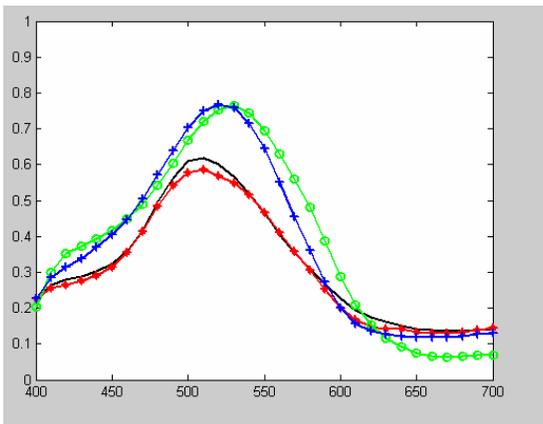


Figure 4. A real (curve without marking) and generated reflectance functions by proposed (marked “\*”), Wiener (marked “o”), and HF (marked “+”) methods.

When using the simulated data, the same amount of quantisation error and noise were introduced for testing all methods. In this case, the proposed method performed the best in terms of median measure for the spectra and colorimetric difference. On the contrary, the HF method is best in terms of maximum measure of colorimetric difference. However, all methods gave a much smaller difference in performances using the simulated data than that using the camera data.

When testing different methods using the real camera data, the difference between the proposed method and the Wiener or the HF method is much larger, i.e. the proposed method is much better than the other methods. The proposed method used the training data set to characterise the camera and to obtain the mapping matrix  $W$  at the same time. The spectral reflectance function can then be predicted using the matrix and training data set. The only source of error comes from the estimation of the mapping matrix  $W$ . While for the Wiener and HF methods, the camera’s weight matrix  $Q$  must be first estimated. This could introduce large errors. Additionally, it is known that the camera’s signal may not be

linearly correlated with the SPD of the illuminant, sensors and reflectance as expressed by Eq. (2). In order to have a fair linear relationship, gamma correction to the camera signals is needed. However, we did the gamma correction while estimating the matrix  $Q$  at the same time. Comparing the results in Tables 1 and 2 for the DC data set, the estimating error for finding  $Q$  is amplified when reconstructing reflectance using the Wiener and HF methods. This indicates that both methods are sensitive to the errors in estimating the matrix  $Q$ . This phenomenon can be easily understood, since both methods have no characterisation stage and they just based on equation (2) and the training data set. If Eq. (2) is heavily violated, the reconstructed reflectance function would largely deviate from the true reflectance function.

## Conclusions

This paper describes a new method for generating spectral reflectance functions based on camera signals. Unlike the Wiener and HF methods, the new method does not need to know or to estimate the camera's sensors. However, it requires a training data set including camera's signals and their corresponding spectral reflectance functions. The method first characterizes the camera and estimates the matrix  $W$  to map the camera's signal to its reflectance function simultaneously. After the matrix  $W$  is found, the constrained least squares problem is then used for generating the reflectance, which is always within the desired spectral reflectance function range. Testing different methods using the real and simulated data, the comparison results showed that the proposed method outperformed both the Wiener and HF methods.

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