# Extending Printing Color Gamut by Optimizing the Spectral Reflectance of Inks

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## Abstract

A computer simulation was performed to investigate the optimum combinations of three- and four-chromatic inks in order to maximize the color gamut for halftone printing. A virtual printing model was created based on the Yule-Nielsen-spectral-Neugebauer equations and Kubelka-Munk turbid media theory and was used to predict the spectral reflectances and colorimetric coordinates of prints. In this manner, the spectral properties of a printer's entire color gamut were predicted from the spectral properties of ink and paper. A new method was developed to derive plausible ink spectral properties based on two Gaussian functions. The results showed that the optimum three-chromatic inks, as expected, were cyan, magenta and yellow. For four chromatic inks, several optimal results were found.

## Introduction

A printer's color gamut is the volume of the color solid defined colorimetrically produced by a particular set of ink and paper, which contains all the colors that the printer can produce. The color gamut is affected by many factors including the ink formulation, printing process, substrate, and halftoning or lay down technique. In this research, we will concentrate on the effect of the number of inks and their spectral properties. Generally, printers with just four process inks, cyan, magenta, yellow and black, only can produce a relatively small color gamut, limiting color reproduction accuracy. For colors that are out of gamut, color-mapping algorithms are required, which by definition, result in reduced accuracy. A large color gamut is always desirable for color reproduction.

One effective method to expand the color gamut is increasing the number of inks. Most multi-ink printing research includes conventional process color inks, cyan, magenta, yellow and black (CMYK), plus other colors, 1-9 as summarized in Table I. The addition of inks to the traditional CMYK four inks generally chooses intermediate hues, such as red, green and blue since it is difficult to produce good greens, reds and blues with traditional CMYK inks.

In general, inks used for ink-jet printers can be selected from among many potential inks. However, such trial-and-error approaches are inefficient, and there remain questions as to how to guarantee a truly optimum ink set. An alternative approach is to develop a computer simulation system based on a printer model to select the optimum inks.

Viggiano and Hoagland[5] created a small system to select colorants for six-color lithographic printing. They used two criteria to optimize the ink set. One was the gamut volume that was image-independent, and the other was the fraction of colors within a digital image that were contained within the gamut. The experimental results showed that a printer with six inks provided a significantly larger volume than four inks. At the same time the optimized ink set also increased the fraction of colors within the gamut.

In the field of photography, Ohta[6-8,10] carried out a series of studies in order to search for the optimum combinations of three and four dyes in transmission-type color films and in reflection-type color prints by a computer simulation method. He optimized the peak wavelengths and breadths of the spectral density of the dyes to maximize color gamut.

Takayuki[11-13] performed a similar analysis for dye-based ink-jet printing. He evaluated different printing models and selected the Kubelka-Munk model in order to predict the color gamut created by hypothetical dye sets. Only one-color ramp data were used to evaluate the printing model, so the accuracy of the model for overprints was not disclosed. In a personal communication, Takayuki introduced more recent results. He used symmetric and asymmetric cubic-spline functions to model the ink's spectral density properties and optimized ink curves by adjusting the peak wavelength and width to maximize color gamut. The advantage of this method is that only two parameters are needed to simulate a spectral ink curve, which can simplify the optimization procedure. However, these functions can only simulate single peak spectra in density space. As a consequence, green inks cannot be simulated, for example. This limited the range of inks that were chosen.

In this research, a new method is described to synthesize ink spectra in reflectance space. When combined with a virtual printing model, it was possible to optimize the spectral properties of any number of printing inks.

**Table I. Summary for Multiple Ink Printing Research.**

| Author | Küpper[1] | Ostromoukhov[2] | | Boll[3] | Takaghi[4] | Viggiano[5] | Ohta[8] | Takayuki[9] |
|---|---|---|---|---|---|---|---|---|
| Ink number | 7 | 7 | 9 | 7 | 9 | 6 | 4 | 4 |
| Ink 1 | Cyan | Cyan | P* 801C | Cyan | Purple | Cyan | Cyan | Cyan |
| Ink 2 | Magenta | Purple | P* 802C | Magenta | Magenta | Magenta | Magenta | Magenta |
| Ink 3 | Yellow | Yellow | P* 803C | Yellow | Yellow | Yellow | Yellow | Yellow |
| Ink 4 | Black | Black | P* 804C | Black | Black | Black | Purple | Orange |
| Ink 5 | Red | Red | P* 805C | Red | Red | Purple | | |
| Ink 6 | Green | Green | P* 806C | Green | Green | Orange | | |
| Ink 7 | Blue | Blue | P* 807C | Blue | Blue | | | |
| Ink 8 | | | Blue | | Orange | | | |
| Ink 9 | | | Black | | Gray | | | |

P*-PANTONE

## Virtual Printing Model

When the ink optimization is performed, a printing model is required to accurately predict the spectral reflectance of printings with arbitrary inks since it is impossible to print and measure all possible combinations of inks for optimization. A virtual model was developed based on the Yule-Nielsen Spectral Neugebauer model (YNSN)[14] and Kubelka-Munk (K-M) theory.[15]

It is well known that the YNSN model and its cellular extension can achieve high prediction accuracy.[16] In a recent modeling of a six-color ink-jet printer,[17] the average spectral RMS error was less than 0.5% and the average $\Delta E_{00}$ was less than 1.0. However, this level of performance required a number of actual printed samples. For the current research, a model was required that only required knowledge of the spectral reflectance factors of paper and ink at full area coverage. The YNSN model was chosen as the basic model to optimize the ink set because of its reasonable accuracy and simple form.

Stollnitz[18] proposed a method to model the spectral reflectance factor of overprints printed on top of an opaque support. Tzeng[19] used Kubelka-Munk turbid media theory for translucent inks printed on top of a highly scattering support. In this research we assumed that transparent ink-jet inks would penetrate the paper support, yielding a homogenous colored layer. Accordingly, the opaque K-M equations were used:

$$R_\lambda = 1 + \left(\frac{K}{S}\right)_\lambda - \left[\left(\frac{K}{S}\right)_\lambda^2 + 2\left(\frac{K}{S}\right)_\lambda\right]^{1/2} \quad (1)$$

$$\left(\frac{K}{S}\right)_\lambda = \frac{(1 - R_\lambda)^2}{2R_\lambda} \quad (2)$$

where (K/S) is the absorption and scattering ratio, calculated using Eq. (3):

$$\left(\frac{K}{S}\right)_{\lambda,primary} = \left(\frac{K}{S}\right)_{\lambda,paper} + \left(\frac{K}{S}\right)_{\lambda,ink1} + \left(\frac{K}{S}\right)_{\lambda,ink2} + \cdots + \left(\frac{K}{S}\right)_{\lambda,inkn} \quad (3)$$

Based on these Neugebauer primaries predicted by the Kubelka-Munk model, the YNSN model was created. The YNSN model can be used to predict the spectral reflectance of overprints with different inks. The YNSN model is written as follows:

$$R(\lambda) = \left(\sum_i F_i R(\lambda)_i^{1/n}\right)^n \quad (4)$$

where $R(\lambda)_i$ is the macroscopic spectral reflectance of the $i^{th}$ color type at 100% area coverage, $n$ is the Yule-Nielsen exponent, and $F_i$ are the fractional area coverages of each microscopic color type. The fractional areas are determined as a product of random variables, shown in Eq. 5. These probabilities, when used for printing, are attributed to DeMichel.[20]

$$F_i = \prod_j \begin{pmatrix} a_j & \text{If ink j is in Neugebauer Primary i} \\ (1 - a_j) & \text{If ink j is not in Neugebauer Primary i} \end{pmatrix} \quad (5)$$

where $a_j$ is the effective area coverage of ink j.

## CIELAB94 Color Space

The goal of this research was to search for the optimum ink combinations that maximized the size of the color gamut. In order to evaluate the effect of printing gamut size, a more uniform color space was desirable than CIELAB space. A correction was derived based on the CIE94 color difference equations to express the color gamut in a more uniform color space. The following equations were used to calculate new values, designated as $L_{94}$, $a_{94}$, and $b_{94}$. This correction was introduced by Luo[21] in his LLAB color space:

$$\begin{aligned} L_{94} &= L^* \\ C_{94} &= \frac{\ln(1 + 0.045C^*_{ab})}{0.045} \\ h_{94} &= \arctan\left(\frac{b^*}{a^*}\right) \\ a_{94} &= C_{94}\cos(h_{94}) \\ b_{94} &= C_{94}\sin(h_{94}) \end{aligned} \quad (6)$$

The color gamut volume, $V$, can be expressed as function of $R_{ink}$ and factorial area coverages:

$$V = f\left(R_{ink}, a_{factorial}\right) \tag{7}$$

The objective function, maximizing the color gamut volume, can be expressed as:

$$R_{ink} = \max(V) \tag{8}$$

where $f$ represents the function calculating color gamut volume from basic spectra of inks and factorial area coverage, $V$ represents the volume of color gamut, $R_{ink}$ is the reflectance spectra of printer inks and $a_{factorial}$ is factorial area coverage.

## Experimental

### Creation of the Virtual Printing Model

The parameters of the virtual printing model were optimized based on a modified Epson Pro 5500 ink-jet printer. The light cyan and magenta were replaced with pigmented orange and green inks yielding a CMYKGO ink set. Samples were printed on Epson photo quality ink jet glossy paper, model number KA3N20MDK. The spectral reflectance of this paper was also used as the reflectance of the substrate in the virtual model.

The (K/S) of the paper was calculated from its reflectance, as shown in Eq 2. The (K/S) of single inks were calculated from the measured reflectance of these inks printed on paper and paper's reflectance. The n value of YNSN model was optimized to minimize the spectral root mean square (RMS) error between predictions and measurements based on printed one color ramps.

The performance of the virtual printer model was evaluated according to this printer and its CMYKGO ink set. 600 test samples were printed and their reflectance was measured using a GretagMacbeth Spectrolino spectrophotometer. Colorimetric values were calculated for the illuminant D50 and the CIE 1931 2° standard observer.[21] The YNSN model with measured Neugebauer primaries was also used to predict these test samples, the results shown in Table II. The color difference $\Delta E_{00}$ was calculated. The predicted spectra were parametrically corrected[22] such that a perfect match was obtained for *D50*, then their $\Delta E_{00}$ was calculated for illuminant *A* and used as a metameric index (MI under *A*).[23] The correction was also performed for illuminant A and metamerism evaluated for *D50* (MI under *D50*). The model results using the YNSN model with primaries estimated using K-M theory were reasonable. The spectral predictions using K-M theory were very similar to actual measurements.

### Spectral Reflectance of Hypothetical Inks by Gaussian Function

The spectral reflectance of an ink was modeled by two Gaussian functions with six parameters: two peak wavelengths, two bandwidths and two peak heights. The Gaussian functions are shown in Eq. (9):

$$R = \begin{cases} h1 \cdot \dfrac{e^{-(x-p1)^2}}{2 \cdot w1^2} + h2 \cdot \dfrac{e^{-(x-p2)^2}}{2 \cdot w2^2} & R < R_{\max} \\ R_{\max} & R \geq R_{\max} \end{cases} \tag{9}$$

where h1 and h2 represent the heights, p1 and p2 represent the peak wavelengths and w1 and w2 represent the widths at half height. Reflectances larger than a defined maximum were truncated and set to this maximal value. Finally, the curve was smoothed in the region of truncation. Constraints were defined according to real Epson inks. In this research, the maximum and minimum reflectances were 0.86 and 0.006, respectively. This function was evaluated by simulating real chromatic inks curves. Table III shows the spectral RMS and colorimetric errors between real and simulated ink spectra. Figure 1 shows the reflectance of the real (dashed line) and simulated (solid line) magenta. This Gaussian function could accurately simulate most kinds of ink curves in reflectance space.

**Table II. Performance Comparison Between YNSN Model and Virtual Printer**

| | $\Delta E_{00}$ | | Spectral RMS (%) | | MI (D50→A) | | MI (A→D50) | |
|---|---|---|---|---|---|---|---|---|
| | Mean | Max | Mean | Max | Mean | Max | Mean | Max |
| Measured primaries | 2.4 | 8.7 | 1.3 | 6.6 | 0.5 | 2.8 | 0.5 | 2.8 |
| Estimated primaries | 2.9 | 11.6 | 1.4 | 5.4 | 0.7 | 3.4 | 0.7 | 3.8 |

**Table III. The Spectral RMS Errors and Colorimetric Errors of Simulated Ink Curves.**

| | Cyan | Magenta | Yellow | Green | Orange |
|---|---|---|---|---|---|
| Spectral RMS | 2.26% | 2.13% | 1.60% | 1.49% | 1.5% |
| $\Delta E_{00}$ | 1.3 | 0.58 | 0.87 | 0.61 | 0.72 |

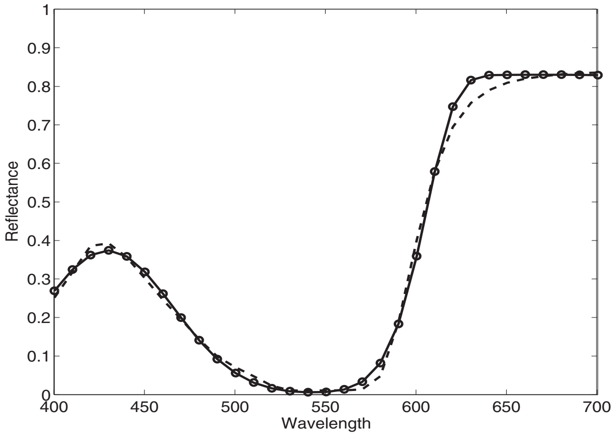

*Figure 1. Spectral reflectance factor of real (solid line with circle marker) and simulated (dashed line) magenta ink-jet ink.*

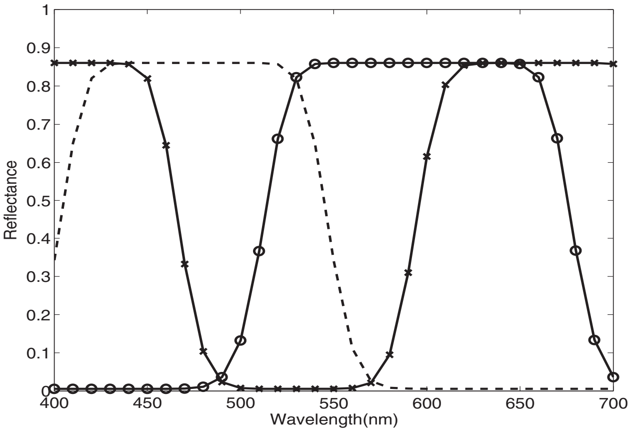

*Figure 2. Spectral reflectance of the three optimized inks. Cyan-dashed line; magenta-solid line with x marker and yellow- solid line with circle markers.*

## Printing Gamut

The color gamut and its volume can be calculated with factorial area coverage data. For example, we selected 11 steps from 0% to 100% area coverage in intervals of 10% for each ink. By combining these steps of four colors, there were $11^4$=14,641 samples.

According to the area coverage, corresponding spectral reflectances were calculated by the virtual printer model, tristimulus values calculated for illuminant D50 and the 1931 observer, and $L_{94}$, $a_{94}$, and $b_{94}$ calculated. The volume of printing color gamut was calculated using the convex hull algorithm in Matlab.

## Optimization Results

An ink combination was optimized to maximize the printing gamut. This problem is classified as a constrained nonlinear optimization and was solved by a genetic algorithm.[25] The optimization results are described in the following sections. The optimization results were verified with different starting values and was proved to be global optimization results.

### *The Optimum Combination of Three Inks*

First, three ink combinations were optimized. Figure 2 shows the optimized spectral reflectance for the three inks. Its printing gamut was compared with the gamut composed by real three inks, cyan, magenta and yellow, used in the Epson printer. These gamuts were also compared with the gamut of sRGB,[24] which was calculated with 1% flare, to indicate the extent they matched each other. Figure 3 and Figure 4 show the comparison of printing gamut at lightness levels of 40 and 70 L*. The dashed line represents the gamut of the optimized inks and the solid line represents that of typical inks. The dash-dot line represents the sRGB gamut. The printing gamut created by the typical inks is $1.34 \times 10^5$ units in CIELAB94 space and $1.95 \times 10^5$ units for the optimized ink set. The color gamuts created by the optimum combination of three inks are 1.5 times larger than the real set.

### *The Optimum Combination of Four Inks*

The method was used to optimize four-ink combinations that maximized the printing gamut. Two sets of four-ink combinations were found that achieved very similar gamuts.

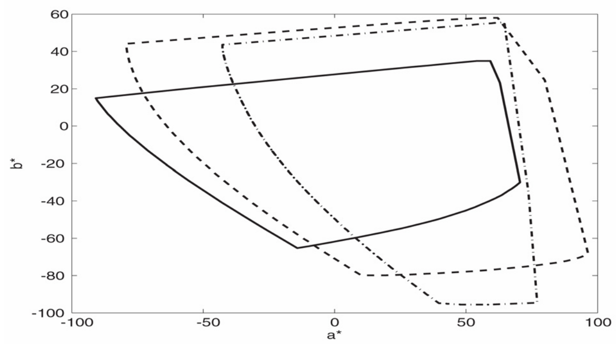

*Figure 3. Printing gamut comparison for the three optimized inks (dashed line) and typical inks (solid line) and sRGB gamut (dash-dot line) at lightness level 40.*

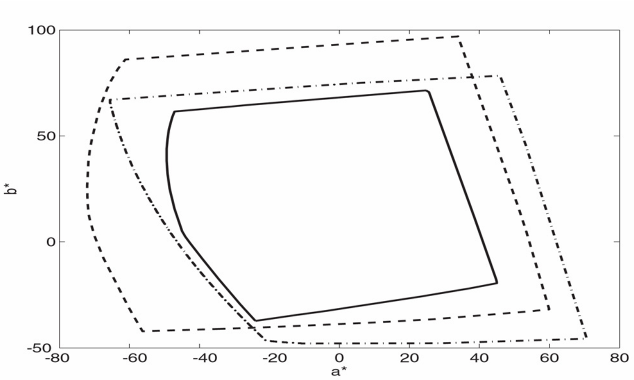

*Figure 4. Printing gamut comparison the three optimized inks (dashed line) and typical inks (solid line) and sRGB gamut (dash-dot line) at lightness level 70.*

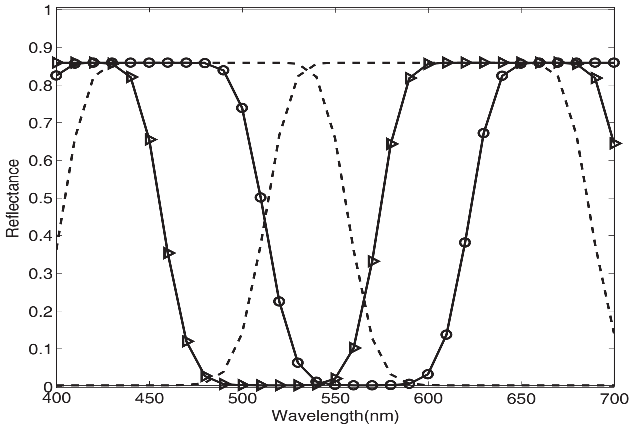

*Figure 5. Spectral reflectance of optimized four inks. Magenta ink—solid line with triangle markers; Purple ink – solid line with circle markers; cyan, yellow—dashed lines*

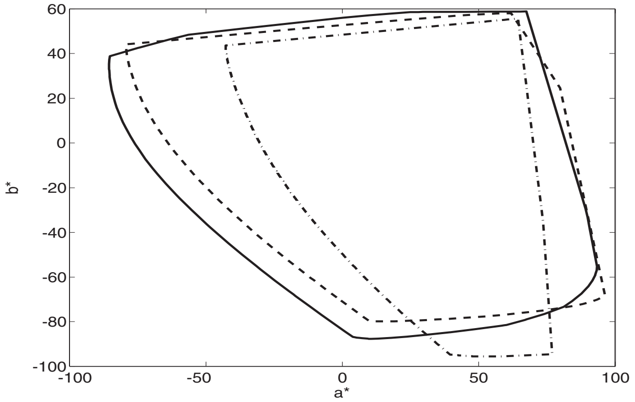

*Figure 6. Printing gamut comparison between four optimized inks (solid line) and three optimized inks (dashed line) and sRGB gamut (dash-dashed line) at lightness level 40.*

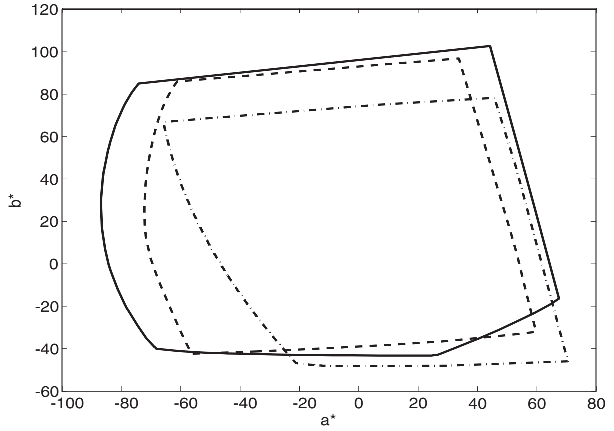

*Figure 7. Printing gamut comparison between the four optimized inks (solid line) and three optimized inks (dashed line) and sRGB (dash-dot line) at lightness level 70.*

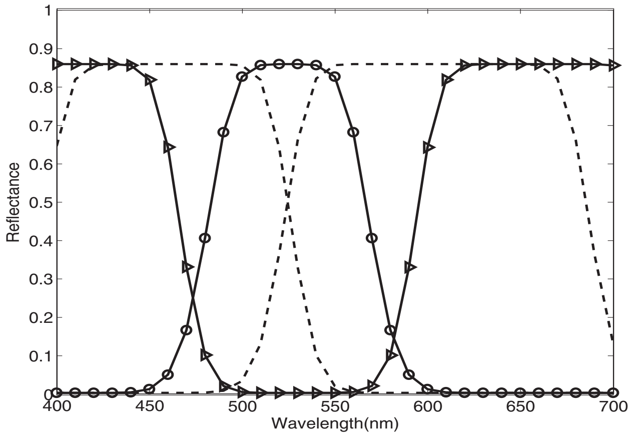

*Figure 8. Spectral reflectance of second four ink combination. Magenta ink—solid line with x markers; green ink – solid line with circle markers; cyan, yellow—dashed lines*

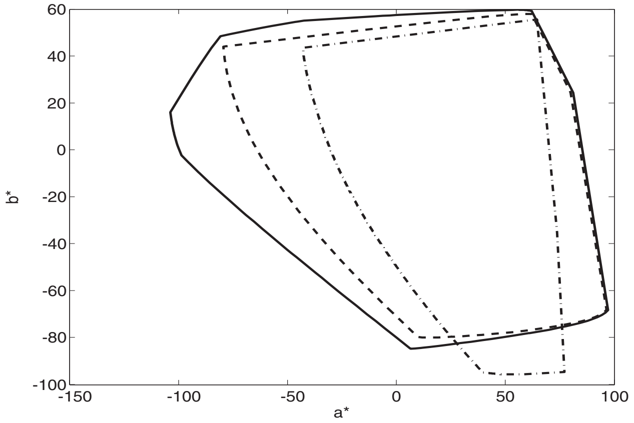

*Figure 9. Printing gamut comparison of the four optimized inks (solid line) and three optimized inks (dashed line) and sRGB (dash-dot line) at lightness level 40*

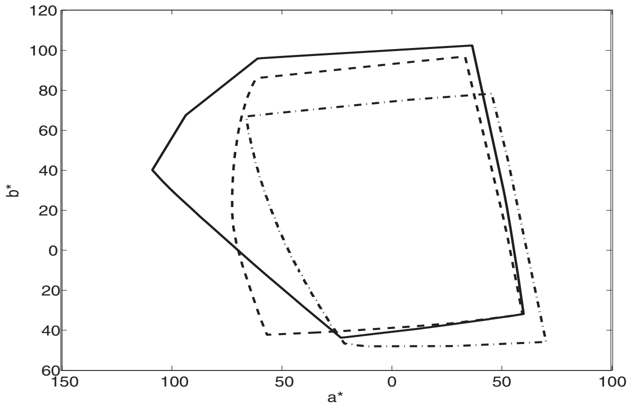

*Figure 10. Printing gamut comparison of the four optimized inks (solid line) and three optimized inks (dashed line) and sRGB gamut (dash-dot line) at lightness level 70*

Figure 5 shows the spectral reflectance of these inks. The added ink is some kind of purple. Figure 6 and 7 show the printing gamut comparison among optimized three inks, this four ink combination and sRGB with lightness levels of 40 and 70 L*.

Another four-ink combination achieved a similar gamut. Figure 8 shows the spectral reflectance of these inks. A green ink was added to cyan, yellow and magenta. Figure 9 and 10 show the printing gamut comparison between the optimized three inks and this four ink combination at lightness levels of 40 and 70. We can find that the printing gamut increased significantly in the green direction, negative a*.

Table IV shows the summary of printing gamut volumes for typical three inks, optimized three inks, the two sets of optimized four inks and sRGB. From Table IV, we find that adding one-ink to a typical three-ink system can increase printing color gamuts by approximately 14%. The difference of volume size between two printing gamuts for four-ink combinations was very small.

**Table IV. Printing Gamut for Different Inks Combinations**

| | Typical 3 inks | Optimized 3 inks | Optimized 4 inks 1 | Optimized 4 inks 2 | sRGB |
|---|---|---|---|---|---|
| Gamut Volume | $1.34x10^5$ | $1.95x10^5$ | $2.22x10^5$ | $2.20x10^5$ | $1.75x10^5$ |

## Conclusions

The optimum three-ink combination and four-ink combinations were explored. The optimum three-ink combination was stable, that is, the results were always obtained at very different starting values. These optimized results correspond with previous results by Ohta and Takayuki. These results confirm that for three-color subtractive systems, the optimum colorants can only be cyan, magenta and yellow.

The optimization of four inks resulted in two different ink combinations, which can achieve almost similar volume size of printing gamuts, while different inks combinations will increase the printing gamut in different directions. The four-ink system can give around 14% increase of printing gamut. However, in this research an ink limitation was not considered. We think that under the limitation of ink amount, the added ink will contribute more to the color gamut. In this research the black ink was not included into the ink set because the affect of the black ink to the size of the printing gamut was negligible. The black ink will be considered when the limitation of ink amounts is applied.

In multi-ink printing systems, there are usually more inks than colorimetric coordinates. As a consequence, many colors can be matched using more than one combination of inks. That makes it possible to create a set of ink combination for minimizing the color inconstancy of prints. The color constancy is the general tendency of the color of an object to remain constant when the level and color of the illumination are changed. In a future study, the author will use color constancy as a criterion to optimize the ink spectra. The black ink will be considered because it can improve the color constancy of prints, significantly. More limitation will be applied on primary ink spectrum to simulate real inkjet printing.

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## Biography

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