Color conversion for multi-primary displays using a spherical average method

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Abstract

Multi-primary color display devices, which have more than four primaries, require more than two sets of control signals to reproduce specific tristimulus values. However, rapid changes in the control signals, even when the reproduced tristimulus values change smoothly, is considered to cause degradation of image quality. Smooth multi-primary color decomposition, which means that the control signals change smoothly when the tristimulus changes smoothly, is required. To meet this requirement, we propose a new method for smooth multi-primary color decomposition based on the spherical average of signal values.

1. Introduction

Existing analog/digital image systems receive and display images using a combination of three primary colors (red, green, and blue) because the human visual system consists of three sensors with different spectral sensitivities. However, a display device based on three primary colors cannot cover the entire color gamut perceived by the human eye. It is therefore widely acknowledged that technologies to collect accurate color spectrum information using multi-band cameras and reproduce color faithfully using multiple primary color displays are essential if high-fidelity color reproduction is to be achieved.

The technology for color reproduction of an object using multiple primary colors is referred to as multi-primary color decomposition. It is classified into two color reproduction technologies: one focuses on physical colors (spectral distribution) and the other focuses on the colors perceived using human tristimulus values (XYZ).

Color reproduction along a color spectrum that is close to the color spectra reflected by an illuminated object is the technology used to display colors in a spectral space. However, although ideally the spectral distribution should be the same as that of the illuminated object, neither photography, printing, nor television can achieve this using a finite number of primary colors.

In contrast, the human visual system consists of three sensors with different spectral sensitivities. Colors with different spectral distributions look the same if the tristimulus values (XYZ) created by these sensors are the same. Using this phenomenon of metamerism, we can achieve high-fidelity display of colors by producing colors with the same tristimulus (XYZ) as that of the object. While color reproduction based on multi-primary colors expands the color gamut as opposed to three primary colors, we then face the problem of not being able to determine a unique multi-primary color decomposition.

Let $p_i(\lambda), i = 1, \cdots, N$ be the spectral distribution of N primary colors, and $0 \le s_i \le 1, i = 1, \cdots, N$ be the control signal. The tristimulus value x_1, x_2, x_3 within the color gamut is given as:

$$x_i = \sum_{j=1}^{N} s_j \int m_i(\lambda) p_j(\lambda) d\lambda, \quad i = 1, 2, 3, \tag{1}$$

where $m_1(\lambda), m_2(\lambda), m_3(\lambda)$ represent the color matching functions (CMF). Using a signal vector $s = (s_1, \cdots, s_N)^T$ and 3xN matrix $P = (p_{ij})_{3\times N}$ consisting of the primary tristimulus value $p_{ij} = \int m_i(\lambda) p_j(\lambda) d\lambda$, then the tristimulus value $x = (x_1, x_2 . x_3)^T$ is calculated as

$$x = Ps, \quad 0 \le s \le 1. \tag{2}$$

The problem of not being able to determine a unique multi-primary color decomposition can be mathematically described as follows. Since the control signal s of a primary color has $N - 3$ degrees of freedom for the three-dimensional tristimulus value x of an object, Eq. (2) can yield multiple solutions. Applying other constraints to Eq. (2) before solving it provides a key to multi-primary color decomposition.

Once the spectral distribution of an object is obtained, one possible approach is to apply the constraint of approximation of the spectral space to Eq. (2), which results in good color reproduction [1],[2]. Considering that visual sensory characteristics differ between individuals and between age groups, there is another approach to reduce color mismatching caused by variations in CMF data [3], [4].

There is a need to reproduce color in a three-dimensional tristimulus value space for only tristimulus image data. Accordingly, several multi-primary color decomposition methods, such as matrix switching (MS) [5], linear interpolation of the equi-luminance plane method (LIQUID) [6], and the metameric black method (MB) [7], have been developed.

However, the matrix switching method may result in abrupt changes in control signal values because different linear conversions are applied to different regions in the color gamut of multi-primary displays [8]. Abrupt changes in control signals are known to result in image quality degradation since the human visual system emphasizes discontinuity in changes in the tristimulus value of a color stimuli.

To improve the continuity of control signals, the LIQUID and MB methods were proposed [6], [7]. In particular, the MB method shows that the signals generated can change smoothly over the whole color gamut [8]. For a given tristimulus value, the solution of Eq. (2) is a $N-3$ dimensional set. When the tristimulus values change smoothly, the solution set also changes smoothly. The MB method takes advantage of this property. In fact, the center of gravity of the solution set was selected as the control signal.

However, for some tristimulus values near the surface of the color gamut, the shapes of the solution set do not change so smoothly. We know that only one set of control signals exists for any tristimulus value on the surface of the color gamut. This surface condition must be considered in attempting to generate smooth control signals over the whole color gamut of a multi-primary display. This paper proposes a new method for multi-primary color decomposition that features smooth changes in control signals even when the tristimulus value is near the surface of the color gamut.

2. Spherical average decomposition

In the following, the above issues are described mathematically. Let $\Omega = \{x \mid x = Ps, 0 < s < 1\}$ be an open set of tristimulus values that can be displayed using multiple primary colors. Ω is an $N \times (N-1)$ surface convex polyhedron. On its surface $\partial\Omega$, the control signal $\tilde{s}(x) = (\tilde{s}_1(x), \cdots, \tilde{s}_N(x))^T, x \in \partial\Omega$ is unique. If the number of primary colors, N, is greater than 3, the color gamut contains several control signals $s(x)$ that satisfy Eq. (2). It is desirable to have smooth control signals $s(x)$ among these control signals.

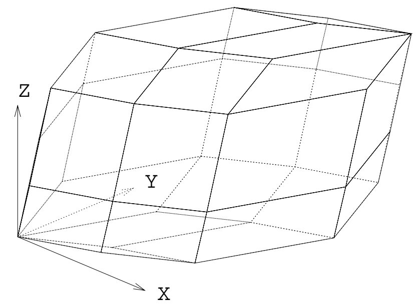

Figure 1: *Display gamut for six-primary color display*

In the tristimulus values space, for any two colors x_A and x_B, any color x_O along the straight line connecting x_A and x_B can be generated from x_A and x_B as follows:

$$x_O = \frac{|x_B - x_O|}{|x_B - x_A|} x_B + \frac{|x_A - x_O|}{|x_B - x_A|} x_A. \quad (3)$$

Suppose $s(x_A)$ and $s(x_B)$ are control signals for x_A and x_B. Then the control signal $s(x_O)$ can be calculated as follows using the same coefficient as in Eq. (3):

$$s(x_O) = \frac{|x_B - x_O|}{|x_B - x_A|} s(x_B) + \frac{|x_A - x_O|}{|x_B - x_A|} s(x_A). \quad (4)$$

Both the MS and LIQUID methods are based on this principle. $s(x_O)$ is also smooth in the direction of the straight line connecting x_A and x_B, but it cannot maintain this smoothness in other directions.

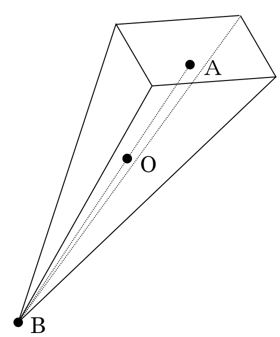

Figure 2: *Example of matrix switching method*

To provide smoothness in all directions, the tristimulus value x_O within the color gamut of Ω is generated using the spherical average of the sphere $S_r(x_O)$ with a radius of r and center x_O instead of applying Eq. (3) as follows:

$$x_O = \frac{1}{\omega_r} \int_{S_r(x_O)} x ds_x. \quad (5)$$

The above equation can be rewritten as follows for convenience:

$$\int_{S_r(x_O)} (x - x_O) ds_x = 0. \tag{6}$$

Let the radius r be so small that the sphere $S_r(x_O)$ is contained within the color gamut Ω. For each x on the surface of $S_r(x_O)$, there is y on the surface of the color gamut Ω such that

$$\frac{x - x_O}{|x - x_O|} = \frac{y - x_O}{|y - x_O|} \tag{7}$$

is satisfied.

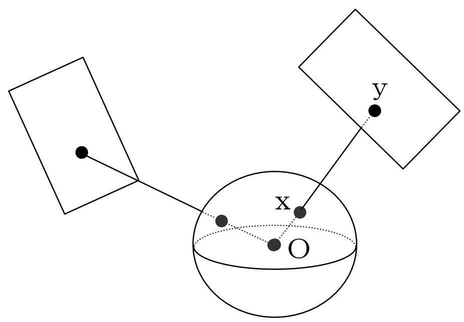

Figure 3: *Unit sphere and gamut surface*

Note that

$$ds_x = \frac{|x - x_O|^2}{|y - x_O|^2} \frac{(y - x_O) \cdot n}{|y - x_O|} ds_y, \tag{8}$$

where n is the outward normal of y on the surface $\partial\Omega$, then Eq. (6) can be converted to the integral on the surface of the color gamut Ω,

$$\int_{\partial\Omega} \frac{y - x_O}{|y - x_O|} \frac{(y - x_O) \cdot n}{|y - x_O|^3} ds_y = 0. \tag{9}$$

Define $A(x_O)$ as follows:

$$A(x_O) = \int_{\partial\Omega} \frac{(y - x_O) \cdot n}{|y - x_O|^4} ds_y, \tag{10}$$

x_O can be obtained by

$$x_O = \frac{1}{A(x_O)} \int_{\partial\Omega} y \frac{(y - x_O) \cdot n}{|y - x_O|^4} ds_y. \tag{11}$$

Now we define $u(x_O)$ as

$$u(x_O) = \frac{1}{A(x_O)} \int_{\partial\Omega} \tilde{s}(y) \frac{(y - x_O) \cdot n}{|y - x_O|^4} ds_y, \tag{12}$$

where $\tilde{s}(y)$ is a control signal to generate the tristimulus value $y = P\tilde{s}(y)$ on $\partial\Omega$. Lastly, we multiply Eq. (12) by matrix P to obtain

$$\begin{aligned} Pu(x_O) &= \frac{1}{A(x_O)} \int_{\partial\Omega} P\tilde{s}(y) \frac{(y - x_O) \cdot n}{|y - x_O|^4} ds_y \\ &= \frac{1}{A(x_O)} \int_{\partial\Omega} y \frac{(y - x_O) \cdot n}{|y - x_O|^4} ds_y \\ &= x_O, \end{aligned}$$

which means that the control signal $u(x_O)$ generates the tristimulus value x_O. It is interesting that the control signal $u(x_O)$ is found by taking the spherical average of all the control signals on the surface of the color gamut. In the next section, we describe several experiments that were carried out to evaluate the smoothness of the control signal $u(x_O)$ when the tristimulus value x_O changed smoothly.

3. Evaluation

In multi-primary display systems, any discontinuity in the control signals is considered to have an adverse effect on the quality of the images [8]. We therefore focused on evaluating the smoothness of the control signals.

In our experiments, we used a six-primary DLP projection display. The spectral distribution of the six-primary display is shown in Figure 4, where the primaries are labeled P (purple), B (blue), C (cyan), G (green), O (orange), and R (red).

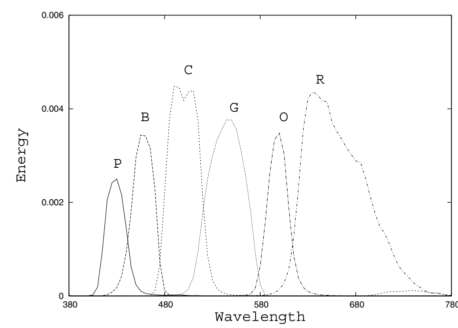

Figure 4: *Spectral distribution of six-primary display*

First, we evaluated the distribution of one-order differentials derived from our method and the MB method. The norm of the one-order differentials of a control signal in a $L^*a^*b^*$ uniform color space is defined as follows:

$$D(L^*, a^*, b^*) = \| \bigtriangledown s(L^*, a^*, b^*) \|^2, \tag{13}$$

where the gradient operator $\bigtriangledown$ is defined as $(\frac{\partial}{\partial L^*}, \frac{\partial}{\partial a^*}, \frac{\partial}{\partial b^*})^T$. We selected one million tristimulus values uniformly over the gamut, and calculated one-order differentials for each of these values. For small differentials (less

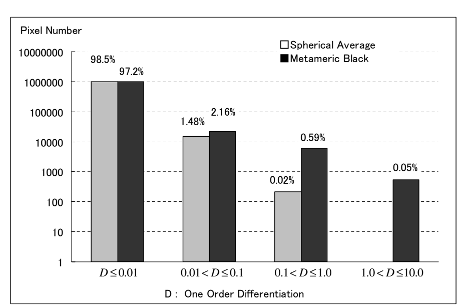

Figure 5: *Differential distribution of control signals*

than 0.01), we obtained 98.6% of the tristimulus values using our method, and 97.2% using the MB method. But for larger differentials between 0.1 and 1.0, we obtained 0.59% of the tristimulus values using the MB method, and only 0.02% using our method. These results show that our method has advantages over the MB method in achieving smooth changes in control signals. The distribution of the one-order differentials is shown in Figure 5,

Next we show three gradation patterns defined in a $L^* H^*\ C^*$ color space, where L^* indicates lightness, C^* chroma, and H^* hue. In these gradation patterns the color changes along with one of L^*, H^*, C^*, and the other two are constants.

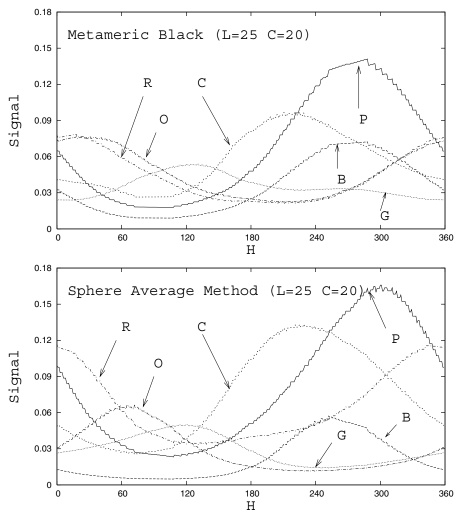

Figure 6: *Device control signals (H modulation at L=25 and C=20)*

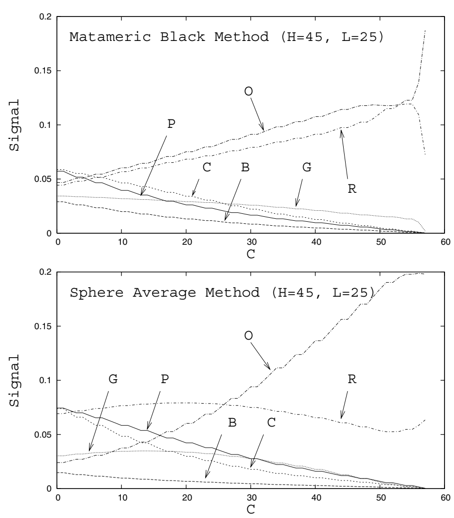

Figure 7: *Device control signals (C modulation at H=45 and L=25)*

When the hue component $0 \leq H \leq 360$ is modulated by the unique lightness $L = 25$, and chroma $C = 20$, the signals generated by both methods change smoothly (see Figure 6). In this case all the colors are not near the surface of the color gamut and we know that the set solution does not change as smoothly near the surface.

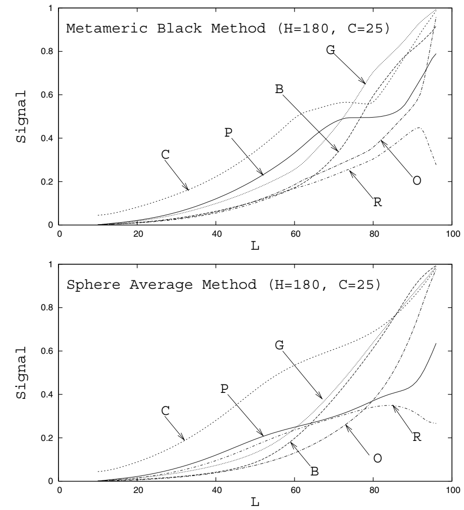

Figure 8: *Device control signals (L Modulation at H=180 and C=25)*

As Fig. 7 shows, when the chroma component $0 \leq C \leq 56$ is modulated by the unique lightness $L = 25$, and hue $H = 45$, the signals generated by our method change smoothly, but the signals generated by the MB method do not change smoothly when the primary colors R and O change near $C = 56$.

When the lightness component $0 \leq L \leq 100$ is modulated by the unique hue $H = 180$, and chroma $C = 25$, the signals generated by our method also change smoothly, as Fig. 8 shows, but the signals generated by the MB method do not change smoothly as the primary color R changes near $L = 90$.

The results showed that control signals generated by our method changed smoothly over the whole color gamut, and better than the MB method which lost smoothness at the edge of the color gamut.

4. Summary

We proposed a multi-primary color decomposition method which achieves greater smoothness than existing methods and evaluated its performance in terms of smoothness. In our experiments, the smoothness of our method was shown to be better than that of the MB method at the edge of the color gamut. This success was based on generating control signals that took the spherical average of all the control signals on the surface of the color gamut.

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Biography

Hiroshi Kanazawa received his Ph.D. in Mathematical and Computing Sci. from Tokyo Institute of Technology, Tokyo, Japan, in 1998. He joined USE Co., Ltd. in 1999. Since 2003 he has been a researcher at the Akasaka Natural Vision Research Center, NiCT, Tokyo, Japan.