A Radial Sampling of the OSA Uniform Color Scales

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Abstract

The OSA Uniform Color Scales were derived using a unique geometry for the physical samples. Regular rhombohedral packing allows each sample to be compared to twelve other equally distant samples. While this sampling scheme provides an efficient geometry for sample comparison and allows multiple cleavage planes, it obscures the underlying perceptual attributes. However, it is relatively straightforward to compute a radial sampling of data points in OSA. This radial sampling results in a distance from the achromatic axis and an angular quantity and can be used to compare other color spaces. This paper presents a method and considerations for computing the radial sampling. The utility of this data is demonstrated by comparing the perceptual uniformity of five different color spaces.

Introduction

Color spaces and color order systems provide a means to organize and describe color. A uniform color space is one in which equal differences in any part of the space correspond to equal perceptual differences. There is an abundance of prior and ongoing research in the area of uniform color spaces. The OSA Uniform Color Scales[1-6] or UCS is one such color space and was the result of extensive psychophyisical investigation and other considerations. The UCS exists both as physical samples and in equation form. The samples are based on a regular rhombohedral sampling scheme. This sampling results in twelve equally spaced neighbors for any given color and yields the closest uniform three-dimensional sampling. This sampling scheme has a number of advantages, including visualization of color differences along multiple cleavage planes and efficient sampling for psychophysical evaluation. However this sampling scheme can make it more difficult to visualize color space uniformity. For example the OSA UCS space has been compared to other color spaces and color order systems,[7] such as CIELAB[8] or the Munsell Book of Color,[9] or as a qualitative demonstration of perceptual uniformity for a physiologically based system of photometry and colorimetry.[10] However the radial structure of the Munsell Book of Color and the regular rhombohedral structure of the OSA UCS can make visual comparisons difficult. However,

aside from computational considerations there is no specific limitation or restriction on calculating alternative sampling schemes for the OSA UCS. This paper reviews the OSA UCS, the forward transformations and then proposes a radial sampling of the OSA UCS. Finally as an example of how this data can be used, the CIELAB, CIELUV,[8] CIECAM97s,[11] IPT[12,13] and CIECAM02[14] color spaces are compared to each other using the radial OSA UCS data.

Previous research using the OSA UCS has made use of planes taken from the regular rhombohedral sampling to assess the CIELAB color space.[15] Additional research has considered trends in hue angle differences for several color spaces, including OSA UCS.[16] This work complements previous work by providing a radial sampling scheme for the OSA UCS that allows alternative visualizations of trends in the data and the direct calculation of statistics on the polar coordinates. The sampling scheme is not intended for use as a look up table for image processing[17,18] but as a possible data set for comparing color spaces or other applications.

OSA Uniform Color Scales

The history of the development of the Optical Society of America's Uniform Color Scales is documented elsewhere.[1-6] In summary, extensive visual psychophysics was conducted to derive a highly perceptually uniform color space and resulted in both a physical set of samples and a set of forward equations. The OSA UCS makes use of the 10 degree 1964 CIE Standard Observer and consists of a lightness axis and two opponent color axes. The lightness axis is called L and ranges from about –9 to 5 and is centered around a middle gray. The yellow-blue axis is j and ranges from about –15 to 15. Finally, the red-green axis is g and ranges from about –20 to 15 as well. Note that the g axis is reversed relative to the other color spaces to be considered later in this paper.

A polar form of the space can be computed and the resulting angle and distance from the achromatic axis correspond roughly to hue angle and chroma, although these perceptual quantities were not modeled directly during the experimentation. A relatively recent modification[19] of the Uniform Color Scales was considered but not incorporated in order to provide compatibility with the larger body of research and publications making use of the original

equations. Furthermore, the procedure for computing the radial sampling can be applied regardless of whether the original or revised Uniform Color Scales are used.

The OSA USC exists as a book of physical samples arranged according to the regular rhombohedral scheme. The space also is defined by table of Ljg values and the corresponding 10-degree XYZ values for a D65 white point. Finally, there are equations that can be used to compute Ljg coordinates given 10 degree XYZ values. However, there is no closed form inverse for calculating tristimulus values given Ljg values. The following section reviews the forward transformation for computing Ljg values given 10 degree XYZ values. Next a standard numeric method is briefly described for use as an inverse transformation.

Forward Transformation

The forward transformation starts with 10 degree tristimulus values and computes intermediate variables Y_0 and L. Chromaticity values for the stimulus are also required and these can be computed from the XYZ values.[8] Next the 1964 XYZ values are converted to a new RGB space using a 3 by 3 matrix. After computing another intermediate variable C, the Ljg coordinates are computed. Calculation of L makes use of a cube-root non-linearity and has a correction for the crispening effect. Computation of j and g also make use of a cube-root non-linearity and opponent differences computed from the red, green and blue channels. The cube-root non-linearity is the same for all of the channels and unlike CIELAB, does not have a linear section at the lower end of the scale.

$$Y_0 = Y\left(4.4934\,x^2 + 4.3034\,y^2 - 4.276\,xy - 1.3744\,x - 2.56439\,y + 1.18103\right) \quad (1)$$

$$\Lambda = 5.9\left[Y_0^{1/3} - \frac{2}{3} + 0.042(Y_0 - 30)^{1/3}\right] \quad (2)$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 0.799 & 0.4194 & -0.1648 \\ -0.4493 & 1.3265 & 0.0927 \\ -0.1149 & 0.3394 & 0.717 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (3)$$

$$C = \frac{\Lambda}{\left[5.9\left(Y_0^{1/3} - \frac{2}{3}\right)\right]} \quad (4)$$

$$L = \frac{(\Lambda - 14.4)}{2^{1/2}} \quad (5)$$

$$j = C\left(1.7R^{1/3} + 8G^{1/3} - 9.7B^{1/3}\right) \quad (6)$$

$$g = C\left(-13.7R^{1/3} + 17.7G^{1/3} - 4B^{1/3}\right) \quad (7)$$

Given Ljg coordinates, polar coordinates can be computed using standard mathematical equations.

$$\rho = \left(j^2 + g^2\right)^{0.5} \quad (8)$$

$$\phi = \tan^{-1}\left(\frac{g}{j}\right) \quad (9)$$

where ρ roughly corresponds to chroma or the distance from the achromatic axis or j=0, g=0 point and ϕ roughly corresponds to hue or the angular location relative to the axes. It is important to note that modeling chroma and hue was not an explicit objective of the visual experiments and for this reason they are referred to as ρ and ϕ and not C and h. However, as will be seen in the discussion there is a good agreement between these quantities and perceptual attribute correlates of some color appearance models. Therefore while care is taken in this paper not to discuss OSA UCS chroma and hue scales, the ρ and ϕ quantities can be systematically sampled to provide a radial data set that can be used to assess perceptual uniformity of other color spaces. This is an important point since while the OSA UCS was not constructed to explicitly provide perceptual attribute correlates, such as chroma and hue, the resulting uniform radial sampling scheme provides a powerful independent test of other color spaces.

Inverse Transformation

There is no set of closed form inverse equations for computing the 10 degree XYZ values that result from a given set of Ljg coordinates. However, standard numeric methods can be applied, such as the Newton-Raphson method or tetrahedral interpolation. The Newton-Raphson approach has been used for other color transformations[20] and are is a possible method for consideration. This is an iterative algorithm and can be written:

$$(L_0, j_0, g_0) = f\left(X_i, Y_i, Z_i\right) \quad (10)$$

$$(L_1, j_1, g_1) = f\left(X_i + \Delta X, Y_i, Z_i\right) \quad (11)$$

$$(L_2, j_2, g_2) = f\left(X_i, Y_i + \Delta Y, Z_i\right) \quad (12)$$

$$(L_3, j_3, g_3) = f\left(X_i, Y_i, Z_i + \Delta Z\right) \quad (13)$$

$$\begin{bmatrix} X_{i+n} \\ Y_{i+n} \\ Z_{i+n} \end{bmatrix} = \begin{bmatrix} (L_1-L_0)/\Delta X & (L_2-L_0)/\Delta X & (L_3-L_0)/\Delta X \\ (j_1-j_0)/\Delta Y & (j_2-j_0)/\Delta Y & (j_3-j_0)/\Delta Y \\ (g_1-g_0)/\Delta Z & (g_2-g_0)/\Delta Z & (g_3-g_0)/\Delta Z \end{bmatrix}^{-1}\begin{bmatrix} L-L_0 \\ j-j_0 \\ g-g_0 \end{bmatrix} + \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} \quad (14)$$

where L, j, and g are the input OSA USC coordinate, ΔX, ΔY, and ΔZ are set to 3 and X_i, Y_i, and Z_i are initial estimates. These initial estimates can be made by using an approximate fit of the Ljg values to corresponding CIELAB values and can be calculated:

$$\hat{L}* = 5.4212\,L + 61.136 \qquad (15)$$

$$\hat{a}* = (-0.0989\,g^2) - 6.2408\,g + 0.6616 \quad (16)$$

$$\hat{b}* = (0,0507\,j^2) + 6.4797\,j + 1.2249 \quad (17)$$

The approximate CIELAB values can then be converted back to tristimulus values using the standard equations.[8] and used as the initial estimates. The calculations are repeated until some minimum error is achieved or a maximum number of iterations has occurred.

This method is mostly effective but use of forward-difference numerical derivatives may be problematic in some cases[†], such as the zero L plane and in fact an alternate method can be used to invert the equations. Specifically, given an initial estimate from equations 15 through 17, a random cloud of points is computed with a given radius. The point with the smallest resulting error is then used as the new center and the radius of the cloud of points is reduced. This process is repeated until some minimum error or some maximum number of iterations is reached.

Radial Sampling

The OSA UCS lightness or L was sampled at nine levels corresponding L* steps of 10 from 10 to 90 and is roughly similar to Munsell Value steps of 1 from 1 to 9. This resulted in L values of –9.16, -7.47, -5.78, -4.07, -2.33, -0.48, 1.78, 3.59 and 5.37. The step size for ϕ was then chosen to be 30 degrees starting at 0 degrees. This resulted in twelve different values for ϕ ranging from 0 to 330. The final consideration was how to sample ρ. In this case the boundary of real Munsell colors was used as an approximate limit. For example, figure 1 shows the resulting radial data in the CIELAB color space for L values of –0.48. The graph shows a* as the x-axis and b* as the y-axis and the corresponding limit for real Munsell colors is plotted as a dotted line. Other graphs at additional lightness levels show similar properties in which the extent of the radial sampling is approximately within the boundary of real Munsell colors. Other boundaries could have been used but this limit provides a rough correspondence with the real Munsell colors with respect to the range of the data.

The result is a 560 point data set, which balances the size of the data set with a sufficient degree of inter-point spacing for visualization purposes. The data is available electronically via the world wide web[‡] and consists of a table of Ljg values and corresponding 10-degree XYZ values. The resulting data was verified to be accurate to three decimal places relative to the forward transformations shown in equations 1 through 7.

[†] The author is grateful to J. A. S. Viggiano of Acolyte Color Research for bringing this issue to his attention and helpful discussion of alternative solutions, such as the use of a version of Powell's hybrid method.

[‡] http://www.hpl.hp.com/personal/Nathan_Moroney/osa-radial-560.dat
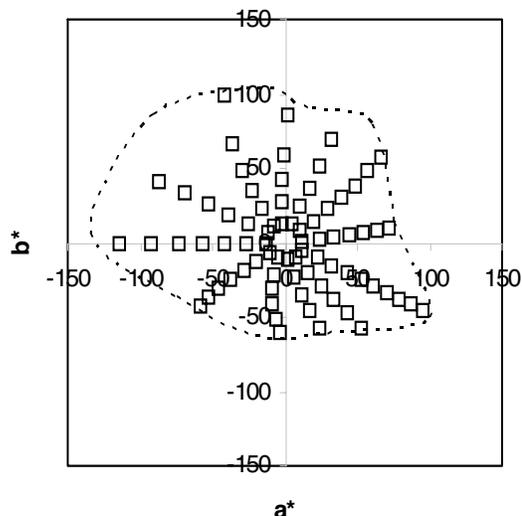
Figure 1. Plot of the OSA UCS data for L = -0.48 or roughly L* of 60 in the CIELAB color space with the corresponding approximate Munsell Value = 6 real colors boundary shown as a dotted line.

Assessing Color Space Uniformity

As an example of how this data could be used, five different color spaces are compared using the radial OSA UCS data. The CIELAB, CIELUV, CIECAM97s, IPT and CIECAM02 color spaces are assessed in figures 2 through 7. These figures show the radial OSA UCS data in each of the color spaces as viewed with the corresponding lightness axis going into the page and the opponent-color axes as the abscissa and ordinate. Figure 2 shows the original sampling scheme in the OSA UCS Ljg space. This sampling uses uniform steps in ρ and ϕ to create a series of co-centric circles with irregular limits, dependent roughly on the boundary of real Munsell colors. This arrangement of points should be used as a reference for the results shown in figures 3 through 7.

Figure 3 shows the results for the CIELAB color space where the x-axis is a* and the y-axis is b*. The data show a large degree of spread for the blue region of the color space. This is the lack of blue constancy that has been observed for CIELAB in which lines of constant hue tend to shift from blue to purple as the chroma is decreased.[21] This curvature is also seen when Munsell constant hue blue data is plotted in CIELAB.[22] However, there is very little spread in the data along the 150 degree and 330 degree hue angles. In fact, the b* axis can be well fit to j and it is primarily the disagreement between a* and g that accounts for the differences between the OSA UCS and CIELAB.
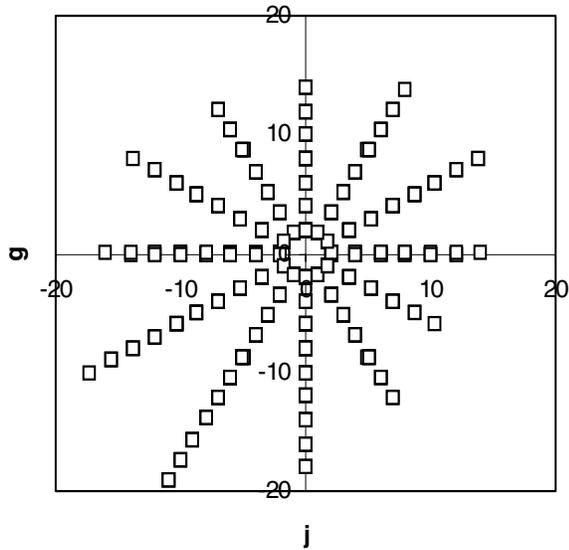
Figure 2. Plot of the radial OSA UCS data in the UCS Ljg space.
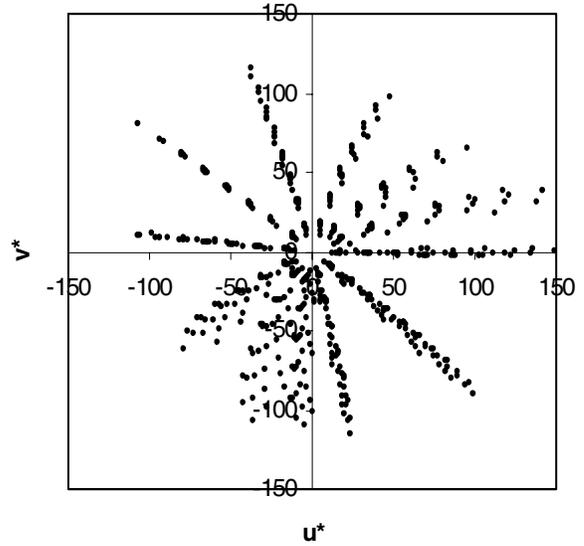
Figure 4. Plot of the radial OSA UCS data plotted in the CIELUV color space.
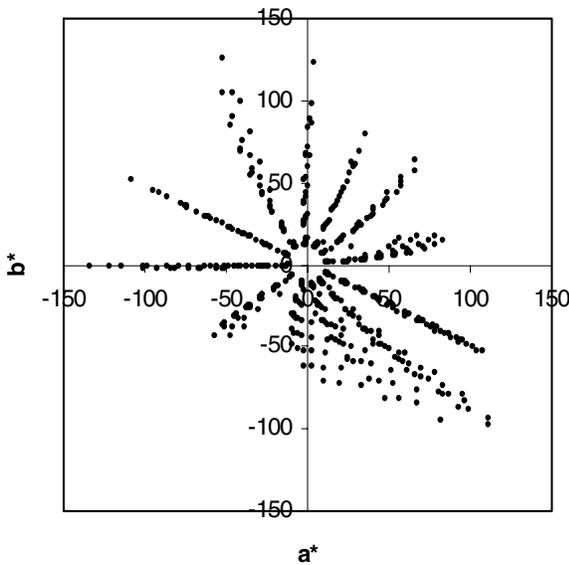
Figure 3. Plot of the radial OSA UCS data plotted in the CIELAB color space.
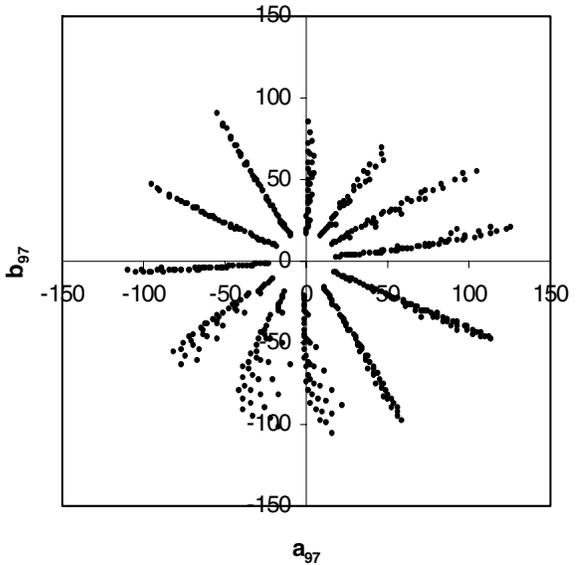
Figure 5. Plot of the radial OSA UCS data in the CIECAM97s JCh color space.

Figure 4 is a plot of the radial OSA UCS data in CIELUV. The x and y axes are u* and v*, respectively. Again there is a considerable degree of spread in the data for the blue region. The spacing of the hue angles is better than CIELAB and the hue uniformity or space between angles is better than for CIELAB.

The results for the CIE interim color appearance model CIECAM97s are shown in figure 5. In this case the axes are the a_{97} and b_{97} coordinates or the red-green and yellow-blue opponent color axes. This plot shows both an improvement in hue uniformity and hue constancy for the blue region. While it is an improvement relative to CIELAB and CIELUV, there is still some degree of spread for the blue region. Furthermore, there is fairly large initial chroma step as shown with the larger first constant ρ data relative to results shown for all of the other color spaces.
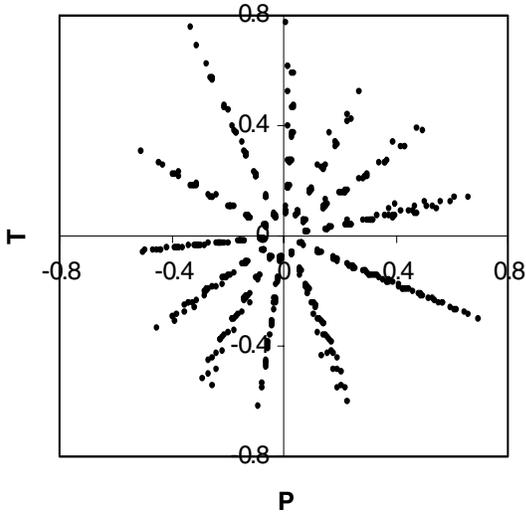
Figure 6. Plot of the radial OSA UCS data in the IPT color space.
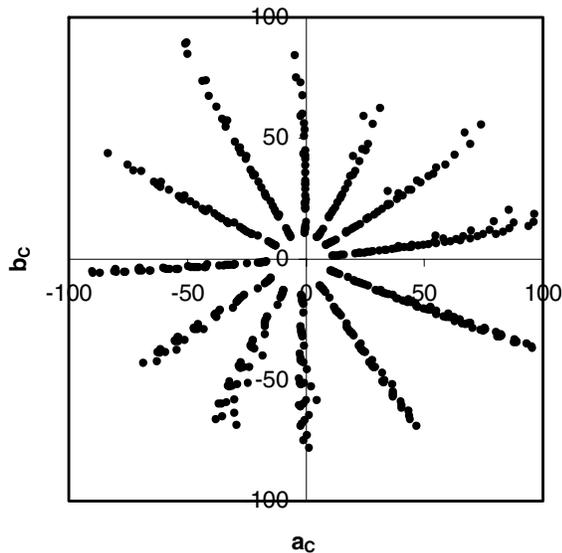
Figure 7. Plot of the radial OSA UCS data in the CIECAM02 JCh color space.

The IPT color space was optimized for hue constancy and use for digital imaging. Figure 6 shows the radial OSA UCS data plotted in the IPT space where the x-axis is the P or protan axis and the y-axis is the T or tritan axis. Given the simplicity of the equations, the results shown in Figure 6 are quite good. There is a minimal degree of spread for all of the hue angles. The chroma scale appears somewhat asymmetric in the first constant ρ data is not quite as circular relative to the results for the other color spaces. This is a qualitative assessment and requires additional analysis. It is also not entirely unexpected given that IPT was optimized for hue but not necessarily for chroma. The yellow and blue axes are

also not quite orthogonal as they are for CIECAM97s and CIECAM02.

Finally, figure 7 shows the results for CIECAM02 the revisions to the CIECAM97s model proposed by CIE technical committee 8-01. The results for CIECAM02 also show minimal spread at all hue angles. In addition the first ρ ring is smaller than that shown for CIECAM97s. These results are quite encouraging given that the OSA UCS can be used as a completely independent data set for evaluating color space uniformity. The OSA UCS was not used for the derivation of any of these color spaces but is in close agreement with current state-of-the-art color spaces and color appearance models. It is likely that more detailed, quantitative analyses can also be performed, such as those that have been carried out using the Munsell data.[23]

Conclusions

The OSA UCS has been reviewed and a radial sampling of data was computed using an iterative inverse. The data is provided for testing purposes and as an example CIELAB, CIELUV, CIECAM97s, IPT and CIECAM02 are compared. The radial OSA UCS data are in good agreement with IPT and CIECAM02. The poor blue linearity for CIELAB and CIELUV are clearly shown and improvements in CIECAM97s can be seen. It is hoped that the derivation of a radial OSA UCS data set will increase its use as an independent validation of color space uniformity and other applications.

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Biography

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