

Backward Model for Multi-Primary Display Using Linear Interpolation on Equi-luminance Plane

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Abstract

Backward model for multi-primary display, which gives digital count in order to present a given color stimulus value XYZ on a screen, is proposed in this paper. Performance of the proposed method is evaluated with 6-primary display. Error in predicting XYZ of colors projected on a screen was 0.8 on average and 2.9 at maximum.

Backward Model for Multi-Primary Display

Volume and shape of device gamut are one of important factors that determine color reproducibility of a display system. The sRGB display covers 76% of (Pointer + SOCS) gamut,^{1,2} which is a typical example of color distribution of reflective natural object.³ Therefore, device gamut expansion is required in order to reproduce the whole natural scene.⁴ TAO* has developed 6-primary DLP™ projector, which has covered almost 100% of (Pointer + SOCS) gamut as shown in Fig. 1.³

To realize accurate color reproduction, colorimetric control of a display system is another important issue. In other words, backward model that provides digital count to display a given color stimulus XYZ is required. Backward model is given as an inversion of the following forward model, in case linearity and additivity are ensured on colorimetric characteristics of display system:⁵

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_d = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{bias} + \begin{bmatrix} X_1 & X_2 & \cdots & X_n \\ Y_1 & Y_2 & \cdots & Y_n \\ Z_1 & Z_2 & \cdots & Z_n \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_n \end{bmatrix} \quad (1)$$

$$\Leftrightarrow \mathbf{C} = \mathbf{K} + \mathbf{M}\mathbf{P}$$

where $[X \ Y \ Z]_d'$ is tristimulus value of target color, $[X \ Y \ Z]_{bias}'$ is tristimulus value of bias component, $[X_i \ Y_i \ Z_i]'$ (i.e., the component of \mathbf{M}) is tristimulus value of the i -th primary ($i = 1, 2, \dots, n$) without bias. Additionally, scalar S_i ($0 \leq S_i \leq 1$)

associated with linear description of the i -th primary is given as follows:

$$S_i = TRC(d_i) \quad (2)$$

where d_i is digital count to drive a display system, $TRC()$ is tone reproduction curve.

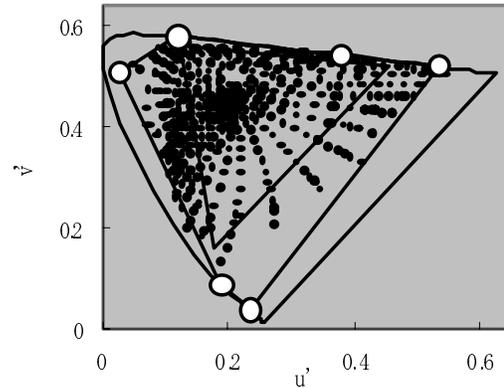


Figure 1. Gamut of 6-primary DLP™ projector (hexagon with open circles) compared with Pointer+SOCS gamut (solid dots) and sRGB gamut (triangle).

Color management system requires calculating digital count d_i in order to present $[X \ Y \ Z]_d'$ on a screen. Backward model is in charge of this function with the following equation:

$$\mathbf{P} = \mathbf{M}(\mathbf{C} - \mathbf{K}) \quad (3)$$

where \mathbf{M} is the generalized inverse matrix of \mathbf{M} . Since \mathbf{M} is $(3 \times n)$ matrix, the inversion contains a degree of freedom. Therefore, multi-primary display can present a given color stimulus XYZ with several combinations of digital count. Figure 2 shows the examples of 6-dimensional digital counts in order to display $[X \ Y \ Z] = [20 \ 20 \ 20]$ on a screen.

If the degree of freedom is not under controlled in calculating the inversion, image quality is apt to be damaged due to discontinuity of differential coefficient. As

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the result, the discontinuity causes spatial or temporal noise to reproduction image. In case $n = 3$, the inverse matrix of \mathbf{M} is unique. Therefore, a degree of freedom is a particular problem on designing backward model for multi-primary display system.

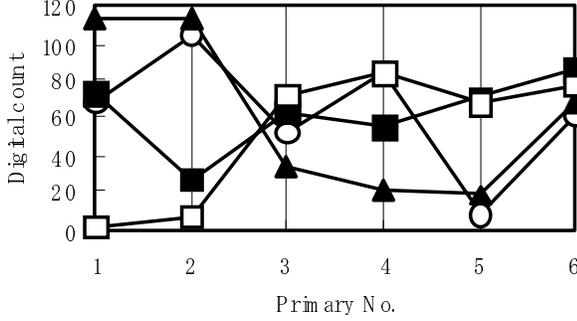


Figure 2. Examples of combinations of six-primary digital counts corresponding to $[X\ Y\ Z] = [20\ 20\ 20]$.

Linear Interpolation on Equi-Luminance Plane

This paper proposes a unique inversion using linear interpolation on equi-luminance plane. In order to realize a unique inversion, the linear interpolation is composed with three points, which correspond to A, B and C in Fig.3. In XYZ space, target color $[X\ Y\ Z]_d$ is described with the three points as follows:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_d = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{bias} + \begin{bmatrix} X_a & X_b & X_c \\ Y_a & Y_b & Y_c \\ Z_a & Z_b & Z_c \end{bmatrix} \begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix} \Leftrightarrow \mathbf{C} = \mathbf{K} + \mathbf{UQ} \quad (4)$$

where $[X_a\ Y_a\ Z_a]'$, $[X_b\ Y_b\ Z_b]'$, and $[X_c\ Y_c\ Z_c]'$ are tristimulus value without bias of point A, B and C respectively. In scalar space, the corresponding point $[S_1\ S_2\ \dots\ S_n]_d$ to the target color is expressed as follows:

$$\mathbf{S} = \begin{bmatrix} S_{1,a} & S_{1,b} & S_{1,c} \\ S_{2,a} & S_{2,b} & S_{2,c} \\ \vdots & \vdots & \vdots \\ S_{n,a} & S_{n,b} & S_{n,c} \end{bmatrix} \begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix} \Leftrightarrow \mathbf{S} = \mathbf{PQ} \quad (5)$$

where $[S_{1,a}\ S_{2,a}\ \dots\ S_{n,a}]'$, $[S_{1,b}\ S_{2,b}\ \dots\ S_{n,b}]'$ and $[S_{1,c}\ S_{2,c}\ \dots\ S_{n,c}]'$ are scalar of point A, B and C respectively. Backward model is established by substituting \mathbf{Q} in Eq.4 to \mathbf{Q} in Eq.5 as follows:

$$\mathbf{S} = \mathbf{PU}^{-1}(\mathbf{C} - \mathbf{K}) \quad (6)$$

The proposed method assigns one of the three points on a neutral axis for smoothness of continuous modulation on digital count. Since the point A is on the neutral axis KW as

shown in Fig. 3, $S_{i,a}$ is consistently above zero except bias component regardless the point B and C. The point B and C are assigned on gamut surface as illustrated in Fig. 4, which shows a color solid of 4-primary display. Ajito has developed backward model based on pyramid interpolation, in which the base is on gamut surface. In case the base consists of primary and secondary color, digital count on channels unrelated to the primary and the secondary color becomes zero. On the other hand, the proposed method seems to be better than the pyramid interpolation in terms of smoothness of continuous modulation on digital count.

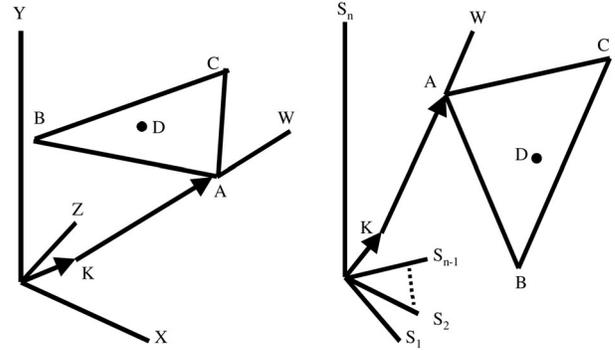


Figure 3. Linear interpolation with A, B and C in XYZ and scalar S_i space: D is a target color. W is white. K is bias.

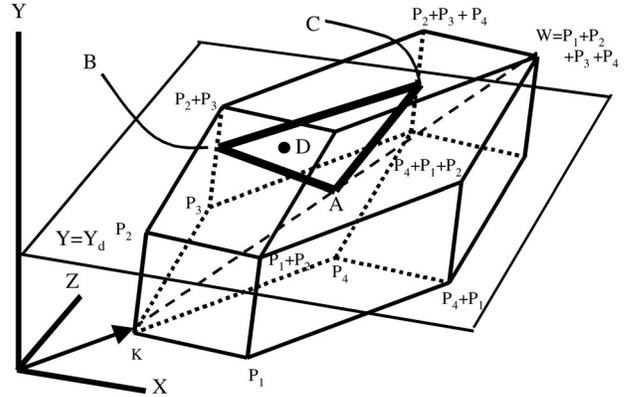


Figure 4. Linear interpolation on equi-luminance plane for 4-primary display: triangle ABC is on the Y_d plane. Point A is on neutral axis KW.

The proposed method assigns the three points A, B and C on equi-luminance plane in order to access them with their lightness Y. The display gamut is sliced with the lightness of the target color Y_d as shown in Fig. 6. Ridgelines connected with vertices (i.e. the point $P_1, P_2, P_3, P_4, P_1+P_2, \dots, P_4+P_1+P_2, W$ in case of Fig.4) on the display gamut have intersection points to Y_d equi-luminance plane. *Ridgeline Y map*, which indicates the lightness of the vertices as shown in Fig. 5, is established to find out the

ridgelines that have the intersection point. Among the intersection points, two points that enclose the target color $[X\ Y\ Z]_d^t$ with the point A are chosen as the point B and C.

Both ends of the ridgeline that includes the point B or C (i.e. $H_B, L_B / H_C, L_C$) are used in order to calculate the XYZ and the scalar of the point B or C with the equations in Fig. 5.

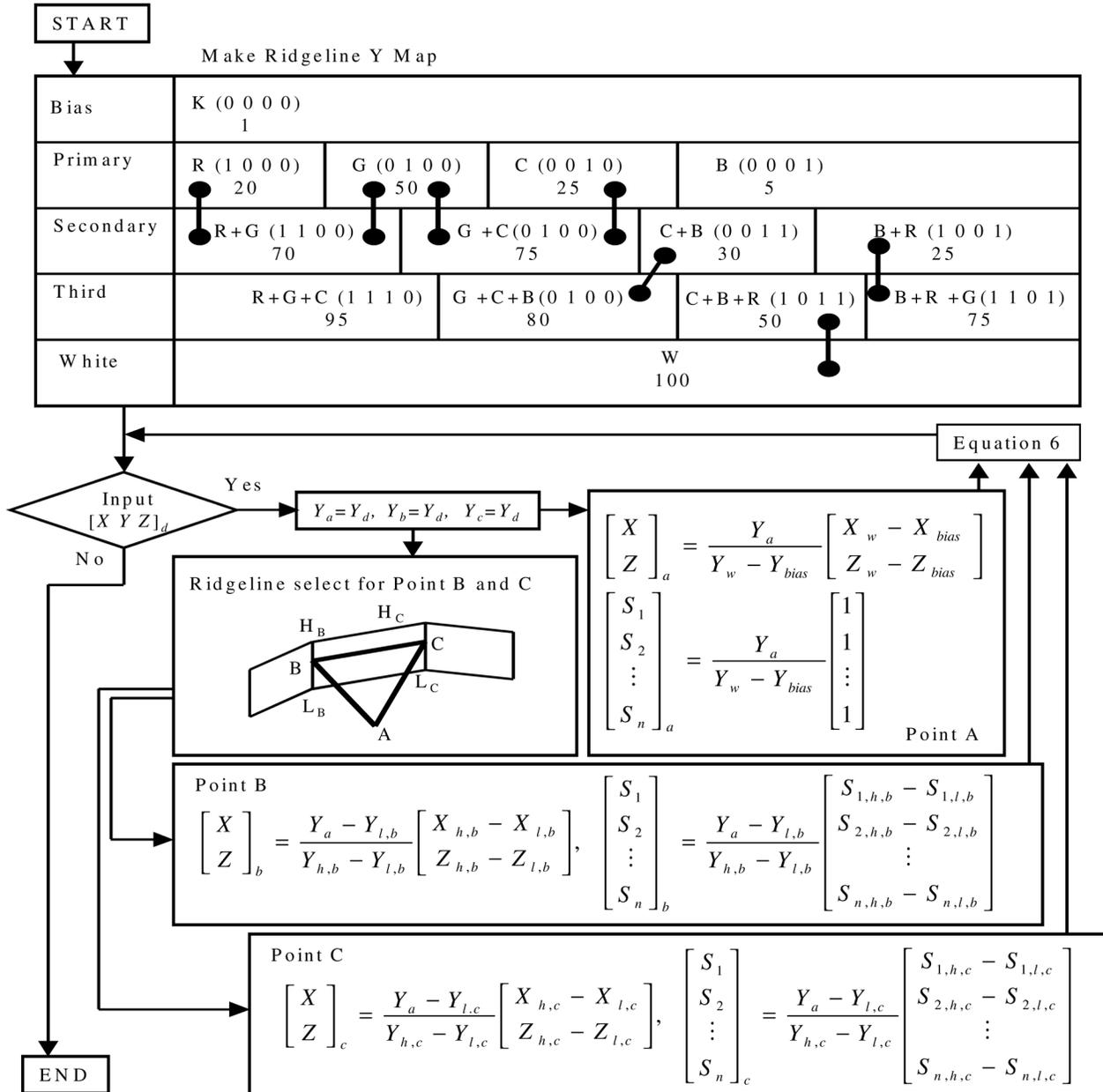


Figure 5. Workflow of the linear interpolation on equi-luminance plane: Ridgeline Y map is illustrated for 4-primary display, of which lightness Y is 20, 50, 25, 5 for R, G, C, B primary respectively. In case the lightness of the target color Y_d is 60, the ridgelines, which indicate as the line with two solid circles, have the intersection point with $Y_d=60$ plane. Drawing in "Ridgeline select for Point B and C" is extracted from Fig. 7.

Required data for establishing the backward model using the proposed method are TRCs, bias $[X\ Y\ Z]_{bias}'$, and primary colors $[X_i\ Y_i\ Z_i]'$. In case $i=3$, ICC display profile is sufficient for the proposed method to provide the required data. Unfortunately, the current ICC display profile is not enough for multi-primary display.

Introduction of neutral TRC is useful for the proposed method to heighten prediction accuracy, because interpolation for point A can be performed in more concentrated region as illustrated in Fig. 7. It is also useful to add measured TRCs for secondary color and up to $(n-1)$ -th color. However, The neutral TRC is more useful, because 1) human vision system is more sensitive to achromatic component than chromatic one and 2) size of display profile becomes smaller

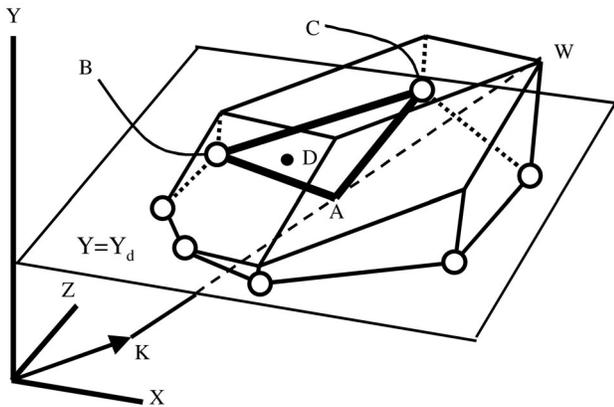


Figure 6. Intersection points (open circle) between ridgelines of gamut surface and Y_d equi-luminance plane: The seven intersection points are on the ridgelines that indicate in Ridgeline Y map shown in Fig. 5.

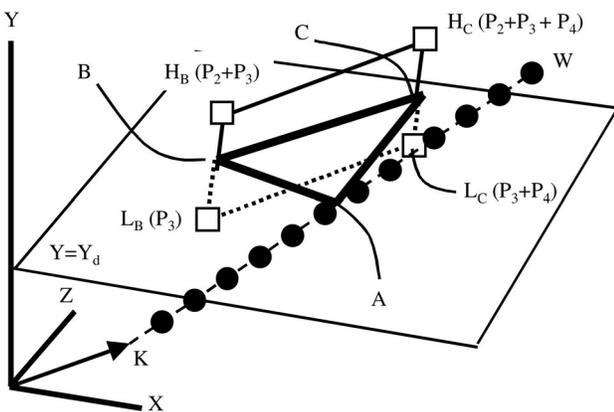


Figure 7. Neutral TRC for interpolating the point A: The solid circles are measured data. The Point A is interpolated with the solid circle neighbors. H_b and L_b (or H_c and L_c) are both ends of the ridgeline that includes the point B (or C).

Experiments

Performance of the proposed method was evaluated by predicting tristimulus value projected on a screen with 6-primary DLP™ projector (Panasonic TW-Z01CN1DZ1). 84 sets of $[X\ Y\ Z]_d'$ were chosen randomly inside of the projector as shown in Fig. 8. The proposed method predicted digital counts so that the projector displayed the 84 sets of $[X\ Y\ Z]_d'$ on a screen. Projected colors with the projector by the predicted digital count were measured with spectrophotometer (Topcon SR-2). Measured colors are shown in Fig. 8. Prediction error in CIELAB is listed in Table 1. Error distribution is also shown in Fig. 9.

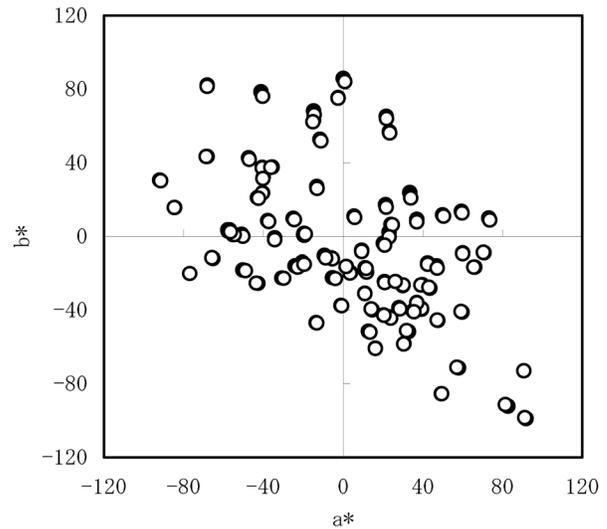


Figure 8. Measured (solid circle) and predicted (open circle) colors.

Table 1. Predicted Error in CIELAB.

Average	Maximum	Minimum	Std.
0.83	2.88	0.11	0.55

Consideration

Prediction error in chromatic component (i.e. a^* or b^*) is larger than achromatic one (i.e. L^*) as shown in Fig. 9. This result can be explained that the dynamic range of the chromatic component (i.e. from 0 to 100) is larger than the achromatic component (i.e. from -120 to 120, approximately). Maximum error is occurred in dark blue region, where the chroma is above 130. Visual system is more sensitive to achromatic difference at a sharp edge and in temporal modulation rather than chromatic one. Therefore, the proposed method is robust for spatial and temporal noise.

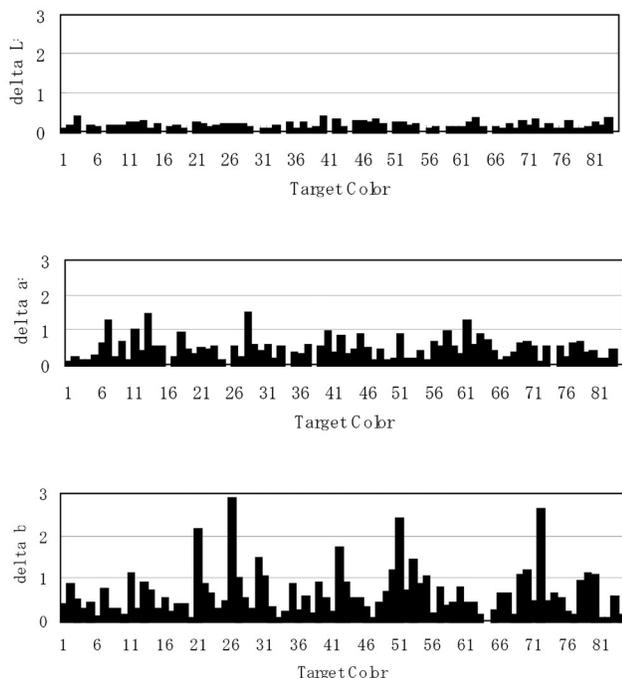


Figure 9. Prediction error distribution in terms of the absolute value of ΔL^* , Δa^* and Δb^* .

Conclusion

Backward model for multi-primary display was proposed and its performance was evaluated. Linear interpolation on equi-luminance plane has higher priority to achromatic component rather than chromatic one on prediction

accuracy. Implementation of the proposed method to color management system is practical in terms of computing power and device profiling.

References

1. M. R. Pointer, "The gamut of real surface colors," *Color Research and Application*, **5**, 3, 145-155 (1980)
2. TR X 0012, Japanese Industrial Standards Association (1998)
3. H. Motomura et al., "Development of six-primary HDTV display system", *Proc. EuroDisplay*, in press (2002)
4. M. Yamaguchi et al., "Color image reproduction based on the multispectral and multiprimary imaging: Experimental evaluation", *Proc. SPIE* **4663** (2002)
5. R. Berns, "A generic approach to color modeling", *Color Research and Application*, **22**, 318-325 (1997)
6. Ajito et al, "Color conversion method for multiprimary display using matrix switching", *Optical Review*, **8**, 3, 191-197 (2001)

Biography

Hideto Motomura received his M.S. degree in Image science and technology from Chiba University, Japan, in 1990. In the same year, he joined Matsushita Research Institute Tokyo Inc. in Kawasaki, Kanagawa. From 1997 to 1999, he was Visiting Scientist of Center for Imaging Science, Rochester Institute of Technology. He is Staff Researcher of Advanced Technology Research Laboratory, Matsushita Electric Industrial Co., Ltd. Currently, he is on loan to Telecommunication Advancement Organization of Japan as Researcher of Akasaka Natural Vision Research Center. He is a member of CIE TC8-03, gamut mapping.