The Colors of the Deep Sky

Thor Olson
Electronics for Imaging
Eagan, Minnesota

Abstract

The term *deep sky object* is popular among astronomers to describe those faint patches among the stars that can be found only with optical assistance. The equipment needed to take pictures of deep sky objects is within the reach of dedicated amateur astronomers. In such low light levels there strictly is no color, but there is a natural desire to make these images look as they might appear if our vision were sensitive enough to perceive color. This is a component of esthetic presentation, and the use of color for this subject is extremely useful in education.

CCD imaging equipment, because of its linearity, allows us to record and measure the spectral energy in a specific band determined by a filter placed in front of the sensor. By making several such recordings through red, green, and blue filters, a full color composite can be created. Of particular interest to astrophotographers are emission nebulas, whose spectra frequently comprise the dominant lines of hydrogen and ionized oxygen. These are present in a scene along with the wideband blackbody emissions of stars. Stars that shine into clouds of interstellar dust show yet other characteristic color. What is the representation of the recorded channels to portray a colorimetrically correct picture of these deep sky objects? This is the topic that will be addressed in this paper.

Background and Fundamentals

It is a well-known principle in color reproduction theory that to faithfully reproduce a scene containing arbitrary spectra, the detector must span the vector space of the human visual system. In practice, real world detectors are limited by the physics of filters and transducers, and true colorimetric systems are not commercially available.

In the case of astrophotography, there is an opportunity to make very close reproductions of the scene. It must be stated here however that the “true” representation of the color of these extended objects is a deception. Were we to see them with our own eyes, even at the eyepiece of a powerful telescope, we would see only pale hints of color if we see any at all. The nebulae and galaxies that we can see and record are faint extended objects in space. If we could get closer to them, they would appear larger, but also fainter, as the light they emit comes from an ever larger, more diffuse area. There is no vantage point where a human could ever directly see their color. Hence, by definition, they have no color.

Nevertheless, we can imagine the light being amplified, or our eyes being sensitized in some way that if we were to see color, what it would be. Further, we can render it using the channels recorded by a CCD detector through its filter set. What makes this possible, in spite of the general requirement for colorimetric detectors, is that the light we are recording does not contain arbitrary spectra as in a sunlit scene on our planet, it is actually very simple; it can be modeled with only a few known and constant spectral components.

To determine the requirements for detecting and rendering astrophotos with colorimetric accuracy, we will use the tools of linear vector spaces. An excellent and concise presentation of this topic and its application to color reproduction is given by Horn.[1] He proves a number of useful color reproduction theorems, starting with detector requirements. While the detector response to reproduce the appearance of arbitrary spectra needs to span the human visual functions, there is no such requirement on the primaries of the display device, a happy situation that has allowed many diverse technologies to be used on the output side of reproduction.

Many popular astrophoto scenes contain objects which have well-known spectra. Obviously there are stars, which can be portrayed for visual purposes by spectra of black body radiators.[2] There are also emission nebulas, large regions of excited gas that emit light at specific characteristic wavelengths. A dominant emission is the primary line of hydrogen, H-alpha at 656nm. Also found are H-beta, at 486nm, and ionized oxygen, OIII at 501nm. A third large category of colorful objects are reflection nebulas, regions of interstellar space populated with dust that reflects the light from nearby stars. We will discuss the colorimetric rendering of these objects, focusing primarily on emission nebula, but the theory developed will accommodate the broadband sources of stars and reflection nebula as well.

As a first simple attack on the problem, assume there are only three components of the light in a scene, say two line sources, H-alpha and OIII, and the uniformly flat spectrum, “E”, representing all wideband sources (stars).

The spectrum of the light in this scene will comprise three weighted components:

$$S(\lambda) = \sum_{i=1}^{3} F_i f_i(\lambda) \quad (1)$$

where $f_1 = 1$
$f_2 = \delta(656)$
$f_3 = \delta\,(501)$

and F_1, F_2, and F_3 are the "amounts" of each of the source illumination functions.

We will take three recordings of this spectrum through three filters. Each filter, combined with the detector spectral sensitivity, has a response of $r_j(\lambda)$. Each of these channels sees the light source spectra differently, their output will be:

$$\mathbf{r}_i = \sum_{j=1}^{3} \mathbf{f}_j \int f_j(\lambda) \cdot r_i(\lambda) d\lambda \quad (2)$$

In vector notation:

$$\mathbf{r} = \mathbf{H}\,\mathbf{f} \quad (3)$$

where **r** and **f** are column vectors and the elements of matrix **H** are:

$$h_{ij} = \int r_i(\lambda) f_j(\lambda) d\lambda \quad (4)$$

A human eye sees the spectrum through three channels as well, their responses are the color matching functions $e_i(\lambda)$. The three outputs of this system are tristimulus values:

$$\mathbf{e}_i = \sum_{j=1}^{3} \mathbf{f}_j \int f_j(\lambda) \cdot e_i(\lambda) d\lambda \quad (5)$$

The vector version is:

$$\mathbf{e} = \mathbf{G}\,\mathbf{f} \quad (6)$$

With the elements of **G**:

$$g_{ij} = \int e_i(\lambda) f_j(\lambda) d\lambda \quad (7)$$

Solving equations (3) and (6):

$$\mathbf{e} = \mathbf{G}\,\mathbf{H}^{-1}\,\mathbf{r} \quad (8)$$

So we can compute the tristimulus **e** of this (spectrally limited) scene which produced detector output **r**.

When we want to present the scene on a display that uses three primary (typically RGB) light sources, characterized by the vector equation:

$$\mathbf{e} = \mathbf{C}\,\mathbf{p} \quad (9)$$

We solve for the drive levels of the primaries, **p**, for the desired tristimulus:

$$\mathbf{p} = \mathbf{C}^{-1}\,\mathbf{e} \quad (10)$$

And if we set the output tristimulus vector to the tristimulus of the original scene, we obtain our desired result:

$$\mathbf{p} = \mathbf{C}^{-1}\,\mathbf{G}\,\mathbf{H}^{-1}\,\mathbf{r} \quad (11)$$

The matrices can be combined into a single operator, **B**, that relates the detector channels to the display channels:

$$\mathbf{p} = \mathbf{B}\,\mathbf{r}; \qquad \mathbf{B} = \mathbf{C}^{-1}\,\mathbf{G}\,\mathbf{H}^{-1} \quad (12)$$

So we now have a simple means to obtain an image for a specific display (or RGB color space), directly from the recorded channels of a given multiband image sensor. It will be colorimetrically correct to the degree that the scene can be represented by the basis functions assumed in $f_i(\lambda)$.

What if the scene is more complex than the simple 3-component set of light sources we assumed above? Say we wanted to include a third or fourth emission line, or, as will be described later, the broadband sources are represented by a superposition of three bases instead of the simple "E" flat spectrum. The additional components are included in a larger model for the source spectrum:

$$S(\lambda) = \sum_{i=1}^{N} F_i f_i(\lambda) \quad (13)$$

To resolve the higher N-dimensional spectrum, we need N independent detector channels, each with its particular spectral sensitivity, $r_j(\lambda)$. The detectors need not single out each component of the illuminant spectrum, but collectively they must span its vector space. This is so that the relation between them (equations 2-4) can be inverted, that is, $\mathbf{H}^{-1}$ must exist.

The transform matrix **G** that obtains the tristimulus vector is evaluated as before by equation 7, but now there are N cross products with each color matching function, and the matrix becomes 3xN. It is always used in this form, no inverse, or pseudo-inverse is needed. The solution for the overall transform **B**, between detector channels and display channels yields a similar 3xN matrix.

Some Suitable Illumination Source Models

In addition to line emission sources in astrophotos, we encounter two other major types of illumination. Stars, even though their detailed spectra are quite complex, are well-represented by a blackbody emission spectra. And reflection nebula, which are really just regions of space that reflect the light from nearby stars. Can these objects be approximated by a superposition of linear basis spectra?

Figure 1 shows the chromaticity locus of blackbody spectral power distributions. Over the region 3000K° to T_{inf}, the chromaticity follows a gentle path. If we take these endpoints and find a suitable third spectral "primary" to mix in, we could approximate the spectral shapes found along this path. Figure 2 shows a candidate: the spectral waveform that when added to those of 3000K° and T_{inf} yields "E", the flat spectral distribution. This set of basis waveforms will be referred to as the "3Ei" set indicating their linearly independent components (3000K°, E, T_{inf}). E is actually the "whitepoint" for this three-component system. Figure 1 includes the chromaticity "gamut" of 3Ei, which contains the blackbody locus very efficiently. The 3Ei basis set can represent all of the stars in an astrophoto (at least those that have an effective blackbody temperature between 3000K° and infinity).

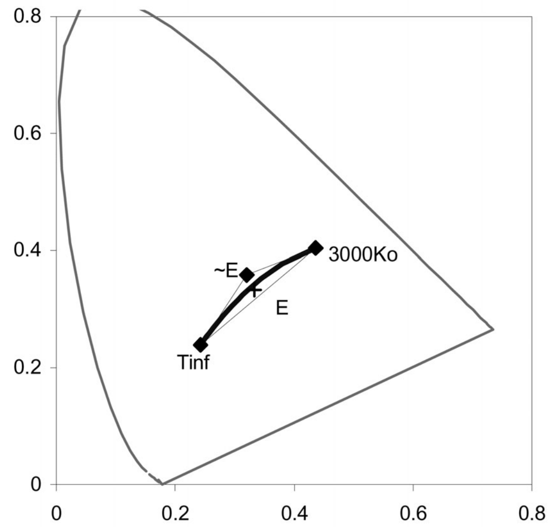

Figure 1. The locus of blackbody colors from 3000 K^o to T_{inf} on the chromaticity diagram. Point, ~E is the location of a complementary power distribution which, when combined with the endpoints results in a "whitepoint" of E, the flat equi-power spectrum. The resulting gamut of this three primary system is an efficient container for the blackbody locus.

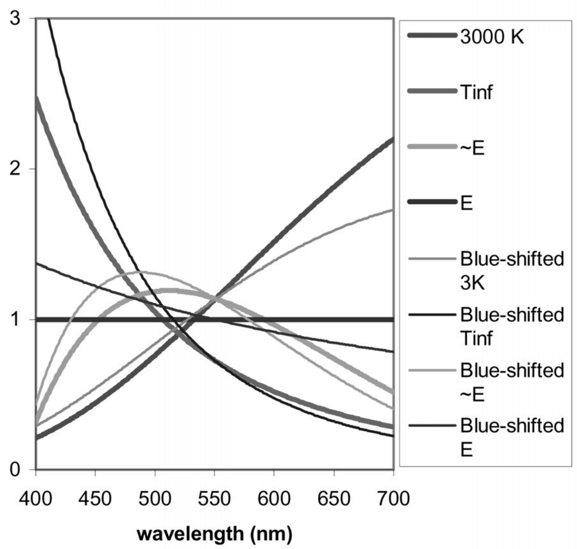

Figure 2. The 3Ei basis spectra (bold curves). They sum in equal parts to form the flat E spectrum, and blackbody spectral shapes are approximated by suitable sums of them. Also shown, the effect of Mie scattering: the 3Ei basis waveforms are blue-shifted (thin lines) by the 1/lambda scattering of reflection nebula material.

When stars are in a region of relatively dense dust and gas in the interstellar medium, their light is scattered, giving rise to deep sky objects called *reflection nebula* (as opposed to emission nebula whose energized gas is actively emitting light). Astronomers have learned that these regions are dominated by Mie scattering: the light is preferentially reflected according to a $1/\lambda$ law. The light source could be a star of any temperature, and reflection nebula are collectively wide ranging and colorful deep sky objects.

Because they are illuminated by their nearby stars, reflection nebula spectra should be a composite of them, modified by their $1/\lambda$ characteristic. Applying the scattering function to the 3Ei basis spectra, we obtain the second set of spectra in figure 2, blue-shifted versions of the blackbody spectra. Their visual response and location on the chromaticity diagram is shifted only slightly from the blackbody colors, but it is important to maintain this distinction, the detector channels will see these spectra rather differently than people.

We now have an assortment of basis functions to use for the illumination model of the scene. The 3Ei set is appropriate for stars, their blue-shifted versions for reflection nebula, and delta functions at the appropriate wavelengths for emission nebulas. Not all are needed in every scene, and new ones may be required for more exotic situations. Because the scenes have known spectra, we can represent them with these linear models, and obtain a simple method to display their image in a colorimetrically-correct manner. An example of this is given in the next section.

Application

To test this attractively simple result and illustrate its application, a set of astronomical image frames were obtained as source data. The image is of the Veil Nebula, a supernova remnant comprising a shell of ionized gasses that emit in H-alpha, H-beta and O-III spectral lines. The images are by astrophotographer Mike Cook who used an SBIG ST10 CCD detector equipped with three wideband red, green and blue filters, and three narrowband filters that isolate and pass the emission lines.

There are other factors that influence the sensitivity functions $r_j(\lambda)$ of the detector. These include the spectral sensitivity of the silicon photodetector array, and the "atmospheric extinction", that characteristic of the air that makes the sun look yellow in a sky of blue, reddening as it approaches sunset.[3] The net detector response of the six channels is shown in figure 3.

The 3Ei spectral basis set (described prior) was used to represent the stellar light sources. Another model of illumination for the line-spectra emissions was made from a simple set of delta functions at the wavelengths 486, 501, and 656nm. Keeping with three-letter acronyms, these bases form the "a3b" (H-alpha, O-3, H-beta) set.

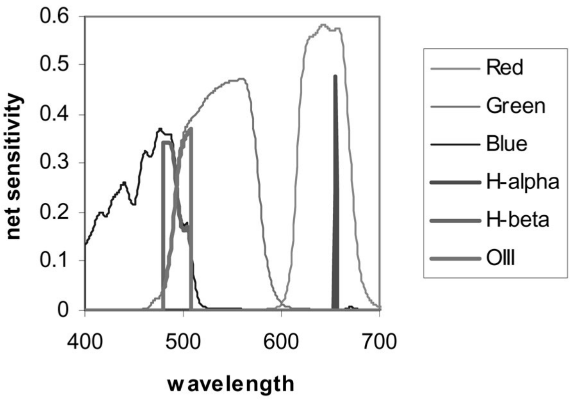

Figure 3. Net filter and detector response. This includes the effects of the silicon sensitivity and of atmospheric extinction. The bold curves are the narrowband filter responses, the light curves show the wideband RGB filter set.

Looking first at the wideband red, green, and blue rgb (lowercase rgb) filtered images, we can form the various cross-product integrals of the detector response with the light source. If we use the 3Ei broadband light source model we get a 3Ei-to-rgb version of **H**. Similarly, we can form **G**, the crossproducts of the 3Ei broadband light source bases with the color matching functions. Selecting AdobeRGB for the RGB display space (uppercase RGB) for **C** gives us everything we need to evaluate equation 12. The resulting matrix is:

$$B_{rgb-3Ei-RGB} = C^{-1}GH^{-1} = \begin{bmatrix} 2.470 & 1.237 & -0.352 \\ -0.093 & 2.985 & -0.514 \\ -0.100 & -0.376 & 4.120 \end{bmatrix}$$

This matrix is essentially just a transform from the color filter primaries, rgb, to the display space primaries RGB. An RGB scanner would (should) use a similar system.

If we go through the same procedure but using the a3b emission-line light source model, we get the matrix:

$$B_{rgb-a3b-RGB} = \begin{bmatrix} 0.665 & -0.583 & -0.353 \\ -0.101 & 1.614 & 0.330 \\ -0.012 & -0.161 & 1.649 \end{bmatrix}$$

These weighting matrices can now be applied to the rgb image frames. In practice, CCD images are obtained with varying exposures times. To maintain calibration, the data must first be divided by the relative exposure so that the image channels are in a common scale. Other practical issues that must be addressed include establishing the zero level of the data, and removing any residual scale factors. Fortunately CCD sensors are linear detectors, easing the effort to bring the data into a calibrated linear signal space.

Figures 4a and b (color plate) shows the application of these color conversion matrices to a portion of the Veil image. After performing the operation, the resulting image data was scaled to where the low level emission signal becomes visible, though the high intensity stars saturate. The first image shows a picture typical of many photographs of the Veil Nebula, a delicate red cloud hanging among the stars. The stars are correctly rendered, but the nebula itself is not.

The second image shows the same source data, but the line emission transform was applied. In this case the nebula is correctly rendered, but the stars are all blue-green! This is actually their correct representation in this system: a broadband source like a star would contribute roughly equal amounts of energy at the three emission line wavelengths. The chromaticity location of equal spectral amplitude levels of the a3b primaries is shown in figure 5, just outside the blue-green edge of the AdobeRGB gamut.

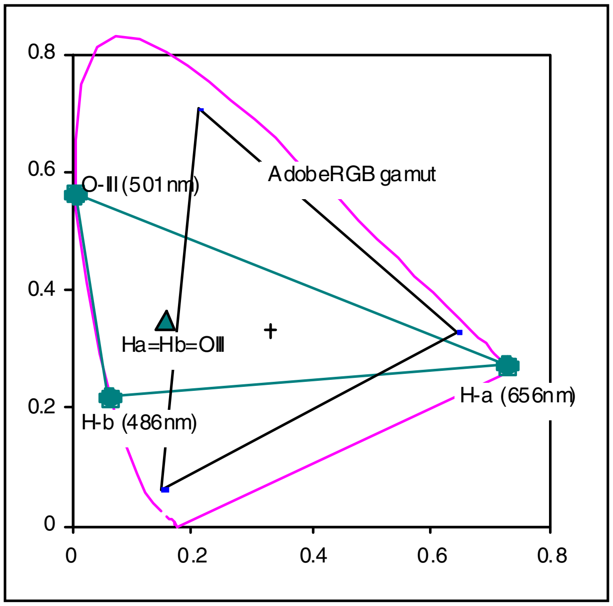

Figure 5. Emission nebula containing only Ha, Hb, and OIII emission lines can make colors having chromaticities anywhere within their three spectral loci. An equal amplitude mix of the three, either from a region of space where the three emissions are truly equal, or from three narrowband observations of a broadband blackbody source (i.e. any star in the view), would make a color whose chromaticity lies just outside the gamut of the AdobeRGB display space.

While the above example shows that one can obtain the correct nebula colors from a wideband rgb image, it is beneficial from the standpoint of signal to noise ratios, to render the nebula from a set of narrowband detector images. The same analysis applies, this time the crossproducts are with the narrowband filter responses. For Mike Cook's system, the matrix result is:

$$B_{a3b-a3b-RGB} = \begin{bmatrix} 0.800 & -0.614 & -0.372 \\ -0.118 & 1.699 & 0.347 \\ -0.013 & -0.169 & 1.736 \end{bmatrix}$$

The image (figure 6) has the same characteristically blue-green stars, but the details and amplitude in the nebula itself are more pronounced, and the strong colors of the emission lines stand out.

How can one obtain both correct nebula colors and correct star colors? A six-dimensioned system to achieve this is the next step in this investigation, but the difficulties in matching signal and noise characteristics between channels is a problem yet to be solved. In the meantime, a spatial filtering method has been successfully used.

If one could identify the broadband sources (the stars) in the image, one could select the appropriate image (e.g. figure 4a or 6) from which to select rendered pixels and merge them in a composite result. It is tempting to use a color-selective method to generate a spatial mask, by identifying all those out of gamut blue-green pixels, for example, that result from stars being interpreted as a3b sources. This was found to be not reliable, since there are legitimate a3b sources in the nebula region that look the same.

Instead, a spatial median filter was used to remove small point-like sources. The complement of this filtered image identifies the stars. By appropriate scaling, this becomes a useful mask, permitting a mixing between the two source images and obtaining a picture that has a colorimetrically rendered nebula among correctly-tinted stars (figure 7).

The image is an unusual portrayal of this famous deep sky object. The energy associated with the leading shock front is vividly shown by the blue-green emissions of ionized oxygen, with the red, lower energy hydrogen trailing behind it. Most images easily capture the red but under-represent this color of green (color film is nearly blind to this spectral line, and silicon has great sensitivity to deep red).

Conclusion

While the application of colorimetric rendering methods to astrophotographs does not advance the science of astronomy, it does enhance the ability to present and explain it. The educational aspects and the esthetic component of visual accuracy are benefits to be gained. The principles for exact reproduction are well known but usually not of widespread or of practical interest because of the spectral complexity in most subjects. Here is a subject that is spectrally simple (except perhaps to a professional astronomer) and actually *can* be exactly rendered.

Acknowledgements

I am indebted to Mike Cook for contributing his full resolution image frames to this project and guiding me through an explanation of their exposure. Some of Mike's other technically challenging activities are presented on his website: http://www.af9y.com.

References

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also see

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Biography

Color engineer by day, Thor Olson is an amateur astronomer who spends clear nights attempting to capture images of faint fuzzy objects, and cloudy nights wondering what they really look like. His background is in physics and electrical engineering, and spent much of his career designing high resolution film recording equipment for presentation, prepress, and motion picture applications. His current work has been more print-centric, most recently developing user interfaces to help graphic artists get what they expect from modern printers.