# A Physically-Based Reflection Model for Glossy Appearance

*Yinlong Sun*
*Dept. of Computer Sciences, Purdue University, W. Lafayette, Indiana*
*(sun@cs.purdue.edu)*

## Abstract

Glossy appearance is commonly observed on natural and fabricated objects. This paper proposes a physically-based model to account for this behavior. Our basic consideration is that a glossy surface typically consists of a thin transparent layer above the substrate material. We assume that the top surface is very smooth and the interface between the transparent and substrate materials is a Gaussian height field. The numerical results and rendered images generated using our model have successfully demonstrated the features of glossy appearance. This work has eliminated the arbitrariness on choosing parameters in previous empirical method such as Phong model. Since all involved parameters are physically measurable, our model can be strictly verified by experiment. This model has a capability of achieving high accuracy for applications in computer graphics, computer vision, color science, and appearance design.

## Introduction

Many objects in reality demonstrate glossy appearance.[1] Such examples include metallic and ceramic enamelware, furniture, floors, and many biological surfaces (leaves, fruits and so on). Therefore, accurately modeling the behavior of gloss appearance is an important problem in many areas including computer graphics, image analysis, computer vision, color science, and appearance design.

A glossy object typically shows a bright highlight in the mirror-reflection direction and the highlight appears in the color of the illuminating light, which is often white. In the mean time, other areas on the object (where the mirrorreflection condition is not satisfied) demonstrate colors that inherently associated with the bulk material properties of the object. Moreover, when the viewing angle approaches the glazing condition, the highlight intensity increases dramatically and the overall color of the object appears substantially desaturated.

Figure 1 shows the physical structure of the boundary surface of a glossy object. The entire boundary typically involves two materials: a thin transparent layer on the top and some opaque bulk material in the bottom as the substrate. The top transparent material can be enamel, oil, ice, water etc. The substrate material can be metal, ceramic, clay, wood and so on. For glossy appearance, the top surface (between the air and the transparent layer) is very smooth, while the bottom surface (between the transparent layer and the substrate material) quite rough. Thus the specular highlight on a glossy object is caused by the reflection from the top surface (ray 1). On the other hand, the complex light path, which includes one reflection from the bottom surface and two transmissions through the top surface (ray 2), generates the color that is associated with the substrate material.

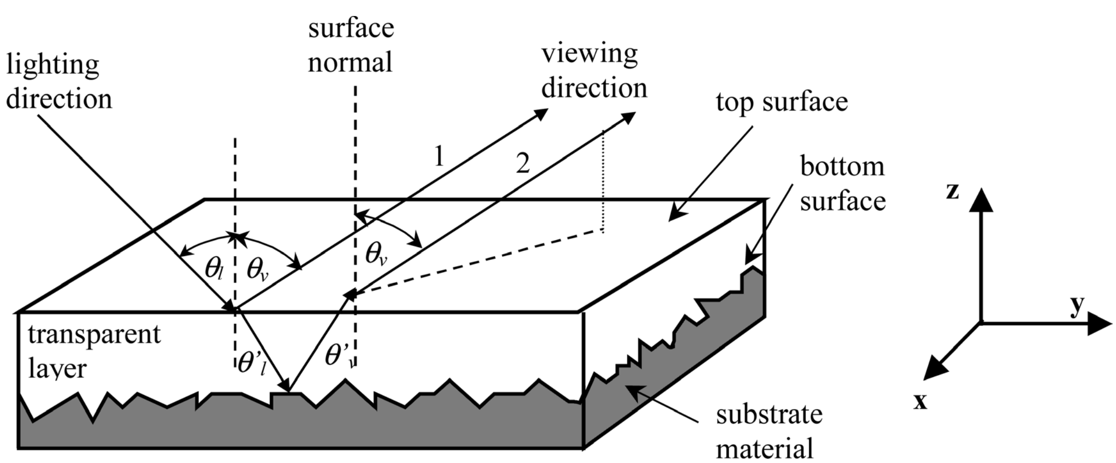


*Figure 1. The typical physical structure and geometry notations for reflection at a surface with glossy appearance.*

This paper develops a physically-based reflection model to address the glossy surface behavior. The critical component is on light reflection at the rough interface between the top transparent layer and the substrate. To handle this we will adopt the approach of microfacet theory for rough surfaces. Besides, based on the consideration of the physical structure of a glossy surface, we make reasonable assumptions that allow us to derive the analytical form for the overall glossy reflection. The numerical and rendered results generated using our derived model have successfully demonstrate the characteristics of glossy appearance.

## Related Work

Light reflection is often described using a simple empirical function[2] and this approach has been extended to include more parameters such as surface smoothness[3] and anisotropy.[4] However, the accuracy is not adequate. As a physicallybased approach, one may consider that a simplex surface such as metallic consists of many planar, perfectly specular, and isotropic microfacets.[5,6] Based on this assumption, Cook and Torrance[7] described specular reflection in terms of a product of Fresnel coefficient, a shadowing and casting factor, and a microfacet statistical distribution. This model not only improves the accuracy, but also successfully demonstrates Fresnel color shift that is observed on metallic objects. Oren and Nayar[8] applied a similar idea to non-Lambertian surfaces by assuming microfacets as Lambertian instead of specular. Significant research has also been dedicated on computing the shadowing factor.[9–13]

Alternatively, Kajiya[14] proposed to derive reflection based on Kirchhoff theory, which calculates an electromagnetic field by integrating a Green's function over the space boundary. This method is well represented by the work of He et al.,[15] where many factors including surface statistics, subsurface scattering, and polarization are considered. Recently, Stam applied this approach to rendering diffraction.[16] Kirchhoff theory is attractive because it may generate analytic solutions of simple form.[17] However, it is difficult to handle self-shadowing and multiple scattering, which create important shading effects.[18]

A variety of techniques have been proposed to model multi-layer reflection. These include the Kubelka-Munk model by Haase and Meyer,[19] Monte Carlo simulation of subsurface scattering by Hanrahan and Krueger[20] and by Pharr and Hanrahan,[21] the Abeles matrix formalism by Icart and Arques,[22] and reflection model based on appearance standards by Westlund and Meyer.[23] Marschner et al.[24] have measured BRDFs of various surfaces. Westin[25] proposed to generate BRDFs by simulating light scattering among microfacets.

## Glossy Reflection Model

Now we will derive a physically-based reflection model for glossy appearance based on the surface structure as shown in Figure 1. Our assumptions include

1. The top surface (between the transparent layer and the air) is very smooth. Thus the self-shadowing effect on the top surface is neglected.
2. The transparent layer is very thin and therefore the volume absorption of light in it is negligible.
3. The bottom surface (between the transparent layer and the substrate material) is homogenous and isotropic, and its height field is Gaussian.
4. Multiple reflections between the top and bottom surfaces are ignored.

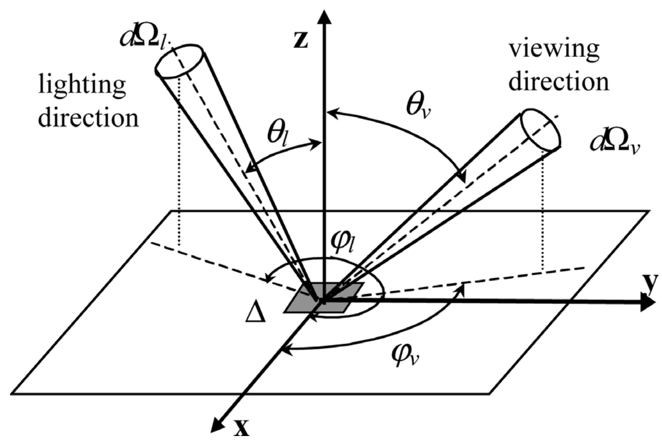


*Figure 2. Geometry and notations for BRDF.*

Mathematically, surface reflection is generally described by a bi-directional reflectance distribution function (BRDF). This function depends on both the incident direction), ($\theta_l$ $\varphi_l$) and the outgoing direction), ($\theta_v$, $\varphi_v$) as well as light wavelength $\lambda$ (Figure 2). Thus, we express the overall BRDF for glossy reflection as

$$\rho(\theta_l, \varphi_l, \theta_v, \varphi_v, \lambda) = \rho_1(\theta_l, \varphi_l, \theta_v, \varphi_v, \lambda) + \rho_2(\theta_l, \varphi_l, \theta_v, \varphi_v, \lambda) \quad (1)$$

where the two terms account for the contributions from ray types 1 and 2 in Figure 1, respectively. For the first term, according to the microfacet theory,[5] one-bounce reflection can be expressed as a product of the Fresnel coefficient, a self-shadowing factor, and a statistical distribution function that is related to the surface orientation.[7] Since the self-shadowing effect is neglected for the top surface,

$$\rho_1(\theta_l, \varphi_l, \theta_v, \varphi_v, \lambda) = \frac{\overline{R}(\delta/2, \lambda)}{16\pi \cos^3\theta \cos(\delta/2)} s^2 \exp(-s^2 \tan^2\theta/4) \tag{2}$$

where

$$\overline{R}(\delta/2, \lambda)$$

is the Fresnel coefficient averaged over all polarizations for incident angle δ/2, and the rest corresponds to the probability distribution for the top surface for slope tanθ.[17] In Eq. (2), parameter *s* describes the surface smoothness and is defined as

$$s = \frac{\tau}{\sigma}. \tag{3}$$

where σ and τ are the height deviation and correlation length of the top surface. In other words, larger *s* implies a smoother surface.

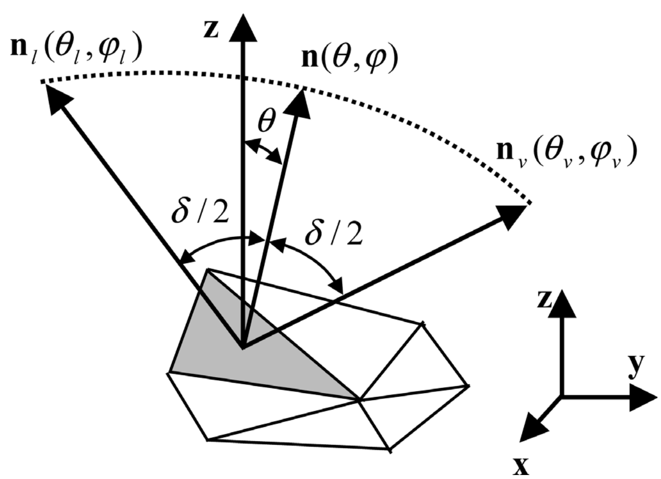


*Figure 3. Microfacet scattering geometry.*

As shown in Figure 3, δ is the angle between the lighting and viewing directions, and θ is the polar angle of the half vector that equally divides the lighting and viewing directions. Thus we have the following relation:

$$\cos\delta = \sin\theta_l \sin\theta_v \cos(\varphi_l - \varphi_v) + \cos\theta_l \cos\theta_v \tag{4}$$

and

$$\cos\theta = \frac{\cos\theta_l + \cos\theta_v}{2\cos(\delta/2)} = \frac{\cos\theta_l + \cos\theta_v}{\sqrt{2 + 2\cos\delta}}. \tag{5}$$

Now let us consider the reflection at the bottom surface. Since the bottom surface is rough, we need to include the surface self-shadowing effect. In other words, the shadowing and casting factors should be maintained. Therefore, the BRDF for reflection at the bottom surface should have the following form:

$$\rho'(\theta_l', \varphi_l', \theta_v', \varphi_v', \lambda) = \frac{R(\delta'/2, \lambda)\Gamma_0(\theta_l')\Gamma_0(\theta_v')}{16\pi \cos^3\theta' \cos(\delta'/2)} s'^2 \exp(-s'^2 \tan^2\theta'/4) \tag{6}$$

where the primed notations apply specifically to the bottom surface. Here $\Gamma_0(\theta_l')$ *and* $\Gamma_0(\theta_v')$ are the shadowing factors for the lighting and viewing directions.

Since the top surface is very smooth, we can apply the Snell's law to link the polar angles (see Figure 1):

$$\sin\theta_l = n \sin\theta_l', \quad \sin\theta_v = n \sin\theta_v', \tag{7}$$

where *n* is the refractive index of the material in the transparent layer. Note that δ′ in Eq. (6) is different from δ and should be determined from

$$\cos\delta' = \sin\theta_l' \sin\theta_v' \cos(\varphi_l - \varphi_v) + \cos\theta_l' \cos\theta_v'. \tag{8}$$

In our previous study,[26] we have derived that the shadowing factor for a homogeneous, isotropic Gaussian height field has the following analytic form:

$$\Gamma_0(\theta) = \exp\left[-\frac{\tan\theta}{\sqrt{2\pi}s} \exp\left(-\frac{s^2}{4\tan^2\theta}\right)\right]. \tag{9}$$

As shown in Figure 4, with fixed parameter *s*, $\Gamma_0(\theta)$ pproaches from 1 to 0 when θ increases from 0 to π/2.

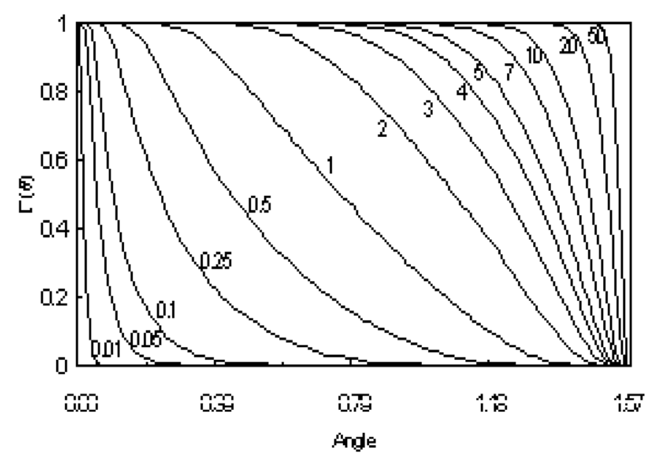


*Figure 4. The angular dependency of shadowing factor for different surface smoothness s. The value of s is shown near the corresponding curve.*

On the contribution to the overall glossy reflection from ray type 2 (Figure 1), the BRDF for the bottom surface $\rho'(\theta_l', \varphi_l', \theta_v', \varphi_v', \lambda)$ should be multiplied by two Fresnel coefficients for the transmissions through the top surface. Thus, the entire BRDF is:

$$\rho(\theta_l,\varphi_l,\theta_v,\varphi_v,\lambda) = \rho_1(\theta_l,\varphi_l,\theta_v,\varphi_v,\lambda) + \overline{T}(\theta_l,\lambda)\overline{T}(\theta_v,\lambda)\rho'(\theta_l',\varphi_l',\theta_v',\varphi_v',\lambda) \tag{10}$$

where $T(\theta_l, \lambda)$ and $T(\theta_v, \lambda)$ are the transmission Fresnel coefficients averaged over all polarizations.

To apply our derived model, we need to combine Eqs. (2), (6) and (10). Such computations will require the smoothness parameters $s$ and $s'$ for the top and bottom surfaces, the refractive index of the transparent material, and the spectral properties (i.e. the complex refractive index or optical constants) of the substrate material. Since these parameters are physically measurable, the accuracy of the derived equations can be verified with experimental measurements.

## Numerical and Rendered Results

Figure 5 presents the numerical results of the BRDFs for glossy appearance, as a two-dimensional function of the viewing angle and wavelength. The top plot is the BRDF of a pure copper surface for $s = 1$, and $\theta_l = \pi/4$, $\varphi_l = \pi$. The plot shows a peak near $\theta_v = \pi/4$, $\varphi_v = 0$, which is the perfect mirror-reflection condition. The bottom plot is the BRDF for the same conditions except that the copper plate is enamalized, i.e., covered by a thin transparent layer. Its difference from the top plot is that the spectral color at the highlight is significantly desaturated. Moreover, the highlight area is broader and extends significantly over the region of large $\theta_v$. This effect is related to the Snell's law, because this law maps large polar angles to small ones. Accordingly the reflection intensity increases, that is, smaller polar angles have higher values of $\Gamma_0(\theta)$.

Figure 6 shows some rendered copper plates to test our derived model for different lighting and viewing angles. We use the material data (optical constants) for the copper that are obtained from measurement.[27] The texture appearance on a plate depends on the difference in the smoothness parameter $s$ (larger $s$ generates brighter color). In the top row, since the plates are pure copper, i.e. not covered with any transparent layer, the plates do not appear glossy. In the bottom row, in contrast, the plates are enamelized, i.e. covered with a transparent layer (the refractive index is assumed 1.5). Thus, the plates in the bottom row demonstrate glossy highlights, which are indicated by the white spots at the plate centers. Note that the light source is white and its location ensures that the lighting and viewing angles are the same such that all the highlights occur at the plate centers.

It is interesting to compare the appearances of the rendered image pairs (the corresponding top and bottom images). For any pair, the difference between a pure copper plate and its enamelized counterpart is very obvious. In Fig. 6, we can observe a number of interesting properties. First, the colors of the enamelized plates are significatly desaturated compared to the pure copper plates. This result is due to the fact that the top transparent layer reflects the incident light (which is white) evenly across spectral wavelengths. Second, the color desaturation for an enamelized plate increases with the viewing (or lighting) angle, and this behavior agrees to the fact that the light reflection at the top layer increases with the viewing (or lighting) angle, a consequence of Fresnel's law. For the same reason, when the viewing angle increases, the white highlight becomes brighter while the copper color in other area becomes darker. Finally, at a large viewing angle, i.e., approaching the glazing angle, the pure plate only shows a portion of its texture while the whole pattern on the enamelized plate is still clearly observed. This can be explained by the broadness of the enamelized BRDF for the case (Fig. 5). Note that the disappearance of the texture in the bottom part of the pure copper plate (the top-right image in Fig. 6) is due to shadowing factor $\Gamma_0(\theta_l)$, which vanishes for large $\theta_l$ as shown in Fig. 4.

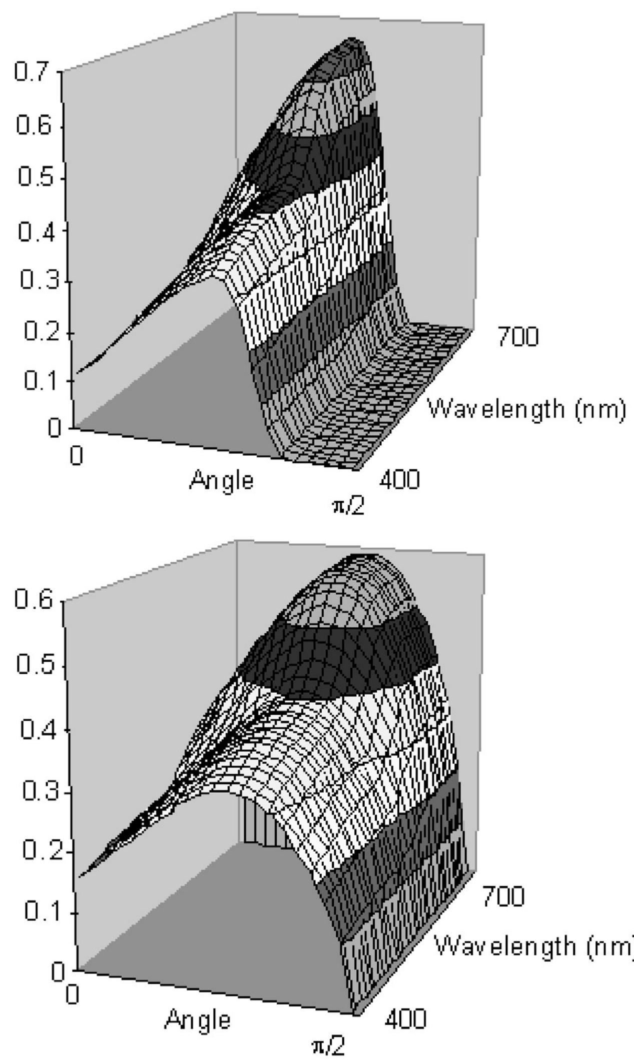


*Figure 5. Plots of BRDFs computed using our derived model. The top plot is the BRDF of a pure copper surface. The bottom one is for the same conditions except that the copper plate is enamalized (see text).*

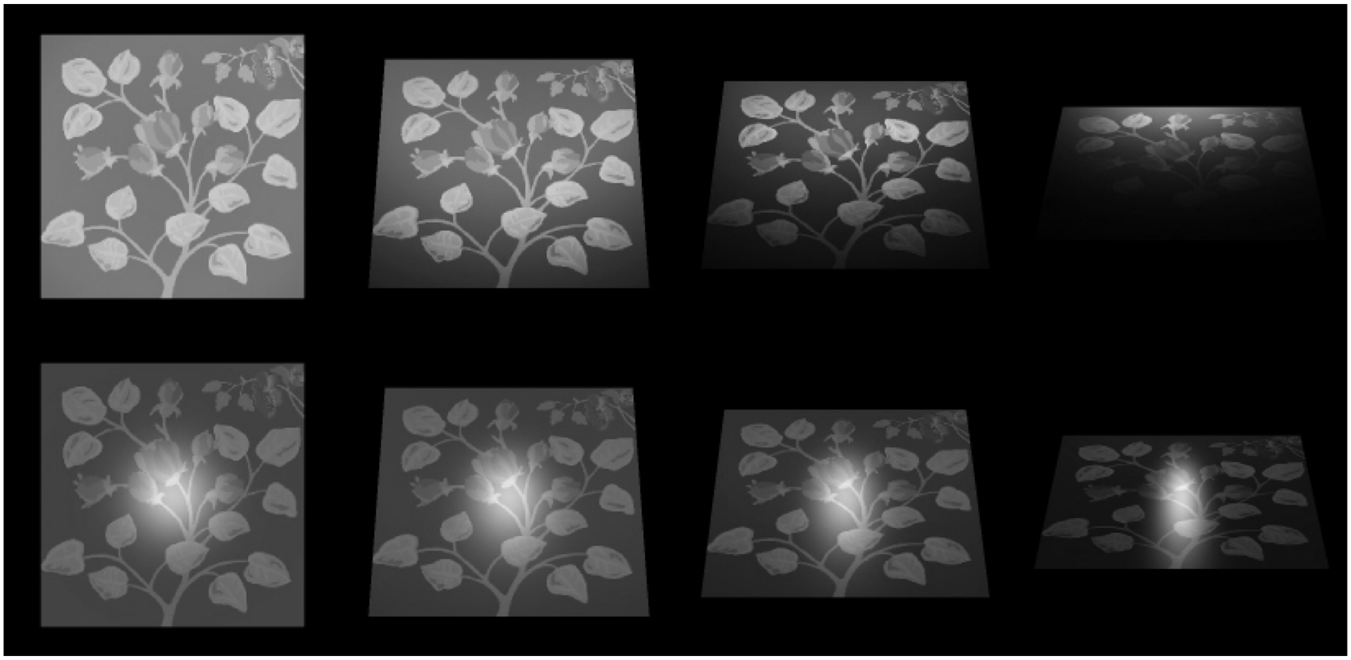

*Figure 6. Rendered copper plates using our derived glossy reflection model. The texture appearance is due to the difference in the smoothness parameter s. The plates in the top row are pure copper, while in the bottom they are enamelized.*

## Conclusions and Future Work

This paper has developed a physically-based reflection model for glossy surface appearance. The model has been derived based on the microfacet theory along with the consideration of the physical structure of glossy surfaces. Using this model, we have rendered pure and enamelized copper plates, and the rendered images have successfully demonstrated the characteristic features of glossy surface appearance.

An important feature of the model developed in this paper is that the model is entirely physically based and has eliminated the arbitrariness in choosing parameters in the common empirical method such as Phong model. Since all the parameters involved are physically measurable, this model can be verified strictly by experiments. Such experiments will involve measurements of surface smoothness,[28,29] material properties (refractive indexes),[27] as well as the overall BRDFs.[24,30]

As future work, our model can be extended in a number of ways. For example, the assumptions for our model can be removed such that general cases can be handled. These include allowing the top surface being rough, considering the volume absorption of light in the top transparent layer, accepting an inhomogeneous, anisotropic, or non-Gaussian bottom surface, and calculating the effect of multiple reflections between the top and bottom surfaces. Furthermore, our model provides a useful basis to deal with those glossy surfaces that involve more complex structures or scattering processes such as multiple layers and translucent materials.

Because our glossy reflection model is entirely physically based, it can be improved for better accuracy in a systematic way. Therefore, it has a great potential in graphics application for achieving high rendering realism. With its improved and verified accuracy, it will also benefit solving the problems in image analysis and computer vision. Finally, an accurate glossy reflection model will help industrial designs to achieve the desired appearances of commercial products such as automobiles, building interiors, furniture, and artistic enamelware.

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## Biography

Dr. Yinlong Sun is an assistant professor in the Department of Computer Sciences at Purdue University. His research interests include computer graphics, scientific visualization, computer vision, and color science. He is particularly interested in combining analytical, numerical, and experimental approaches to solve complex, cross-disciplinary problems. Yinlong received his B.Sc. from Beijing University of China in 1985, and a Ph.D. in Condensed Matter Physics and a Ph.D. in Computer Science from Simon Fraser University of Canada in 1996 and 2000. He is a member of ACM, ACM SIGGRAPH, IEEE, IEEE Computer, and IS&T.