

# Minimal Knowledge versus the Real World

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## Abstract

In this paper we show that Viggiano's Minimal Knowledge assumption can be used for color correction when the illumination is known. Indeed, given a known illuminant we find that real data is well modelled by the Minimal Knowledge assumption. Experiments demonstrate that color correction based on the Minimal Knowledge assumption is similar to that based on real data.

## 1. Introduction

XYZ tristimuli values are needed for accurate color reproduction. Unfortunately color devices are rarely colorimetric. That is, the colors a device sees (e.g. RGBs) are not equal to XYZ tristimulus values. Getting a color device to see tristimuli is called color correction. Typically the correction procedure involves measuring the device response for some calibration set of spectra. A mapping scheme is then derived which takes device RGBs to XYZs. The scheme might involve a look up table with interpolation (e.g. Ref. 1) or alternately (and the focus of this paper) RGBs might be mapped to XYZs using a single linear transform (e.g. Ref. 2).

There are two ways we might find a linear transform. We might carry out a regression based on physical samples or, secondly, given knowledge of the spectral sensitivities of a camera we calculate a transform using a statistical assumption. The advantage of the statistical approach is that it is possible to model an infinite set of spectra and also to incorporate desirable properties that the spectra have to adhere.

The simplest assumption to employ is the Maximum Ignorance assumption.<sup>3</sup> Here it is assumed that spectra are created by a random process where the power at each wavelength varies between -1 and 1. Under these conditions it can be shown that<sup>3</sup> the best least-squares fit transform is simply the linear combination of the RGB sensitivities which lie closest to the corresponding XYZ sensitivities. Unfortunately, negative power spectra do not occur and assuming that they do incurs a penalty: poorer calibration for those color signals that do occur.

Finlayson and Drew noticed this problem and proposed using only the knowledge that spectra are positive and have bounded power<sup>4</sup>: the Maximum Ignorance with positivity assumption (MIP). MIP color correction is better than MI

color correction.<sup>5</sup> One of the advantages of the MIP method is that it generally enforces a correction where white is accurately calibrated.<sup>5</sup>

Recently, Viggiano<sup>6</sup> has argued that we in fact are not maximally ignorant. Spectra tend to have certain predictable 'smoothness' properties. For example, if we measure spectral power at 500 Nanometres and 502 Nanometres we do not generally expect to see much of a difference: the wavelengths are surely correlated. Yet, if we compare measurements at 500 Nm and 600 Nm then we expect there may be a large difference. Viggiano's *Minimal Knowledge* (MK) assumptions makes two predictions about color signal spectra: first, that the correlation between wavelengths depends only on the magnitude of the difference of the wavelengths and second that the degree of correlation does not depend on wavelength location e.g. the correlation between spectra at 450 and 500 Nm is the same as that between 625 and 675 Nm since they are the same Wavelength distance apart.

However, as it stands the MK assumption is not completely defined. Rather, in Viggiano's model there is a control parameter  $\alpha$  which controls the degree of correlation (and so the degree of expected smoothness). Perhaps a more serious criticism of the MK assumption is that we know that real color signal spectra can be highly nonsmooth e.g. spectra captured where the light source is a Fluorescent light.

In this paper we take a second look at the Minimal Knowledge assumption and find there is much to commend it. We begin by making the role of illumination, in camera calibration, explicit and seek only to deliver color correction when the light source is known. By making this assumption we factor out the conditions that might induce non smooth spectra and so provide conditions relative to which the MK assumption might work. However, analysis of real data demonstrates that even when the light is factored out that the MK assumption is not applicable. However, this failure is not catastrophic; indeed, the MK assumption 'almost' holds. We find that wavelengths are correlated as a function of wavelength distance but, for all datasets we are aware of, the degree of correlation increases as a function of wavelength. This observation leads us to define a 'normalised' light, a 'redder' version of the actual light, with respect to which the MK assumption holds.

To demonstrate the applicability of the MK assumption we looked at the statistical properties of real data and found

the MK assumption to approximately hold. Experiments demonstrate that color correction driven by real color signal statistics is not significantly better than that derived under the MK assumptions. This result holds across datasets.

In section 2 we review least-squares color correction. We take trouble to give a detailed treatment of the process and in particular show how least-squares depends on the covariance matrix of a spectral dataset. Section 3 looks at the minimal knowledge assumption. It is shown that this MK assumption can be used if the illumination is known and that the *correlation* matrix of the spectral dataset is used. Experiments are presented in section 4.

## 2. Least-Squares Color Correction

Let  $\underline{X}(\lambda)$  denote the vector of standard observer color matching functions:  $x(\lambda)$ ,  $y(\lambda)$ , and  $z(\lambda)$ . The XYZ tristimulus vector  $\underline{x}$  corresponding to a reflectance  $S(\lambda)$  illuminated by a spectral power distribution  $E(\lambda)$  is equal to,

$$\underline{x} = \int_{\omega} E(\lambda)S(\lambda)\underline{X}(\lambda)d\lambda \quad (1)$$

where the integral is taken over the visible spectrum  $\omega$ . Let us denote the  $m$  (where  $m$  is typically 3) spectral sensitivities of a color device (e.g. color scanner or color camera) as  $\underline{R}(\lambda)$ . The  $m$ -vector device response to  $S(\lambda)$  illuminated by  $E(\lambda)$  is equal to:

$$\underline{r} = \int_{\omega} E(\lambda)S(\lambda)\underline{R}(\lambda)d\lambda \quad (2)$$

We adopt the convention that the visible spectrum can be represented adequately by samples taken 10nm apart over the range 400-700nm (this assumption is routine and forms the basis for the linear systems approach to color vision). Adopting this convention will allow the integrals in equations (1) and (2) to be replaced by summations. It follows that  $\underline{X}(\lambda)$  and  $\underline{R}(\lambda)$  can be represented as 31 x 3 matrices  $\mathcal{X}$  and  $\mathcal{R}$ :

$$\lambda_i = 390 + 10i \quad (i = 1 \dots 31) \quad (3)$$

$$\mathcal{X}_{ik} = X_k(\lambda_i) \quad (4)$$

$$\mathcal{R}_{ik} = R_k(\lambda_i) \quad (5)$$

The double subscript  $ik$  denotes the  $i$ th row and  $k$ th column of a matrix.

Further let  $C(\lambda)$  (the color signal) denote the product function  $E(\lambda)S(\lambda)$  and  $\underline{c}$  its vector approximation:

$$c_i = E(\lambda_i)S(\lambda_i) \quad (6)$$

the single subscript  $i$  indexes the  $i$ th element of  $\underline{c}$ . It follows that we can rewrite equations (1) and (2) as:

$$\underline{x} = \mathcal{X}\underline{c} \quad (7)$$

$$\underline{r} = \mathcal{R}\underline{c} \quad (8)$$

where  $'$  is the the transpose operation.

Let the 31 x  $n$  matrix  $C$  denote a set of  $n$  calibration color signal spectra. Each column of  $C$  contains a single color signal spectrum corresponding to the product of some spectral power distribution with some reflectance spectrum. The human observer and color device response to the entire calibration set are captured by the 3 x  $n$  and  $m$  x  $n$  matrices  $\mathcal{P}$  and  $\mathcal{Q}$ :

$$\mathcal{P} = \mathcal{X}'C \quad (9)$$

$$\mathcal{Q} = \mathcal{R}'C \quad (10)$$

Linear color correction involves mapping the device responses  $\mathcal{Q}$  to the corresponding tristimuli  $\mathcal{P}$ . The least-squares approach to color correction sets out to determine the 3 x  $m$  matrix  $\mathcal{T}$  which best maps  $\mathcal{Q}$  to  $\mathcal{P}$ . Specifically,  $\mathcal{T}$  is chosen to minimize:

$$\|\mathcal{T}\mathcal{Q} - \mathcal{P}\| \quad (11)$$

$\|\cdot\|$  above denotes Root Mean Square Error (the square root of the sum of squared differences between  $\mathcal{T}\mathcal{Q}$  and  $\mathcal{P}$ ). It is well known<sup>7</sup> that the matrix  $\mathcal{T}$  which minimizes (11) is equal to:

$$\mathcal{T} = \mathcal{P}\mathcal{Q}'[\mathcal{Q}\mathcal{Q}']^{-1} \quad (12)$$

In mathematical parlance  $\mathcal{Q}'[\mathcal{Q}\mathcal{Q}']^{-1}$  is called the *pseudo-inverse* of  $\mathcal{Q}$ . Substituting (9) and (10) into (12):

$$\mathcal{T} = \mathcal{X}'\mathcal{C}'\mathcal{R}[\mathcal{R}'\mathcal{C}'\mathcal{C}']^{-1} \quad (13)$$

We can see from (13) that  $\mathcal{T}$  depends only on the 31 x 31 matrix  $\mathcal{C}\mathcal{C}'$  and the 31 x  $m$  device sensitivities  $\mathcal{R}$ .

In order to determine  $\mathcal{C}\mathcal{C}'$  we might proceed in one of two ways. First, we might image a representative set of surfaces under a representative set of lights. The advantage here is that the least-squares regression of (12) is guaranteed to give a good fit for this data. The disadvantage is that it is difficult to define 'representative set' and moreover, we may wish to deliver reasonable performance for non representative data. The second, solution is simply to define the structure of  $\mathcal{C}\mathcal{C}'$  in such a way that it accounts for all spectra we might hypothesise are reasonable. In this approach it is a simple matter to incorporate assumptions about both reasonable and unexpected data.

The Minimal Knowledge assumption (MK) proposed by Viggiano<sup>6</sup> proposes that there is an expected correlation between measurements at different wavelengths. To understand what this means, it is useful to examine the role that  $\mathcal{C}\mathcal{C}'$  plays in more detail. First, note that in the regression formula of Equation (12) that if we substitute  $\mathcal{C}\mathcal{C}'$  by

$$\frac{\mathcal{C}\mathcal{C}'}{N}$$

that the same matrix  $\mathcal{T}$  is calculated (the scalar  $N$  cancels).

$$T = X'CC^t\mathcal{R}[\mathcal{R}CC^t\mathcal{R}]^{-1} = X^t \frac{CC^t}{N} \mathcal{R} \left[ \mathcal{R}^t \frac{CC^t}{N} \mathcal{R} \right]^{-1} \quad (14)$$

The  $ij$ th element of

$$\frac{CC^t}{N}$$

can be written as:

$$\left[ \frac{CC^t}{N} \right]_{ij} = \frac{\sum_{k=1}^N C_{ik} C_{jk}}{N} \quad (15)$$

The right hand side of (15) simply calculates the average of the product of the power at wavelength 4 multiplied by the power at wavelength  $j$  (over  $N$  spectra). The statistical term for this quantity is the covariance of  $i$  with respect to  $j$  (written as  $\sigma_{i,j}$  when  $i \neq j$  and  $\sigma_i^2$  [the variance at wavelength  $i$ ] otherwise). If the covariance term is large then this tells us that the spectral power at wavelength  $i$  is similar to that at wavelength  $j$ . Small covariance indicates a weaker relationship.

We point out that we are using the term covariance in a slightly non-standard way: covariance is usually calculated for variables where the mean is first subtracted (which has not been done here). But, the following argument leads to the conclusion that the term covariance might be adopted. Let us take the color signal  $C$  and add to this the measurements  $-C$ . Clearly, this forces the mean of the combined set to be zero. Moreover, it is straightforward to show that the covariance matrix is defined in (15) (because the mean of the combined data set is 0). Henceforth, we will consider

$$\frac{CC^t}{N}$$

as a covariance matrix and so adopt standard statistical nomenclature:

$$\Sigma = \frac{CC^t}{N} \quad (16a)$$

$$\Sigma_{ii} = \sigma_i^2 \quad (16b)$$

$$\Sigma_{ii} = \sigma_{i,j} \quad (16c)$$

and we rewrite (13) as

$$T = X' \Sigma \mathcal{R} [\mathcal{R} \Sigma \mathcal{R}]^{-1} \quad (17)$$

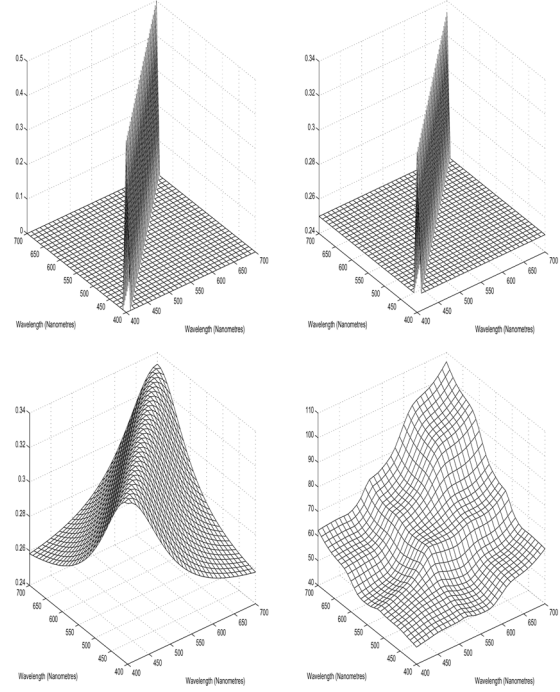


Figure 1. Left to right top to bottom, the correlation structures MI, MIP, MK and the Munsells

### 3. The Minimal Knowledge Assumption

Now that we understand that the driver of least-squares is a covariance matrix it is easy to think about and hypothesise potential covariance structures. For example, under the maximum ignorance (MI) assumption, all spectra with positive and negative power are equally likely and so the covariance between wavelengths is assumed to be 0 and in this case  $\Sigma^{MI}$  is one half of the 31 x 31 identity matrix:  $0.5I$ . Restricting attention only to all positive spectra, the maximum ignorance with positivity assumption,<sup>4</sup> leads to  $\Sigma^{MIP}$  where

$$\Sigma_{ij}^{MIP} = 1/4 (i \neq j) \quad \text{and} \quad \Sigma_{ii}^{MIP} = 1/3.$$

Significantly, enforcing positivity led to much improved color correction.<sup>5</sup>

Viggiano<sup>6</sup> proposed that we might reasonably assume more than simply positivity in defining the covariance structure. Specifically variance terms are the same as MIP:

$$\Sigma_{ii}^{MK} = \Sigma_{ii}^{MIP}$$

but the covariance terms are defined as:

$$\Sigma_{ij}^{MK} = \frac{1}{4} + \left( \frac{1}{12} * \frac{\alpha^2}{\alpha^2 + (\lambda_i - \lambda_j)} \right) \quad (18)$$

where  $\alpha$  is between 0 and 300 Nanometres. In Figure 1 the MI, MIP and MK (for  $\alpha = 100$ ) are shown, as mesh diagrams, in panels top left, top right, bottom left. In the bottom right panel the covariance structure for the Munsells<sup>8</sup> multiplied by a uniform white illuminant are shown.

Looking at Figure 1 the following conclusions might be drawn. First, that the MI and MIP assumptions yield covariance structures far from the covariance structure of real data. Second, that the minimal knowledge assumption is more like real data though even here the structure is visibly quite different. Moreover, the difference cannot be simply accounted for by changing the *smoothness* parameter  $\alpha$ . All covariance structures fitted created from (18) will have the same diagonal (all values equal to 1/3) and all elements in the bands either side of the main diagonal must also be equal. Viggiano noticed this banding behaviour and pointed out that in mathematics (and specifically time series analysis) covariance matrices of this form are called Symmetric Toeplitz matrices. Visual Inspection leads us to conclude that real covariance structures are not necessarily symmetric Toeplitz.

However, let us suppose for a moment that  $\Sigma$  for the real data is symmetric Toeplitz. What happens if we calculate the covariance structure for the same surfaces under a second illuminant? Let us rewrite the color signal matrix  $C$  as

$$C = D(\underline{E})S \quad (19)$$

where  $S$  is the  $31 \times N$  matrix of surface reflectances,  $\underline{E}$  denotes an illuminant vector and  $D()$  places the illuminant along the diagonal of a  $31 \times 31$  diagonal matrix. It follows then that:

$$CC' = D(\underline{E})SS'D(\underline{E}) \quad (20)$$

Clearly, to change to a second illuminant  $\underline{E}'$  we calculate

$$D(\underline{E}') [D(\underline{E})]^{-1} CC' (\underline{E})^{-1} D(\underline{E}') \quad (21)$$

In the example above, shown in Figure 1(d),  $\underline{E}$  is a white illuminant  $E_i = 1$  and so (21) is equal to

$$D(\underline{E}') SS' D(\underline{E}') \quad (22)$$

By the symmetric Toeplitz assumption the diagonal of  $SS'$  has elements with the same value (say  $v$ ). Under the new illuminant the diagonal must equal  $v(\underline{E}')^2$  and so unless  $\underline{E}$  is a white illuminant the new covariance structure cannot be Toeplitz. Rather than casting doubt on the Minimal Knowledge assumption the realisation that its applicability must depend on the illumination gives us a clue to how the minimal knowledge assumption might be applied in practice.

Since we know the diagonal elements of the covariance matrix must be equal in order to be Toeplitz (to meet the minimal knowledge assumptions) we might reasonably seek

an illuminant that makes the diagonal of the covariance have elements all equal to 1. Let us define a special illuminant  $\underline{F}$  such that the  $i$ th element of  $F$  is equal to the reciprocal of the standard deviation of the  $i$ th wavelength:

$$F_i = \frac{1}{\sigma_i} \quad (23)$$

It follows that

$$D(\underline{F})SS'D(\underline{F}) = \begin{bmatrix} 1 & \frac{\sigma_{1,2}}{\sigma_1\sigma_2} & \frac{\sigma_{1,2}}{\sigma_1\sigma_2} & \dots & \frac{\sigma_{1,2}}{\sigma_1\sigma_2} \\ \frac{\sigma_{1,2}}{\sigma_1\sigma_2} & 1 & \dots & & \\ \frac{\sigma_{1,2}}{\sigma_1\sigma_2} & \dots & \dots & & \\ \vdots & & & & \\ & & & \dots & 1 \end{bmatrix} \quad (24)$$

Seen in this way the *illuminant*  $\underline{F}$  takes the covariance structure  $\Sigma$  to its corresponding correlation matrix. The correlation between  $i$  and  $j$  is defined to be

$$\frac{\sigma_{i,j}}{\sigma_i\sigma_j}$$

(the covariance between  $i$  and  $j$  normalised with respect to the standard deviations  $i$  and  $j$ ). The left panel in Figure 2 shows the *illuminant*  $\underline{F}$  and the right panel the covariance structure of the Munsells as a mesh diagram.

From Figure 2 it is clear that the correlation matrix of our reflectance data looks more Toeplitz than before and so the Minimal Knowledge assumption seems more reasonable. Moreover, we arrive at this Toeplitz structure by looking at the surfaces (the Munsells) under a bluish light. From a signal processing vantage (though not necessarily from the human vision perspective) a blue light presents a balanced signal set in the sense that no wavelength appears more important than any other.

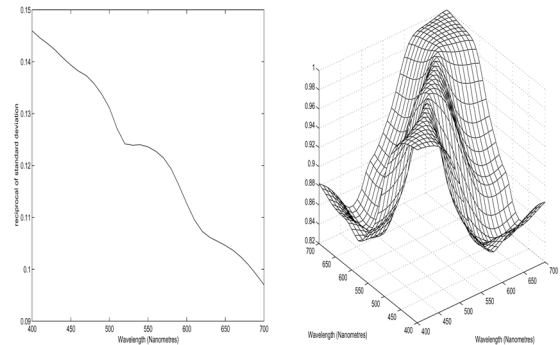


Figure 2. Under illuminant (left panel) the covariance structure of the Munsells (right panel) is more Toeplitz

Let us denote the correlation matrix corresponding to the covariance  $\Sigma$  as  $\Gamma(\Sigma)$ :

$$\Gamma(\Sigma) = D(\underline{F})\Sigma D(\underline{F}) \quad (25)$$

where  $\underline{F}$  is defined in (23). Now suppose take images with respect to a viewing illuminant  $\underline{V}$ . Clearly,

$$CC^t = D(\underline{V})S^tSD(\underline{V}) \quad (26)$$

and this might be rewritten as:

$$CC^t = D(\underline{V}/\underline{F})\Gamma(\Sigma)D(\underline{V}/\underline{F}) \quad (27)$$

where  $\underline{V}/\underline{F}$  denotes the elements of  $\underline{V}$  divided by corresponding elements of  $\underline{F}$ . Simply put, if we wish to map RGBs to XYZs under illuminant  $\underline{V}$  we can define an *normalized illuminant*  $\underline{V}/\underline{F}$  and substitute (26) into (13). Because  $\underline{F}$  is a bluish light,

$$\frac{1}{\underline{F}}$$

must be reddish and so Normalised light is more reddish than the actual illuminant. We make the illuminant more reddish to account for the fact that real reflectance data sets tend to have a higher covariance amongst in the longer wavelengths.

Let us now define a function *Toeplitz* that returns the Toeplitz matrix closest to a given correlation matrix. Such a function is easy to create if one remembers the banded structure of symmetric Toeplitz matrices. Let  $B(\mathcal{M},k)$  denote the  $k$ th band (counting outwards from the diagonal of matrix  $\mathcal{M}$ ).

$$B(\mathcal{M},k) = \{M_{i,i+k}, i = 1 \dots 31 - k\} \quad (k = 0,1,\dots,30) \quad (28)$$

In a Toeplitz matrix all the values in  $B(\mathcal{M},k)$  are the same. So, if we take a correlation matrix and replace all the values in band  $k$  by the mean of the band then the resultant matrix is the Toeplitz matrix which is closest to the original in the least-squares sense. This then is the definition of the function *Toeplitz*. The left panel of Figure 3 shows the correlation matrix of the Munsell dataset and the right hand panel the corresponding closest Toeplitz approximation.

Using the following formula we can measure how the degree to which a correlation matrix is Toeplitz:

$$err = \frac{\|\Gamma(\Sigma) - Toeplitz(\Gamma(\Sigma))\|}{\|\Gamma(\Sigma)\|} \quad (29)$$

where  $\|\cdot\|$  denotes the square root of the sum of squares. For the Munsells we find the data is Toeplitz with an error of less than 0.021 (i.e. the matrix is > 97.9% Toeplitz).

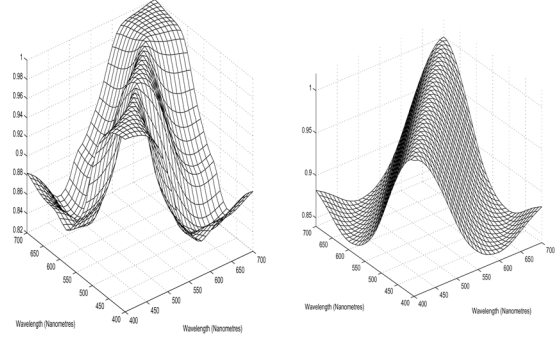


Figure 3. Left Panel: correlation matrix for the Munsells, Right Panel: closest Toeplitz approximation

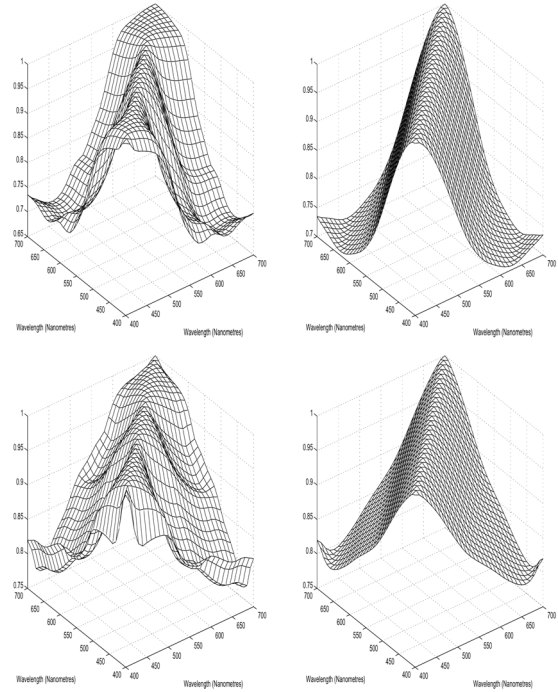


Figure 4. Top left: correlation matrix for Macbeth, Top Right: Toeplitz approximation for Macbeth, Bottom Left: correlation matrix for Object, Bottom Right: Toeplitz approximation for Object

Since the Toeplitz approximation of the correlation matrix gives the best case minimal knowledge conditions we can compute the best regression formula given the Minimal Knowledge assumption by substituting (30) in (13).

$$\hat{C}^t = D(\underline{V}/\underline{F})Toeplitz(\Gamma(\Sigma))D(\underline{V}/\underline{F}) \quad (30)$$

## 4. Experiments

In the left hand panels of Figure 4 are the correlation matrices for The Macbeth color checker<sup>9</sup> and the 170 object reflectances measured By Vrhel et al.<sup>10</sup> The Right hand panels show the closet Toeplitz approximations. It is evident that all the correlation structures are similar and that they are well approximated by a symmetric Toeplitz matrix. Indeed, according to (29) the matrices are 96.68% and 97.9% Toeplitz. Moreover, the Toeplitz structure is similar in all cases.

Let us now test how well adopting the Toeplitz form of the correlation matrices affects correction performance. The  $ij$ th element of Table 1 shows the average CIE Lab error for mapping the RGBs (for a SONY DXC930 Camera<sup>11</sup> and D65 illumination) to XYZs (also illuminant D65) for  $i$ th dataset (Munsell (1), Macbeth(2), Object(3)) where the  $j$ th dataset is used to define the covariance  $CC^t$  in (13). Table 2 repeats the experiment where  $CC^t$  calculated in (30) is used for calibration. Tables 3 and 4 repeat the experiment for CIE A illuminant. Table 3 reports correction driven by the actual correlation matrices. Table 4 correction is based on the Toeplitz approximation (Minimal Knowledge).

**Table 1. Least-squares for D65**

	Munsell	Macbeth	Object
Munsell	1.35	1.36	1.48
Macbeth	1.96	1.88	1.82
Object	1.33	1.19	1.17

**Table 2. Toeplitz least-squares for D65**

	Munsell	Macbeth	Object
Munsell	1.54	1.55	1.42
Macbeth	2.45	2.36	2.1
Object	1.47	1.35	1.24

It is clear that in all cases that Toeplitz approximation delivers similar performance to that achieved by the real data. Moreover, that good results are still achieved when the least-squares transform is calibrated with respect to one data set and applied to a second. That a minimal knowledge assumption that defines correlation only by wavelength distance should model real data so closely and to deliver such good color corrections augurs well for the general applicability of the MK assumption.

## 5. Conclusions

Viggiano proposed a minimal knowledge assumption to model the color signal spectra that might be encountered by a color camera, In this paper we argued that the Minimal Knowledge assumption is not appropriate for unknown viewing illuminants but is appropriate if the lighting conditions are known. When they are known the Minimal Knowledge assumption can deliver calibration performance comparable to the best achievable by least-squares.

**Table 3. Least-squares for illuminant A**

	Munsell	Macbeth	Object
Munsell	1.46	1.52	1.62
Macbeth	2.31	2.24	2.16
Object	1.63	1.42	1.39

**Table 4. Toeplitz least-squares for A**

	Munsell	Macbeth	Object
Munsell	1.73	1.77	2.4
Macbeth	2.83	2.74	2.93
Object	1.8	1.59	1.48

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## Biography

Professor Graham Finlayson leads the colour group at the University of East Anglia. In the last 5 years the Professor Finlayson has published over 100 scientific papers in colour and is currently involved in 4 Patent applications. In 2000 and 2001 he was respectively co-technical and co chair of the 8th and 9th Color Imaging conferences.