A Method for the Evaluation of Color Estimation from Multispectral Data

Yuri Murakami[1], Takashi Obi[2,3], Masahiro Yamaguchi[2,3] and Nagaaki Ohyama[1,3]
[1]Tokyo Institute of Technology, Frontier Collaborative Research Center,
[2]Tokyo Institute of Technology, Imaging Science & Engineering Laboratory Yokohama, Japan
[3]TAO of Japan, Akasaka Natural Vision Research Center, Tokyo, Japan

Abstract

This paper presents a new evaluation method of the color estimation accuracy for a set of multispectral color sensors. In this method, the probability density distribution of the spectral reflectance is assumed, and then it yields the possible range of estimation error in the color space at a specified confidence level. It is shown that the error range in the color space is expressed as the ellipsoids, when the probability density of the spectral reflectance is given by a multi-dimensional Gaussian function. Some examples of the evaluations of multispectral imaging systems are shown for different numbers of bands, and the results are compared with MacAdam ellipse.

Introduction

Since various color imaging devices have appeared recently, including multispectral types,[1-3] it is becoming increasingly important to evaluate the accuracy in the color estimation of those devices every year. Especially for multispectral imaging devices, it should be concerned that the accuracy of the estimated colors for not only recording illuminations or standard observers but also various rendering illuminations or individual observers. However a standard method for the evaluation of a color imaging devices, i.e. spectral shapes of a set of sensors, has not been established.

There have been several attempts to express the color estimation accuracy of a set of sensors by a single metric. Start with Neugebauer's quality factor,[4] then Vora and Trussell,[5] Tajima,[6] Vrhel and Trussell,[7] Wolski et al.,[8] and Sharma and Trussell[9] have proposed respective measures. Since all those measures are a single figure, it can be easily applied in the system optimization problems. From those metrics however, it is difficult to understand how much error can be arisen intuitively. Alternatively, the ensemble average and maximum error of the estimated colors of a set of samples are often used, but it depends on the composition of the samples. In addition, the estimation algorithm used in the evaluation also affects the evaluation results.

This paper presents a new method for the evaluation of the color estimation accuracy of a set of color sensors. If a Luther-conditioned camera is used for image capturing, the object color under the recording illumination can be obtained colorimetric accurately. However, the color under the different illumination from the recording is not decided uniquely. In this paper, this uncertainty is used for the evaluation of the color estimation accuracy of the sensors. To be more precise, the uncertainty is presented as a possible range of the error at a specified probability level, which is independent to neither sample compositions nor estimation method.

Concepts

The color gamut of reflective natural objects, the most famous one of which was Pointer's in 1980,[10] plays an important role in the evaluation of the gamut of display devices or the development of gamut mapping techniques. In contrast, the gamut of the reflectances of natural object has been desired recently, for the purpose of the evaluation of metamerism or the calculation of the gamut of hardcopy taking into account of metamerism. On those requests, spectral gamut has been investigated and several representations of the spectral gamut are proposed.[11-12]

If we have a spectral gamut of objects, the range of the metameric reflectances for a set of sensor responses can be calculated. The reflectances inside the metameric range are indistinguishable to the sensors. At the same time, any reflectance outside this range gives a different response to the sensors from the responses of the inside reflectances. Therefore, the projection of this range to the color space can be the minimum range that certainly contains the true color. In other words, the sensors guarantee that the maximum error is less than the size of this range. From those characteristics, this range can be used for the evaluation of the color sensors on the color estimation accuracy. We call it error range below.

If we have a probability density of the spectral reflectance of natural objects, instead of gamut, the error range is calculated probabilistically. In this case, the error range is expressed as the confidence range that contains the true color with a specified probability level.

In this paper, we present the error range in case of that the probability density of objects is expressed by the multi-dimensional Gaussian function.

Error Range for Gaussian Probability Case

Derivation

Providing that an image capturing system is linear, the capturing process is represented by

$$\begin{aligned}\mathbf{g} &= \mathbf{S}\mathbf{E}_r\mathbf{f} + \mathbf{n} \\ &= \mathbf{H}\mathbf{f} + \mathbf{n},\end{aligned} \tag{1}$$

where $\mathbf{g}$ is a L-dimensional vector representing observed intensity, $\mathbf{S}$ is a N-by-L matrix whose i-th row vector represents the spectral characteristics of the i-th channel of the camera, $\mathbf{E}_r$ is a L-by-L diagonal matrix representing the spectral power distribution of the recording illumination, $\mathbf{f}$ is a L-dimensional vector representing the spectral reflectance of an object, and $\mathbf{n}$ is a L-dimensional vector representing additive noise.

The tristimulus values, e.g. CIE XYZ, of the object under an illumination, whose spectral intensity is given by $\mathbf{E}_v$, is represented by

$$\mathbf{c} = \mathbf{T}\mathbf{E}_v\mathbf{f}\ , \tag{2}$$

where $\mathbf{c}$ is a 3-dimensional column vector of tristimulus, $\mathbf{T}$ is a 3-by-L matrix whose row vector represents the color matching functions corresponding to $\mathbf{c}$.

Let us think about a range that contains the real tristimulus $\mathbf{c}$ in three-dimensional color space at a specified confidence, provided that a vector $\mathbf{g}$ is observed. Here we assume that the probability of $\mathbf{f}$ is described or approximated by Gaussian distribution as

$$P(\mathbf{f}) \propto \exp\left(-\frac{1}{2}(\mathbf{f} - \langle\mathbf{f}\rangle)^T \Sigma_f^{-1}(\mathbf{f} - \langle\mathbf{f}\rangle)\right), \tag{3}$$

where $\langle\mathbf{f}\rangle$ and Σ_f are the average and the covariance of $\mathbf{f}$. If the probability density of $\mathbf{n}$ is also Gaussian,

$$P(\mathbf{n}) \propto \exp\left(-\frac{1}{2}\mathbf{n}^T\Sigma_n^{-1}\mathbf{n}\right), \tag{4}$$

where Σ_n is a covariance matrix of $\mathbf{n}$, and $\mathbf{f}$ and $\mathbf{n}$ are independent, the conditional probability density of $\mathbf{f}$ given $\mathbf{g}$ is[13]

$$\begin{aligned}P(\mathbf{f} \mid \mathbf{g}) &\propto P(\mathbf{f})P(\mathbf{n}) \\ &\propto \exp\left(-\frac{1}{2}(\mathbf{f} - \mathbf{f}^*)^T \Sigma^{*-1}(\mathbf{f} - \mathbf{f}^*)\right),\end{aligned} \tag{5}$$

where

$$\mathbf{f}^* = \Sigma_f\mathbf{H}^T\left(\mathbf{H}\Sigma_f\mathbf{H}^T + \Sigma_n\right)^{-1}(\mathbf{g} - \mathbf{H}\langle\mathbf{f}\rangle) + \langle\mathbf{f}\rangle \tag{6}$$

and

$$\Sigma^* = \left(\mathbf{I} - \Sigma_f\mathbf{H}^T\left(\mathbf{H}\Sigma_f\mathbf{H}^T + \Sigma_n\right)^{-1}\mathbf{H}\right)\Sigma_f\ . \tag{7}$$

Equation (5) indicates that $P(\mathbf{f}|\mathbf{g})$ becomes also Gaussian. Since the tristimulus values $\mathbf{c}$ are obtained by linear projection of $\mathbf{f}$ as Eq. (2), the conditional probability of $\mathbf{c}$ given $\mathbf{g}$ is also Gaussian and its average vector and covariance matrix are

$$\begin{aligned}\mathbf{c}^* &= \mathbf{T}\mathbf{E}_v\mathbf{f}^* \\ &= \mathbf{T}\mathbf{E}_v\left\{\Sigma_f\mathbf{H}^T\left(\mathbf{H}\Sigma_f\mathbf{H}^T + \Sigma_n\right)^{-1}(\mathbf{g} - \mathbf{H}\mathbf{f}) + \langle\mathbf{f}\rangle\right\}\end{aligned} \tag{8}$$

and

$$\begin{aligned}\Sigma_c^{\ *} &= \mathbf{T}\mathbf{E}_v\Sigma^*\mathbf{E}_v\mathbf{T}^T \\ &= \mathbf{T}\mathbf{E}_v\left(\mathbf{I} - \Sigma_f\mathbf{H}^T\left(\mathbf{H}\Sigma_f\mathbf{H}^T + \Sigma_n\right)^{-1}\mathbf{H}\right)\Sigma_f\mathbf{E}_v\mathbf{T}^T.\end{aligned} \tag{9}$$

Then 95% confidence ellipsoid of $\mathbf{c}$ given $\mathbf{g}$ in the color space, is given by

$$(\mathbf{c} - \mathbf{c}^*)^T\Sigma^*_c(\mathbf{c} - \mathbf{c}^*) = 7.81\ , \tag{10}$$

where 7.81 is the 5% of the χ^2 distribution for 3 degrees of freedom. Equation (10) represents the three-dimensional ellipsoid whose center is $\mathbf{c}^*$, and it is the error range which contains the true color $\mathbf{c}$ at 95% confidence.

Equation (10) indicates that the form of the error range is uniform in the color space obtained by Eq. (2). We can also say that it is independent to the estimation method. However, Wiener estimation has a close connection with it. From Eq.(8), it can be said that $\mathbf{c}^*$ is equivalent to Wiener estimate when Σ_f, $\langle\mathbf{f}\rangle$ and Σ_n are used in the estimation. That is, when $\mathbf{c}'$ is obtained through Wiener estimation, the original $\mathbf{c}$ is contained within the ellipsoids $(c - c')^T\Sigma^*_c(c - c') = 7.81$ with 95% probability.

If the size of the error range of a color input device is small, it can be said that it has high color reproduction accuracy. If we consider about an ideal case where a system is noise free and the condition $\mathbf{E}_r\mathbf{H}=\mathbf{E}_v\mathbf{T}$ is satisfied. In this case, all elements of matrix Σ^{*}_c becomes 0, which means possible estimate is settled into one point, original $\mathbf{c}$, without estimation error.

Error Range on u'v' Plane

Error range can be expressed in other color spaces or planes. If we use a uniform color space, the error range varies depending on the color coordinates. As an example, error range on CIE 1976 u'v' plane is shown.

Chromaticity coordinates $\mathbf{u}= (u', v')^T$ is defined as

$$\begin{cases} u' = \dfrac{4X}{X + 15Y + 3Z} \\ v' = \dfrac{9Y}{X + 15Y + 3Z} \end{cases}. \tag{11}$$

If $\mathbf{c}$ is defined as CIE XYZ, the covariance of $\mathbf{u}$ is

$$\Sigma^*_{uv} \cong \mathbf{J}\Sigma^*_c\mathbf{J}^T\ , \tag{12}$$

where

$$\mathbf{J} = \begin{bmatrix} \frac{\partial u'}{\partial X} & \frac{\partial u'}{\partial Y} & \frac{\partial u'}{\partial Z} \\ \frac{\partial v'}{\partial X} & \frac{\partial v'}{\partial Y} & \frac{\partial v'}{\partial Z} \end{bmatrix} \tag{13}$$
$$= \frac{1}{(X + 15Y + 3Z)^2} \begin{bmatrix} 60Y + 12Z & -60X & -12X \\ -9Y & 9X + 27Z & -27Y \end{bmatrix}.$$

Then the 95% confidence ellipse, i.e. error range, is given by

$$(\mathbf{u} - \mathbf{u}^*)^T \Sigma^*{}_u (\mathbf{u} - \mathbf{u}^*) = 5.991 \tag{14}$$

where $\mathbf{u}^*$ is u'v' coordinate calculated from $\mathbf{c}^*$ and 5.991 is the 5% of the χ^2 distribution for 2degrees of freedom.

Computed Examples

We compute error ranges defined by the 95% confidence ellipses on u'v' plane in some conditions. In the computations, the covariance and average of spectral reflectance of 168 samples in Macbeth ColorChecker DC are used as the object statistics. Noise is assumed to be random white noise, and signal to noise ratio (SNR) is defined as

$$SNR = 10 \log \left(\frac{\|\mathbf{g}_w\|^2}{\|\mathbf{n}\|^2} \right), \tag{15}$$

where $\mathbf{g}_w$ is the observed intensity of white reference. Spectral sensitivities of three different cameras are prepared (Fig. 1). First is a three-channel camera (FD420M, Flovel), second is a six-channel camera assembled by two FD420M's and two kinds of filters in our laboratory, and third is a ten-channel camera with ten narrow-band interference color filters. Recording illumination is assumed as CIE D65 standard illuminant and color matching functions are CIE 1931.

Figure 2 shows the results for SNR=50dB and D65 observed illumination. The centers of the ellipses $\mathbf{c}^*$'s are sampled at even intervals in L*a*b color space as a* and b*=$\pm k \times 20$ (k=0,1,2,…) and L*=50 within the available range. From these results, we can see that the shape, size and axes direction of the ellipses are different depending on the chromaticity coordinates. The reason of those variation is the nonlinear relationship between u'v' plane and XYZ color space. On the other hand, not only size but also axes directions of ellipses are different depending on the cameras, which reflect the respective color reproduction accuracy of cameras. We can also confirm that the size of ellipse is reduced as increasing the number of the color channels.

Figure 3 shows the comparisons between the error range and MacAdam ellipses in case of D65 viewing illumination (noise free and SNR=50dB) and F2 viewing illumination (SNR=50dB) at respective white point. We can confirm that the sizes of the 10-channel ellipses are the almost same level as the MacAdam ellipses. In addition, the sizes of 6-channel ellipses are contained by a few times of MacAdam's. We can also see that the SNR-50dB ranges slightly expand from the noise-free ranges, which can be seen that the uncertainty increases by just uncertainty of noise. Since the amount of the added uncertainty is same for every camera system, the influence is noticeable especially when the number of the channels is large.

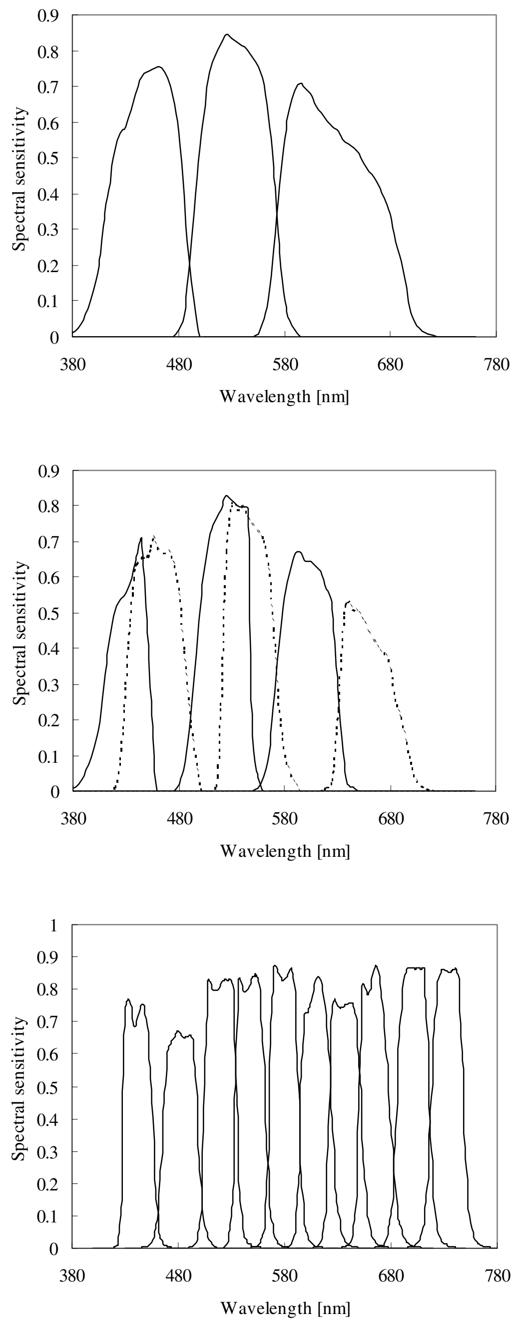

Figure 1. Spectral sensitivities of three-channel (top), six-channel (middle), and ten- channel cameras (bottom).

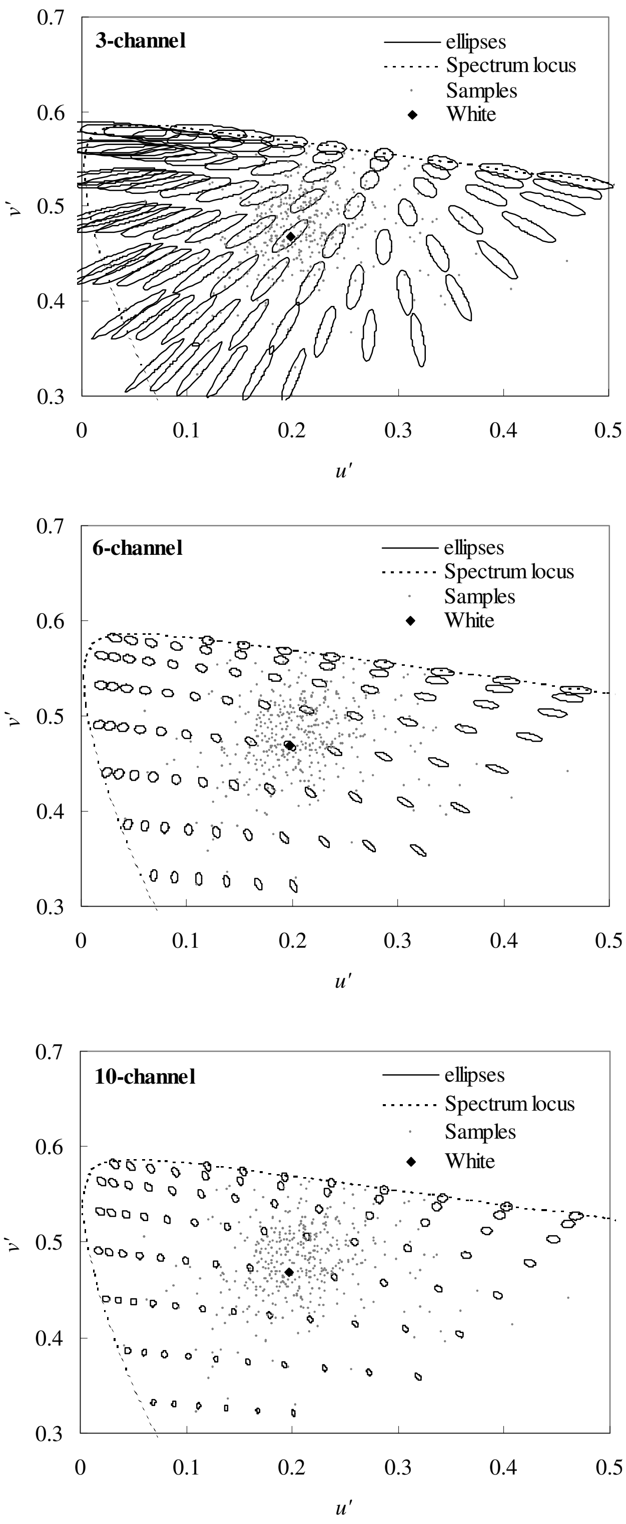

Figure 2. 95% confidence ellipses of estimated colors for three-channel camera (top), six-channel camera (middle), and ten-channel camera (bottom). Gray points are samples used for the calculation of the statistics. The filled square symbol indicates white point of D65.

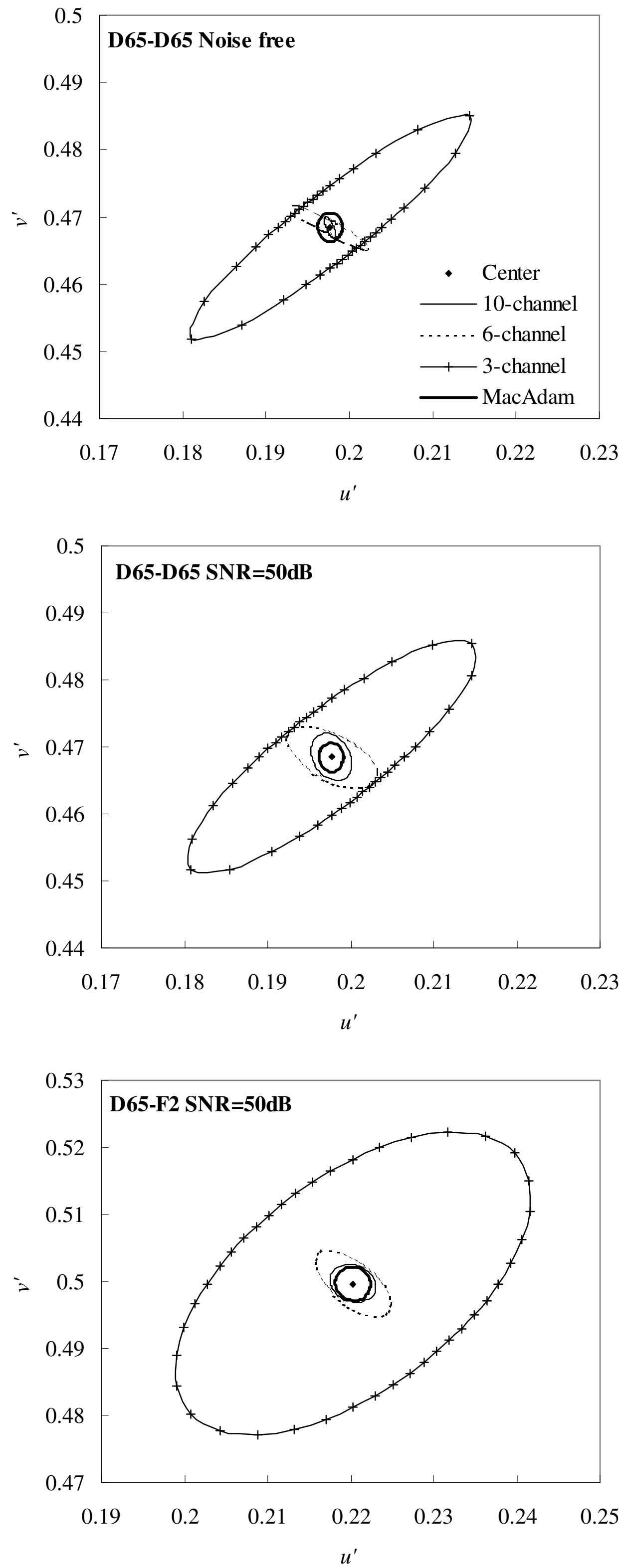

Figure 3. 95% confidence ellipses for 3 kinds of cameras in comparison with MacAdam ellipses.

Conclusion

This paper proposed an evaluation method for multispectral color sensors by an error range that derived from the probability density of the spectral reflectance of objects. When a probability density is expressed by multi-dimensional Gaussian distribution, the error range is expressed as ellipsoid, the size and axes of which are inherent in the spectral shapes of the color sensors. As the results of the evaluation of multispectral imaging systems using the statistics from Macbeth ColorChecker DC, it can be confirmed that the error ranges of ten-channel system are almost same size of MacAdam ellipses.

References

1. H. Haneishi, T. Hasegawa, A. Hosoi, Y. Yokoyama, N. Tsumura and Y. Miyake, "System design for accurately estimating spectral reflectance of art paintings," *Appl. Opt.* **39**, 6621-6632 (2000).
2. Th. Keusen, "Multispectral color system with an encoding format compatible with the conventional tristimulus model," *J. Imaging Sci. Technol.* **40**, 510-515 (1996).
3. F. H. Imai, R. S. Berns and Di-Y. Tzeng, "A comparative analysis of spectral reflecntace estimated in various spaces using a trichromatic camera system," *J. Imaging Sci. Technol.* **44**, 280-287 (2000).
4. H. E. J. Neugebauer, "Quality factor for filters whose spectral transmittances are differenct from color mixture curves, and its application to color photography," *J. Opt. Soc. Amer. A*, **46**, pp821-824 (1956).
5. P. L. Vora and H. F. Trussell, "Measure of goodness of a set of color scanning filters," *J. Opt. Soc. Amer. A*, **10**-7, pp1499-1508 (1993).
6. Tajima Johji, "New quality measures for a set of color setnsors – weighted quality factor, spectral characteristic restorability index and color reproducibility index," *Proc. of 4th CIC*, pp25-28 (1996).
7. M. J. Vrhel and H. J. Trussell, "Filter consideration in color correction," *IEEE Trans. Image Processing*, **3**, 147-161 (1994).
8. M. Wolski, J. P. Allebach, C. A. Bouman and E. Walowit, "Optimization of sensor response functions for colorimetry of reflective and emissive objects," *Proc. of SPIE*, **2170**, pp209-219 (1994).
9. G. Sharma and H. J. Trussell, "Figures of merit for color scanners," *IEEE Trans. Image Processing*, **6**, 990-1001, (1997).
10. M. R. Pointer, "The gamut of real surface colors," *Color Res. Appl.*, **5**, 145-155 (1980).
11. H. Haneishi, Y. Sakuda, T. Honda, "Polyhedral gamut representation of natural objects based on spectral reflectance database and its application," *Proc. of AIC'01*, pp-934-938 (2001).
12. G. D. Finlayson and P. Morovic, "Metamer crossovers of infinite metamer sets," *Proc. of 8th CIC*, pp13-17 (2000).
13. William Menke, "Geophysical data analysis: discrete inverse theory," (Academic Press, INC., San Diego, (1989), 92-99).

Biography

Yuri (Ohya) Murakami received her B.S. degree in mathematical and physical science from Japan Women's University at Tokyo, Japan in 1996 and M.Eng. degree in information processing from Tokyo Institute of Technology at Yokohama, Japan in 1998. Since 2000 she is a research associate in Frontier Collaborative Research Center of Tokyo Institute of Technology and a fellow researcher of Akasaka Natural Vision Research Center, TAO at Tokyo, Japan. Her work has primarily focused on the color image reproduction using multispectral imaging and multispectral image compression.