

# A Simplified Method for the Colorimetric Characterization of Fluorescent Inks

*Ricardo Motta and Joyce Farrell*  
*Hewlett-Packard Laboratories, Palo Alto, California*

## Background

According to Grum<sup>1</sup> luminescence is “the phenomenon of the emission by matter of electromagnetic radiation that for certain wavelengths or restricted regions of the spectrum is in excess of that due to the thermal radiation from the material at the same temperature.” We encounter several different types of luminescence in everyday life such as fluorescence, phosphorescence, cathodoluminescence and chemiluminescence. Fluorescence occurs when light absorbed in one wavelength band generates emissions in longer wavelength bands. This shift towards longer wavelengths is called Stoke’s shift.

Fluorescence is specially prevalent in printed materials. It is commonly associated with the optical brighteners used to whiten paper and the dyes used in the printing inks. Paper has a strong tendency to absorb in the blue region of the visible spectrum (that is why grocery bags are brown.) Optical brighteners work by absorbing in the near UV region, where we do not perceive the decrease in reflectance, and re-emit the energy as blue light, which offsets the paper natural yellowishness. The most common fluorescent dyes are red and magenta. They work by absorbing radiation in the blue green regions, which makes them look more saturated, and re-emit energy in the red end of the spectrum, further increasing the colorimetric purity. Fluorescent yellow dyes are also very common.

Fluorescence has major implications for calibrating color reproduction systems because the spectral reflectance of fluorescent surfaces is **dependent** on the illumination. Unlike non-fluorescing surfaces, which can be described by a  $n$ -dimensional vector of spectral reflectance factors, fluorescing surfaces require at least three vectors. They are called here the diffuse reflectance, the stimulation spectrum and the emission spectrum. Conversely, the fluorescent surface can be described as an  $(n \times n)$  lower diagonal matrix where the diagonal contains the conventional non-fluorescent reflectance and the off-diagonal elements contain the contribution due to fluorescence. Figure 1 illustrates such matrix for a magenta printer ink.

For scanner calibration fluorescence represents a major obstacle. Colorimetric calibration can only be accurately performed for a fixed illuminant, and depending on the combination of detector, light and filters one might find that such illuminant has undesirable white point and metameric properties.

The calibration of printers is no less problematic. While diffuse surfaces can be characterized with simple spectrophotometry, and tristimulus computed later for any desired illuminant, fluorescent surfaces must be characterized with a source that closely resembles the desired illuminant, or by measuring the entire reflectance matrix.

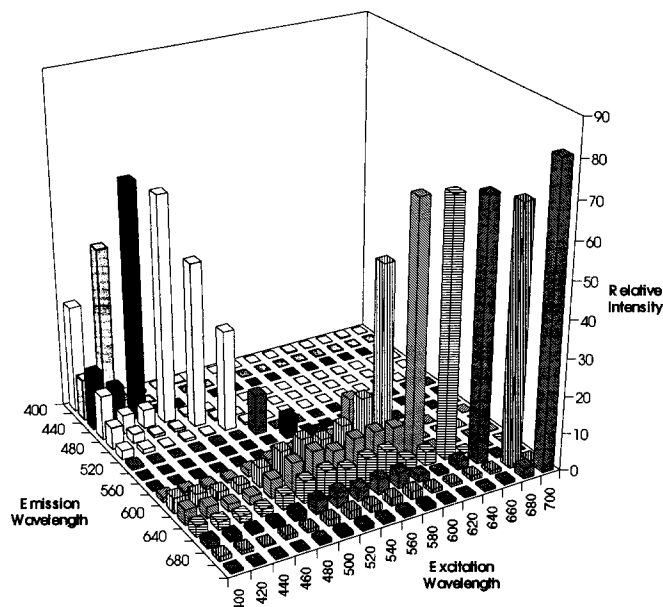


Figure 1. Reflectance matrix for a magenta printing ink

Both are very cumbersome and elaborate tasks, especially if hundreds of samples have to be measured as in most printer calibrations.

The conventional methods for characterizing the spectral reflectance of a fluorescent surface was developed by Donaldson<sup>2</sup> in 1954 and requires measurements of the spectral power distribution of reflected and emitted light at all wavelengths by means of a double-monochromator. In this talk, we describe a variant of Donaldson’s method that allows us to calculate the  $XYZ$  values of a fluorescent surface under a wide variety of illuminants based on a relatively small set of colorimetric measurements.

## Description of Method

Following Wandell<sup>3</sup> we use the term *color signal* to refer to light reflected from surfaces and arriving at a sensing device. The color signal,  $c$ , can be represented as a vector with  $n$  entries representing the radiation values over a range of wavelengths (e.g.  $n=81$  when the wavelengths range from 380nm to 780nm in 5 nm steps). The illuminant,  $e$ , is also a vector with  $n$  entries. But the surface radiance factor,  $S$ , of a fluorescent surface is a matrix with  $(n \times n)$  entries. The color signal,  $c$ , is the product of the surface matrix  $(n \times n)$  and the illuminant matrix  $(n \times 1)$ .

$$c = S e \quad (1)$$

Our method takes advantage of the fact that most illuminants can be described as a linear combination of a

small set of spectral basis functions. To demonstrate the method, we selected 16 filtered tungsten lights to form a spectral basis set from which we can describe arbitrary surface illuminants,  $e$ . The spectral transmissivity of the 16 filters have been selected such that the 16 filtered tungsten lights approximate a wide range of illuminant spectra. In other words,

$$e = Iw \quad (2)$$

where  $w$  is a  $(16 \times 1)$  vector that maps the  $(n \times 16)$  illuminant basis functions,  $I$ , into  $e$ . Thus,

$$c = Se = SIw \quad (3)$$

One can use the spectral radiance of a fluorescent surface measured under the 16 lights to derive an estimate of the color signal of the surface illuminated by  $e$ . The estimates will be accurate for any illuminant,  $e$ , that can be approximated by a linear combination of the 16 spectral basis functions formed by the set of filtered lights.

To demonstrate this latter calculation, we measured the spectral reflectance of a fluorescent surface under the 16 spectral light. This data forms a  $(n \times 16)$  matrix,  $SI$ . Given  $w$ , we can then compute the color signal under the illuminant  $e$  by Equation 3, above.

To compute the  $XYZ$  values of a fluorescent surface under illuminant  $e$ , we can either estimate the color signal of a fluorescent surface under illuminant  $e$ , using the above equation, or simply measure the  $XYZ$  values of the surface illuminated by the 16 spectral lights. Then, the  $XYZ$  tristimulus values that we would expect to measure under illuminant  $e$  is predicted by:

$$x = Mw \quad (4)$$

where  $x$  is a  $(3 \times 1)$  vector containing the predicted  $XYZ$  values,  $M$  is a  $(3 \times 16)$  matrix containing the  $XYZ$  values, either directly measured or computed, and  $w$  is a  $(16 \times 1)$  vector that maps the illuminant basis functions,  $I$ , into  $e$ .

### Optimizing the illuminant spectral basis functions

Obviously, one would like to minimize the number of spectral lights or basis functions that approximate

illuminants Judd *et al.*<sup>4</sup> showed that we could approximate daylight with as few as three spectral basis functions. But we are constrained to physically realizable lights (i.e. no negative numbers) and our set of illuminants that we wish to approximate includes fluorescent and tungsten, as well as daylight. We will need more than three spectral lights, but, as we will show, less than 16.

### Optimizing the illuminant weights

There are two ways to find the weights,  $w$ . One, by minimizing the squared error in wavelength space, i.e.

$$[e - Iw]^2 \quad (5)$$

The second, by minimizing the squared error in  $XYZ$  space, i.e.

$$[(x)(e - Iw)]^2 \quad (6)$$

where  $x$  is a  $(3 \times n)$  matrix containing the  $XYZ$  sensor reponsivities.<sup>5</sup>

Minimizing the equation above leads to better results. We show this by measuring  $XYZ$  values for a fluorescent surface under an illuminant  $e$ . We then predict the  $XYZ$  values for the surface under illuminant  $e$  based on colorimetric measurements of the surface under the 16 spectral lights. The weights  $w$  used in this prediction were obtained in one of two ways outlined above. The latter equation minimizes the perceptual difference ( $\Delta E$ ) between the measured and predicted  $XYZ$  values.

### References

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