Comparisons of Color Mixing Theories for Use in Electronic Printing

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Color Mixing Theory

Roughly speaking, there are two kinds of the color mixing theories. One is formulated for the halftone printing process such as the Neugebauer equations (NE), Yule-Neilsen approximation (YN), and Clapper-Yule multiple internal reflections (CY). The other is based on the subtractive principle such as the Beer-Bouguer law (BB) for the homogeneous medium and the Kubelka-Munk theories (KM) for the turbid medium. In addition, we present an empiri-cal extension of the subtractive models for the halftone printings. We compare these models by applying them to a Canon color copier and an ink jet printer. Brief descrip-tions of these theories with regard to the basic assumptions, formulations, and comparison results, are given in this paper. The emphasis of this study is placed on the spectral modeling of the halftone prints.

Neugebauer equations

The Neugebauer model perhaps is the first attempt to mathematically account for the halftone printing. It is based on broad-band color mixing. Neugebauer recognized that there are eight colors—namely, white, cyan, magenta, yellow, red, green, blue, and black—for constituting any color halftone prints. A given color is perceived as the integration of subtractive primary colors and their 2-color and 3-color overlaps. The incident light reflected by one of the eight colors is equal to the reflectance of that color multiplied by its area coverage. The total reflectance is the sum of all eight colors weighed by the area.^{1,2} An example of a 3-color Neugebauer equation is

$$G = A_w G_w + A_c G_c + A_m G_m + A_y G_y + A_r G_{my} + A_g G_{cy}$$

+ $A_b G_{cm} + A_{cmy} G_{cmy}$ (1)

where G = the total green reflectance

- G_w = the reflectance of the paper measured with green light
- G_c = the reflectance of the cyan ink measured with green light
- G_m = the reflectance of the magenta ink measured with green light
- G_y = the reflectance of the yellow ink measured with green light
- G_{my} = the reflectance of the magenta and yellow overlap measured with green light
- G_{cy} = the reflectance of the cyan and yellow overlap measured with green light
- G_{cm} = the reflectance of the cyan and magenta over lap measured with green light
- G_{cmy} = the reflectance or of the 3-color overlap measured with green light

$$A_w = (1-a_c) (1-a_m) (1-a_y)$$

$$\begin{array}{l} A_c &= a_c \left(1 - a_m \right) \left(1 - a_y \right) \\ A_m &= a_m \left(1 - a_c \right) \left(1 - a_y \right) \\ A_y &= a_y \left(1 - a_c \right) \left(1 - a_m \right) \\ A_r &= a_m a_y \left(1 - a_c \right) \\ A_g &= a_c a_y \left(1 - a_m \right) \\ A_b &= a_c a_m \left(1 - a_y \right) \\ A_{cmy} = a_c a_m a_y \\ a_c &= the single component area of the cyan ink \\ a_m &= the single component area of the yellow ink \\ a_y &= the single component area of the yellow ink \end{array}$$

By assuming that the ink densities are additive (Beer's law), the reflectance of an ink mixture becomes the product of the reflectance of its components, for example, $G_{cmy} = G_c$ $G_m G_y$. Substituting this expression into Eq. (1),we get

$$G = A_w G_w + A_c G_c + A_m G_m + A_y G_y + A_r G_m G_y + A_g G_c$$
$$G_y + A_b G_c G_m + A_{cmy} G_c G_m G_y$$

Similar expressions can be obtained for red and blue. If we replace the broad-band light of a primary color with a narrow spectral radiation for measuring the reflectance, we obtain the spectral Neugebauer equation

$$\begin{split} R(\lambda) &= A_w R_w(\lambda) + A_c R_c(\lambda) + A_m R_m(\lambda) + A_y R_y(\lambda) + \\ A_r R_m(\lambda) R_y(\lambda) + A_g R_c(\lambda) R_y(\lambda) + \\ A_b R_c(\lambda) R_m(\lambda) + A_{cmy} R_c(\lambda) R_m(\lambda) R_y(\lambda) \end{split}$$

where $R(\lambda)$ = the spectral reflectance of the mixed inks at wavelength λ .

The 3-color expression can readily be expanded to four colors by employing the 4-color fractional area expressions given by Hardy and Wurzburg.³

$$\begin{split} R(\lambda) &= A'_{w}R_{w}(\lambda) + A'_{c}R_{c}(\lambda) + A'_{m}R_{m}(\lambda) + A'_{y}R_{y}(\lambda) \\ &+ A'_{r}R_{m}(\lambda)R_{y}(\lambda) + A'_{g}R_{c}(\lambda)R_{y}(\lambda) \\ &+ A'_{b}R_{c}(\lambda)R_{m}(\lambda) + A'_{cmy}R_{c}(\lambda)R_{m}(\lambda)R_{y}(\lambda) \\ &+ A'_{k}R_{k}(\lambda) + A'_{ck}R_{c}(\lambda)R_{k}(\lambda) + A'_{mk}R_{m}(\lambda)R_{k}(\lambda) \\ &+ A'_{yk}R_{y}(\lambda)R_{k}(\lambda) + A'_{rk}R_{m}(\lambda)R_{y}(\lambda)R_{k}(\lambda) \\ &+ A'_{gk}R_{c}(\lambda)R_{y}(\lambda)R_{k}(\lambda) + A'_{bk}R_{c}(\lambda)R_{m}(\lambda)R_{k}(\lambda) \\ &+ A'_{cmvk}R_{c}(\lambda)R_{m}(\lambda)R_{v}(\lambda)R_{k}(\lambda) \end{split}$$

where	A'w	$= (1-a_c) (1-a_m) (1-a_y)(1-a_k)$
	A' _c	$= a_{c}(1-a_{m})(1-a_{y})(1-a_{k})$
	A'm	$= a_m(1-a_c) (1-a_y) (1-a_k)$
	A'y	$= a_y(1 - a_c)(1 - a_m)(1 - a_k)$
	A'r	$= a_m a_y (1-a_c) (1-a_k)$
	A'g	$= a_{c}a_{y}(1 - a_{m})(1 - a_{k})$
	A'b	$= a_c a_m (1 - a_y) (1 - a_k)$
	A' _{cmy}	$= a_c a_m a_y (1 - a_k)$
	A' _k	$= a_k(1 - a_c) (1 - a_m) (1 - a_y)$
	A' _{ck}	$= a_c a_k (1-a_m) (1-a_v)$

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$$\begin{array}{lll} A'_{mk} &= a_m a_k (1 - a_c) \, (1 - a_y) \\ A'_{yk} &= a_y a_k (1 - a_c) (1 - a_m) \\ A'_{rk} &= a_m a_y a_k (1 - a_c) \\ A'_{gk} &= a_c a_y a_k (1 - a_m) \\ A'_{bk} &= a_c a_m a_k (1 - a_y) \\ A'_{cmyk} &= a_c a_m a_y a_k \end{array}$$

Yule-Neilsen model

From the study of halftone process, Yule and Neilsen pointed out that the light does not emerge from the paper at the point where it entered. They estimated that between onefourth and one-half of the light which enters through a white area will emerge through a colored area, and vice versa. Based on this observation, Yule and Neilsen took the light penetration into consideration and derived the spectral equation for the halftone pattern:⁴

$$R(\lambda) = r_{s} + R_{w}(\lambda) (1 - r_{s}) \{ 1 - A [1 - T_{i}(\lambda)] \}^{n}$$

where $r_s =$ the surface reflection

- A = the ink area coverage
- T_i = the transmittance of the ink film
- n =the Yule-Neilsen exponent

The ink area coverage and the effective transmittance are provided by the Neugebauer equation such as the 4color version:

$$\begin{split} A &= A'_c + A'_m + A'_y + A'_r + A'_g + A'_b + A'_{cmy} + A'_k \\ &+ A'_{ck} + A'_{mk} + A'_{yk} + A'_{rk} + A'_{gk} + A'_{bk} + A'_{cmyk} \\ T_i(\lambda) &= [A'_c T_c(\lambda) + A'_m T_m(\lambda) + A'_y T_y(\lambda) + A'_r T_m(\lambda) T_y(\lambda) \\ &+ A'_g T_c(\lambda) T'_y(\lambda) + A'_b T_c(\lambda) T_m(\lambda) \\ &+ A'_{cmy} T_c(\lambda) T_m(\lambda) T_y(\lambda) + A'_k T_k(\lambda) \\ &+ A'_{ck} T_c(\lambda) T_k(\lambda) + A'_{mk} T_m(\lambda) T_k(\lambda) \\ &+ A'_{yk} T_y(\lambda) T_k(\lambda) + A'_{rk} T_m(\lambda) T_y(\lambda) T_k(\lambda) \\ &+ A'_{gk} T_c(\lambda) T_y(\lambda) T_k(\lambda) + A'_{bk} T_c(\lambda) T_m(\lambda) T_k(\lambda) \\ &+ A'_{cmyk} T_c(\lambda) T_m(\lambda) T_y(\lambda) T_k(\lambda)] / A \end{split}$$

Clapper-Yule model

Clapper and Yule developed an accurate account of the halftone process from a theoretical analysis of the multiple scattering, internal reflections, and ink transmissions. The total reflected light is the sum of light fractions that emerge after each internal reflection cycle.^{5,6} Including all these parameters, they derived an analytical expression for the halftone process.

$$R(\lambda) = k_s + \{f_e(1-r_s) f_r[1-A+AT_i(\lambda)]^2\} / \{1-f_e)f_r [1-A+AT_i(\lambda)^2] \}$$

where

 k_s = the specular component of the surface reflection

- $f_r = a$ fraction of light that is reflected at the bottom of the substrate
- $f_e = a$ fraction of light that is emerged at the top of the substrate

Again, the ink area coverage and the transmittance are provided by the Neugebauer equations.

Beer-Bouguer law

The Beer-Bouguer law relates the light intensity to the quantity of the absorbant.⁷ The model is based on the

absorption phenomenon, therefore it is a subtractive theory. It is widely used in the analytical work of liquid solutions.

$$\log [|(\lambda) / |_{0}(\lambda)] = \xi (\lambda) = -(1 / 2.303) K_{d}(\lambda) x c$$

where

- $l_0(\lambda)$ = a monochromatic light intensity before passing through an absorbent
- $l(\lambda) = a$ light intensity after passing through an absorbent
- $\xi(\lambda)$ = absorbance (also referred to as the optical density) of the absorbent
- $K_d(\lambda)$ = absorption coefficient of the absorbent
- λ = the wavelength
- x = the length of light path traversing through the absorbent
- c = the concentration of absorbent

For mixed color films, the absorbance, ξ_m (λ), is obtained by applying the proportionality and additivity rules.

$$\xi_{m}(\lambda) = k_{d_{1}}(\lambda)x_{1}c_{1} + K_{d_{2}}(\lambda)x_{2}c_{2} + \dots + K_{d_{n}}(\lambda)x_{n}c_{n}$$

i = 1, 2,---,n

where

 K_{d_i} = the absorption coefficient of *i*th primary ink

 c_i = the concentration of *i*th primary ink.

Kubelka-Munk theory

The Kubelka-Munk model assumes that the light is being absorbed and scattered in only two directions, up and down. A background is presented at the bottom of the medium to provide the upward light reflection. The derivation of KM formula can be found in many publications.⁸⁻¹¹ The basic form is shown below:

$$R(\lambda) = (1 - R_{g}(\lambda) \{ \alpha(\lambda) - \beta(\lambda) \coth [\beta(\lambda) S(\lambda) x] \}) / (\alpha(\lambda) - R_{g}(\lambda) + \beta(\lambda) \coth [\beta(\lambda) S(\lambda) x])$$

where

$$\begin{split} R(\lambda) &= \text{the reflectance of the film.} \\ R_g(\lambda) &= \text{the reflectance of the background.} \\ \alpha(\lambda) &= 1 + K(\lambda)/S(\lambda) \\ \beta(\lambda) &= [\alpha(\lambda)^2 - 1]^{1/2} = \{ [K(\lambda)/S(\lambda)]^2 \\ &+ 2[K(\lambda)/S(\lambda)] \}^{1/2} \\ \text{coth } [\beta(\lambda)S(\lambda)x] = \{ \exp[\beta(\lambda)S(\lambda)x] + \exp[\beta(\lambda)S(\lambda)x] \} \\ &/ \{ \exp[\beta(\lambda)S(\lambda)x] - \exp[-\beta(\lambda)S(\lambda)x] \} \\ K(\lambda) &= \text{the absorption coefficient} \\ S(\lambda) = \text{the scattering coefficient} \end{split}$$

This expression is the foundation for various twoconstant Kubelka-Munk (KM2) formulas.

Single-constant KM theory

In the limiting case of an infinite thickness, the equation becomes

$$\begin{split} R_{\infty}\left(\lambda\right) &= \alpha(\lambda) - \beta(\lambda) = 1 + [K(\lambda)/S(\lambda)] - \{ \ [K(\lambda)/S(\lambda)]^2 \\ &+ 2[K(\lambda)/S(\lambda)] \ \}^{1/2} \end{split}$$

The single constant, $K(\lambda)/S(\lambda)$, of a multi-component system is obtained by summing ratios of all components.

$$\begin{split} \mathsf{K}(\lambda)/\mathsf{S}(\lambda) &= [\mathsf{k}(\lambda)/\mathsf{s}(\lambda)]_p + c_1 \, [\mathsf{k}(\lambda)/\mathsf{s}(\lambda)]_1 + c_2 [\mathsf{k}(\lambda)/\mathsf{s}(\lambda)]_2 \\ &+ \cdots + c_n \, [\mathsf{k}(\lambda)/\mathsf{s}(\lambda)]_n \end{split}$$

where

 $(k/s)_p$ = the single constant of the substrate $(k/s)_i$ = the single constant of the component i c_i = the concentration of the component i

Each $(k/s)_i$ is, in turn, calculated from measured reflection spectrum of a primary ink.

$$[k(\lambda)/s(\lambda)]_i = [1 - R_i(\lambda)]^2/2R_i(\lambda)$$
 $i = 1, 2, 3, \dots, n$

In this single-constant KM model (KM1), the correction for the refractive index that changes between air and a colored layer is included. A simple correction is to subtract the surface reflection from the measured reflectance, $R_m(\lambda)$.

 $R_{\infty}(\lambda) = R_m(\lambda) - r_s$

Another frequently used approach is Saunderson's correction, 12

$$R_{\infty}(\lambda) = R_{m}(\lambda) - f_{s}] / [1 - f_{s} - f_{i} + f_{i} R_{m}(\lambda)]$$
 where

 f_s = a constant representing the surface reflection

 f_i = a fraction representing internal reflections.

Halftone correction factor

Because the subtractive theories are not developed for the halftone printing, we propose an empirical correction to halftone prints in an effort to relate the computed spectral parameters and device characteristics. This spectral halftone correction factor, $h(\lambda)$, is defined as:

$$h(\lambda) = Q_m(\lambda) / Q_c(\lambda)$$

where Q can be the optical density, KM1 constant, absorption coefficient K, or scatter coefficient S of the KM2 approach. The subscript c indicates the computed quantity where the subscript m indicates the measured quantity from halftone step wedges of a primary color. For a wide spectral band, the definition can be modified as

$$\mathbf{H} = \int \mathbf{Q}_{\mathrm{m}}(\lambda) \, \mathrm{d}\lambda \, / \int \mathbf{Q}_{\mathrm{c}}(\lambda) \, \mathrm{d}\lambda$$

This factor provides a direct connection between the calculated and measured quantities. It is used to reduce any differences between the computation and measurement. For example, it can be used for the continuous-tone printing when the deviation from BB linear behavior is high or simply to improve the accuracy of the data fitting.

Experimental

Two printing devices are used for this study. They are a Cannon Color Laser Copier 500 (CLC-500) utilizing xerographic technology and a continuous ink jet printer using premixed liquid inks.

CLC-500 copier

The CLC-500 is a product of Canon Inc. Japan. Electronic files with known printer *cmy* and *cmyk* values are printed for testing the 3-color mixing and 4-color mixing, respectively. In addition, multilevel intensity wedges, ranging from the solid coverage to near white, of each primary color are used to determine the area coverage, effective transmittance, and halftone correction factors. Halftone tints are made by using the line screen provided by the manufacturer.¹³

Colorimetric data of prints are obtained by a Gretag SPM 100 ($45^{\circ}/0^{\circ}$ measuring geometry) spectrophotometer using a 2° standard observer, illuminant D₅₀, and absolute white scale with black backing. The multi-level intensity wedges are also measured with white backing for determining the K and S of the two-constant KM theory. This instrument outputs the reflection spectrum in a 10 nm interval from 380 to 730 nm. All data are the average of at least three measurements at different locations of a patch. The accuracy of the measurement is discussed in a previous publication.¹⁴

Ink jet printer

The design of this continuous stream ink jet device has been described elsewhere.^{15,16} It has a resolution of 300 spi. Full area coverage is obtained when all addressable pixels within a given area are printed. Five 8-level halftone wedges are printed by premixed inks on Xerox 4024 DP papers. These wedges are a 5% cyan plus 95% magenta mixture, a 90% cyan plus 10% magenta mixture, a 40% cyan plus 60% yellow mixture, a 15% magenta plus 85% yellow mixture, and a 90% magenta plus 10% yellow mixture. Samples are measured by a Macbeth Color-Eye spectrophotometer with black backings and sometimes white backing. This instrument outputs reflection spectrum at 20 nm interval from 400 to 700 nm. Its software calculates tristimulus values, CIELAB specifications, and color difference, ΔE_{ab} , using the CIE 1931 standard observer with D₆₅ illuminant. Two measurements are made for each sample at two different areas of a color patch. Most samples had ΔE_{ab} around 1 or less. The accuracy of the measurement is discussed previously.17

Results and Discussion

Three criteria are used to evaluate the fitting of color models with respect to the experimental results: $\overline{\Delta E}_{ab}$ and ΔE_{rms} , the mean and root-mean-square color differences, and the spectral error ΔR_{rms} expressed as the root-mean-square of the difference between calculated and measured spectra. Table 1 lists the test results under various conditions using the same CLC-500 data. All results are the average of 64 color patches. We treat r_s , f_s , f_i and *n*-expon-ent as adjustable parameters for the best fit to the data.

The NE gives average color differences in low teens for two different Yule-Neilsen *n*-exponents used for computing the area coverage. The lower *n* value gives a better fitting by 0.91 $\overline{\Delta E}_{ab}$ units.

The YN model fits the data in the neighborhood of 8 $\overline{\Delta E}_{ab}$ units (run 3 to run 9 of Table 1). This number is on the order of the CLC-500 printer stability. Varying the surface reflection does not give a significant change in the average color difference. When the printer variability is incorporated into the YN model by varying device/*cmy* values

within the tolerance, we are able to reduce the ΔE_{ab} to 5.45. This is the best improvement we have achieved. It implies that the printer variability is the major source for the modeling error.

Results of the CY modeling are given in run 10 to run 22, they range from 7.5 to 10.0 ΔE_{ab} . Under the same conditions except the Yule-Neilsen factor, the average color difference is improved by more than 1 unit when the higher *n* value is used (see runs 10 & 14 or runs 12 & 19). Decreasing f_s by 0.01 shows a less than 1 ΔE_{ab} improvement (runs 16, 18, and 19). The data fitting improves as f_i increases (runs 11 & 12 or runs 14, 15, 16, & 17).

Without correction, all subtractive color mixing theories fail badly for fitting the xerographic data (runs 23, 24, 25, & 26). The empirical halftone correction, however, is every successful to both BB and KM1 models (runs 27, 28, 29, & 30). A close examination of the KM2 computation reveals that the scattering coefficient, S, is strongly wavelength-dependent. Therefore, the KM2 theory is not suitable for the correction that employs a constant scaling factor across the whole visible spectrum. Results indicate that the fitness is sensitive to the halftone correction curve as shown in runs 28, 29, and 30 of Table 1; the fitting improves from $\overline{\Delta E}_{ab} = 8$ to 5 by adjusting the halftone correction curves. Compared to the effectiveness of the halftone correction factor, other parameters such as the surface and internal reflections become secondary. The adjustment of these parameters accounts for a fraction of an $\overline{\Delta E}_{ab}$ unit.

Run	Model	п	rs	f _s	f_i	ΔE_{ab}	ΔE_{rms}
1	Neugebauer	2.0	-	_	_	12.33	14.86
2	Neugebauer	2.7	-	-	-	13.24	-
3	Yule-Neilsen	2.0	0.0	-	-	7.93	9.09
4	Yule-Neilsen	2.0	0.01	-	-	8.01	9.30
5	Yule-Neilsen	2.0	0.02	-		8.25	9.80
6	Yule-Neilsen	2.7	0.0	-	-	7.85	9.30
7	Yule-Neilsen	2.7	0.01	-	-	7.78	9.31
8	Yule-Neilsen	2.7	0.02	-	-	7.81	9.55
9	Yule-Neilsen	2.7	0.04	-	-	8.69	10.98
10	Clapper-Yule	2.0	0.04	0.01	0.40	10.02	11.15
11	Clapper-Yule	2.0	0.04	0.	0.40	9.40	10.27
12	Clapper-Yule	2.0	0.04	0.	0.50	8.61	9.41
13	Clapper-Yule	2.7	0.04	0.01	0.35	9.11	10.30
14	Clapper-Yule	2.7	0.04	0.01	0.40	8.58	9.70
15	Clapper-Yule	2.7	0.04	0.01	0.45	8.19	9.30
16	Clapper-Yule	2.7	0.04	0.01	0.50	7.91	9.03
17	Clapper-Yule	2.7	0.04	0.01	0.55	7.72	8.85
18	Clapper-Yule	2.7	0.04	0.02	0.50	8.52	9.93
19	Clapper-Yule	2.7	0.04	0.0	0.50	7.54	8.44
20	Clapper-Yule	2.7	0.03	0.0	0.50	7.53	8.43
21	Clapper-Yule	2.7	0.02	0.0	0.50	7.52	8.43
22	Clapper-Yule	2.7	0.01	0.0	0.50	7.51	8.42
23	Beer-Bouguer	-	-	-	-	12.27	13.28
24	KM2	-	0.0	-	-	36.47	40.45
25	KM1	-	0.0	-	-	30.61	32.72
26	KM1S	-	-	0.0	0.6	32.62	34.75
27	Corrected BB	-	-	-	-	7.14	8.60
28	Corrected KM1-1	-	0.0	-	-	8.01	9.73
29	Corrected KM 1-2	-	0.0	-	-	5.83	7.12
30	Corrected KM 1-3	-	0.0			5.01	6.05

Table 1. Summary of CLC-500 modeling results of three primary color mixing

Corrected BB is the Beer-Bouguer model with halftone correction.
KM1S is the single-constant Kubelka-Munk model with Saunderson's correc-

tion.

Corrected KM1 is the single-constant Kubelka-Munk with halftone correction.

Three most effective models for 3-color mixing are selected for 4-color modeling, the results are given in Table II. Each run is the average of 58 color samples. Among these models, the halftone corrected KM1 seems to fit the data best as evidenced by the smaller $\overline{\Delta E}_{ab}$ and spectral error. By comparing these results with those obtained for the 3-color mixing, all three models of the 4-color mixing give a higher error by about 2 $\overline{\Delta E}_{ab}$ units. Further improvements in the data fitting for halftone

corrected BB and KM methods can be realized by properly adjusting the halftone correction curves.

Table III shows the results of ink jet halftone modeling. Color patches are printed by using the clustereddot halftone pattern. The $\overline{\Delta E}_{ab}$ is the average of 40 color patches. All color mixing theories are less successful in modeling the ink jet halftone prints than the xerographic prints. The data suggest systematic errors in the printing process. Nevertheless, better agreements are obtained by

Table II.	Summary	of (CLC-500	modeling	results	of	4-color	mixing
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n	r _s	$\overline{\Delta}\overline{\mathrm{E}}_{ab}$	ΔE_{rms}	ΔR_{rms}
2.0	0.0	10.60	11.22	0.0493
-	-	9.62	10.05	0.0531
-	0.0	7.15	7.76	0.0369
	n 2.0 -	$\begin{array}{ccc} n & r_{\rm s} \\ 2.0 & 0.0 \\ - & - \\ - & 0.0 \end{array}$	$\begin{array}{cccc} n & r_{s} & \overline{\Delta E}_{ab} \\ 2.0 & 0.0 & 10.60 \\ - & - & 9.62 \\ - & 0.0 & 7.15 \end{array}$	$\begin{array}{ccccccc} n & r_{s} & \overline{\Delta E}_{ab} & \Delta E_{rms} \\ 2.0 & 0.0 & 10.60 & 11.22 \\ - & - & 9.62 & 10.05 \\ - & 0.0 & 7.15 & 7.76 \end{array}$

Run	Model	п	r _s	\mathbf{f}_{s}	\mathbf{f}_{i}	$\overline{\Delta}\overline{\mathrm{E}}_{ab}$	ΔE_{rms}
1	Neugebauer	2.7	-	-	-	17.26	-
2	Yule-Neilsen	2.7	0.04	-	-	12.61	-
3	Clapper-Yule	2.7	0.04	0.01	0.4	12.88	-
4	Beer-Bouguer	-	-	-	-	12.29	13.89
5	KM2	-	0.0	-	-	13.49	-
6	KM1	-	0.03	-	-	18.20	-
7	KM1S	-	-	0.03	0.6	18.34	-
8	Corrected BB	-	-	-	-	9.65	11.77
9	Corrected KM1	-	0.0	-	-	7.64	8.05

Table III. Summary of Ink Jet modeling results of premixed inks

using the halftone correction method (runs 8 and 9) that brings the average color difference to the neighborhood of the printer stability.

References

- H. E. J. Neugebauer, Die theoretischen Grundlagen des Mehrfarbendruckes, Z. wiss. Photogr., 36, 73-89 (1937).
- 2. J. A. C. Yule, *Principles of color reproduction*, Chapter 10, pp. 255 John Wiley & Sons, Inc. New York (1967).
- A. C. Hardy and F. L. Wurzburg, Color correction in color printing, J. Optical Soc. Amer. 38, 300-307 (1948).
- 4. J. A. C. Yule and W. J. Neilsen, The penetration of light into paper and its effect on halftone reproduction, *Proc. Tech. Assn. Graphic Arts*, Vol. 4, 65-75 (1951).
- F. R. Clapper and J. A. C. Yule, The effect of multiple internal reflections on the densities of halftone prints on paper, J. Optical Soc. Amer. 43, 600-603 (1953).
- F. R. Clapper and J. A. C. Yule, Reproduction of color with halftone images, *Proc. Tech. Assn. Graphic Arts*, pp. 1-12 (1955).
- 7. G. Wyszecki and W. S. Stiles, *Color Science*, Page 30, 2nd Edition, John Wiley & Sons (1982).
- P. Kubelka, New contributions to the optics of intensely light-scattering materials. Part 1, J. Opt. Soc. Am. 38, 448-457 (1948).
- D. B. Judd and G. Wyszecki, *Color in business, science, and industry*, 3rd Ed., Chapter 3, pp. 397-461, John Wiley & Sons, New York (1975)
- E. Allen, Colorant formulation and shading, in Optical Radiation Measurements, Vol. 2, Chap. 7, pp. 305-315. Edited by F. Grum and C. J. Bartleson, Academic Press, N.Y.(1980).
- H. R. Kang, Kubelka-Munk modeling of ink jet ink mixing, J. of Imaging Technol., 17, 76-83 (1991)
- J. L. Saunderson, Calculation of the color of pigmented plastics, J. Opt. Soc. Am., 32, 727-736 (1942).
- J. H. Riseman, J. J. Smith, A. M. d'Entremont, and C. E. Goldman, *Apparatus for generating an image from a digital video signal*, U. S. Patent 4,800,442 (1989).
- 14. H. R. Kang, Color scanner calibration, J. Imaging Science & Technol., 3, 162-170 (1992).
- 15. R. G. Teumer, *Tilted deflection electrode method and apparatus for liquid drop printing systems*, U. S. Patent 4,347,521 (1982).
- P. A. Crean, J. M. Schneider, and A. F. Lipani, *Bipolar ink jet method and apparatus*, U. S. Patent 4,395,716 (1983).
- 17. H. R. Kang, Water based ink jet ink. Part III: performance tests, *J. of Imaging Sci.*, **35**,195-201 (1991).

Conclusion

In this study, we examined three halftone color mixing theories, two subtractive color mixing theories, and an empirical extension of the subtractive models for the halftone printing. Results indicate that the spectral NE model and two subtractive models are not suitable for modeling either electronic devices. The halftone corrected subtractive models give good agreements to the experimental results for both electronic printing devices with the YN and CY approaches following closely behind. They can fit the data on the order of the CLC-500 printer variability or better. Generally, they fit the CLC-500 data better than the ink jet results and fit the 3-color printing better than 4color printing. Furthermore, this modeling reveals that the primary factor affecting the accuracy of fitting is the printer variability in terms of the toner density fluctuation and the halftone correction curves. Secondary factors are the Yule-Neilsen *n*-exponent, internal reflections, accuracy of the area coverage, and surface reflections.

Granting that some theories are better than others; these models, however, are phenomenon theories. They depend on the characteristics of printing devices and the specific measurements that the spectra of colorants are determined under circumstances as close as possible to the actual situation. Thus, it is not surprising that the most promising approach is the empirical halftone correction factor. This is because the factor is directly associated to the device output such that it already takes into account the characteristics of an individual printing device. Therefore, it tends to shelter the difference in the devices regardless of the process. Such as it is, we believe that more printing devices should be examined to test this halftone correction approach.