Reflectance recovery using localised weighted method

***Yi-Fan Chou*[1,2]*, Vien Cheung*[2]*, Changjun Li*[3]*, M Ronnier Luo*[2] *and San-Liang Lee*[1]**
[1]*Department of Electronic Engineering, National Taiwan University of Science and Technology, Taipei, Taiwan*
[2]*Colour Imaging and Design Centre, School of Design, University of Leeds, Leeds, UK*
[3]*Department of Computer Science, University of Science and Technology Liaoning, Liaoning, China*

Abstract

This paper evaluated four conventional methods for reflectance recovery: smoothness method [1], principle component analysis [2], basis functions with smoothness constraint [3] and Wiener estimation [4,5]. Most of these methods adopt a "learning-based" procedure with a training set. Modifications based on the training set were applied for improving the reflectance recovery performance. This paper described combined methods involving the application of localised training data and localised training data with a weighted matrix to the four recovery methods [1-5]. All these methods were applied to recover reflectance from XYZ values for two datasets. Both the training and testing performance were evaluated in terms of CIEDE2000 colour differences. The results showed that the performance of the methods with localised training data significantly improved. There are also limited improvements by applying the weighted matrix. Overall, the localised weighted method (using a local training set with a weighted matrix) with Weiner estimation method performed the best.

Introduction

It is well known that a spectral match occurs when two samples have the exact same colour regardless of the illuminant or the observer involved. Therefore, the most accurate method to specify a colour is to directly measure the reflectance of the surface colours. Conventionally, spectrophotometer is widely used for measuring surface colours. In comparison with the conventional spectrophotometer, the digital camera is able to measure the curved or non-uniform surfaces by capturing the whole image of an object. Many researchers are working on the reflectance recovery from the captured images of digital cameras. Li and Luo [1] developed a method to recover reflectance with smoothness constraint. Sobagaki *et al.* [2] applied the principal component analysis to obtain three basis functions representing Munsell chips. Cheung *et al.* [3] applied the Li-Luo smoothness method with the basis function to recover reflectance. Babaei *et al*. [4] adopted Wiener estimation plus a weighted matrix for recovery of spectral reflectance. The aim of this paper is to find the best computation methods. Therefore, the reflectance was recovered from XYZ values by using these methods. Because the variations in sets of colour difference are non-normal distribution [6], their performances were evaluated in terms of median and maximum colour difference (ΔE_{00}) [7].

Reflectance Recovery Methods

Four conventional reflectance recovery methods were implemented: smoothness method, basis function, basis function with smoothness constraint, and Wiener estimation methods. These are denoted as SC, BF, BF&SC and WE methods, respectively. All these methods require a training data set except the SC method. The training set can be a data set covering a large colour gamut or a localised data set with just few samples close to the target sample. These training samples can be also weighted by a weighted matrix i.e. larger weight for samples close to the target.

Smoothness Constraint (SC) Method

Li and Luo [1] applied the smoothness constraint condition to estimate the spectral reflectance. When colour samples are given in terms of tristimulus values C of objects ($C^t=(X, Y, Z)$, the transpose of the 3 by l matrix C where l is the number of given samples) under a particular light source and CIE standard colour matching function, the reflectance R of the samples can be estimated by solving the equation:

$$C=MR \qquad (1)$$

where C is a $3 \times l$ matrix that each column vector relates to XYZ tristimulus values of each sample; R is a $31 \times l$ matrix of reflectance values (ranging from 400 to 700nm in 10nm steps) corresponding to l samples; M is a $k \times 31$ matrix that each column contains the wavelength by wavelength product of the spectral power distribution of the illuminant and the colour matching function of the CIE standard observer. Then, the smoothness constraint for the reflectance function was defined as:

$$\frac{\min}{R}\left\|GR\right\|^2,\ 0 \le R_m = R(\lambda_m) \le 1,\ for\ m = 1,2,...n \qquad (2)$$

and $C=MR$

$$\text{where } G = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & & & \\ & -1 & 1 & & \\ & & \ddots & \ddots & \\ & & & -1 & 1 \\ & & & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \qquad (3)$$

where G is a n by n matrix and considered as an operator derived from the squared first derivative of spectral [8, 9].

Basis Function (BF) Method

The objective of the basis function is to describe the vector space using low-dimensional linear models in terms of weighted sums of a small number of basis functions [10-12].

$$R = [\vec{r}_1\ \vec{r}_2\ \vec{r}_3 ... \vec{r}_h ... \vec{r}_l] = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1l} \\ r_{21} & r_{22} & \cdots & r_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nl} \end{bmatrix}_{n\times l} \qquad (4)$$

For a set of l samples having 31 points each, the matrix R in Equation (4) with the column vector $\vec{r}_h$ represents the h_{th} samples. Then, the principal component analysis (PCA) desires to represent each sample using subspace with $\tilde{n}$ dimensions < n. Thus, the best recovery of the vector subspace with a number of vectors lower than the dimension of the vector space can be provided by the eigenvectors [13]. The eigenvectors are so called principal components and also known as basis function. The column vector $\vec{r}_h$ representing the reflectance of the h_{th} sample can be recovered by the linear combination of the principal components:

$$\vec{r}_h = U \cdot W_h = (\vec{u}_1 \cdot w_1 + \vec{u}_2 \cdot w_2 + ... + \vec{u}_n \cdot w_n) \quad (5)$$

where W_h is a $n \times 1$ column vector whose components are the weighted coefficients of the linear combination of the basis functions from the matrix U. In order to reduce the dimension of the vector space, the number of the basis function $\tilde{n}$ can be chosen with $\tilde{n}$ less than n according to the eigenvalues. Thus, the eigenvectors related to small eigenvalues can be deleted and Equation (5) becomes:

$$\vec{r}_h = U \cdot W_h = (\vec{u}_1 \cdot w_1 + \vec{u}_2 \cdot w_2 + ... + \vec{u}_{\tilde{n}} \cdot w_{\tilde{n}}),\ 1 \le \tilde{n} \le n \quad (6)$$

The practical application of PCA for spectral recovery has been carried out by several researchers such as Maloney and Wandell [14], Imai and Berns [15], Li and Luo [16], Shi and Healey [17]. The reflectance samples having 31 points with 10nm interval from 400 to 700nm can be approximated by a weighted sum of basis function:

$$R = U_{\tilde{n}} W \quad (7)$$

where U is a $31 \times \tilde{n}$ matrix consisting of the first $\tilde{n}$ basis functions; W is a $\tilde{n} \times l$ matrix of weights.

Basis Function with Smoothness Constraint (BF&SC) Method

For many applications, the low-dimensional linear models for reflectance recovery whose dimensionality is less or equal to the number of camera channels are inadequate, especially for trichromatic camera with only three channels. Shi and Healey [17] provided an alternative ways to constrain the solution. Recovering reflectance from tristimulus values can be considered as a trichromatic camera with 3 sensors and requiring $\tilde{n}$ -dimensional linear model, the eigenvectors can be divided into two parts: the first 3 principal components and the remaining $\tilde{n}$ - k principal components. The former vectors establish the matrix of basis function U_1, and the latter vectors placed in matrix U_2 include the remaining basis functions. Their weight coefficient matrices are denoted as W_1 and W_2 respectively. Then, Equation (7) can be expanded to:

$$R = U_1 W_1 + U_2 W_2 \quad (8)$$

Equation (8) can be substituted into Equation (1)

$$C = MU_1 W_1 + MU_2 W_2 \quad (9)$$

The W_1 can be expressed in terms of W_2 by multiplying $(MU_1)^{-1}$ by both sides of Equation (9) and substituted into Equation (8), then Equation (8) can be simplified as

$$R = A + BW_2 \quad (10)$$

where

$$A = U_1 (MU_1)^{-1} C, \mathrm{B} = U_2 - U_1 (MU_1)^{-1} MU_2 \quad (11)$$

Following the method discussed in the SC method, the operator or smooth matrix denoted as G can be applied to Equation (10) to have smooth estimations [3]. The smooth R can be obtained by minimizing $\| GR \|$ or $\| G(A + BW_2) \|$. However, further constraint should be added to the minimisation to enforce each column vector $\vec{r}_h$ between 0 and 1. When only three basis functions are applied, the BW_2 is eliminated from Equation (10). Then the smoothness constraint is not working. Therefore, the number of basis functions should be greater or equal to 4 for the BF&SC method.

Wiener Estimation (WE) Method

In order to estimate R from C, a matrix W applied to transfer the tristimulus values to reflectance should be determined.

$$R = WC \quad (12)$$

In Wiener estimation [18, 19], the matrix W can be represented as [20]:

$$W = W_{WE} = K_r M^T (MK_r M^T + K_n)^{-1} \quad (13)$$

where W_{WE} represent the matrix W derived by Wiener estimation; K_r denotes the covariance matrix of reflectance from the training set; K_n is the covariance matrix of noise. If the noise of different channels is independent, the K_n is a diagonal matrix. Then, the estimation of R from C denoted as R_{es} can be obtained:

$$R_{es} = K_r M^T (MK_r M^T + K_n)^{-1} C \quad (14)$$

As the principle component analysis, the Wiener estimation tries to minimize the mean square error between the actual and estimated reflectance. In this study, $K_n = 0$ is assumed.

Localised Training Set

The methods described above require a training set except the SC method. The training set could be the full data set or the selected L number of neighbouring samples. The neighbouring samples were determined by the smallest ΔE^*_{ab} from the testing sample with known tristimulus values under CIE D65 and 1964 standard colorimetric observer. In the present work, the number of the neighbouring samples were investigated and applied to train the models.

Babaei et al's Weighted Matrix

Babaei *et al.* [4] modified the classical Wiener estimation method by introducing a weighted matrix Q.

$$Q = \begin{bmatrix} q_1 & 0 & \cdots & 0 \\ 0 & q_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & q_l \end{bmatrix}_{l \times l} \quad (15)$$

where Q is a square and diagonal matrix, l is the number of samples in training set. The diagonal elements in Q are the inverse of colour difference values between the i_{th} sample in the training set and the test sample. Samples which are closer to the target will have greater weights than those which are far from the target. The weighted Wiener estimation method is presented by multiplying the weighted matrix Q (which is unique for each particular test sample) by training set matrix:

$$K_{rq} = \mathrm{cov}(DQ) \quad (16)$$

where D is 31 by l matrix regarding to the reflectance of the training set. Then, the normal covariance matrix in equation (14) is replaced by K_{rq}

$$R_{es} = K_{rq} M^T (M K_{rq} M^T + K_n)^{-1} C \quad (17)$$

Equation (17) being the weighted form of Equation (14) has the identical dimensionality. For the BF and BF&SC methods, the reflectance functions of training samples multiplied by the weighted matrix are also applied in this research.

Experimental

Two datasets were collected: Munsell book of Color having 1562 glossy paint chips [21], the Professional Colour Communicator (PCC) consisting of 1063 textile samples [22]. The reflectance samples were measured by the GretagMacbeth CE7000A spectrophotometer with the wavelength from 400 nm to 700 nm with 10 nm interval. The XYZ tristimulus values of all sets were calculated under D65 and CIE 1964 standard observer. The reflectance samples were recovered from these XYZ.

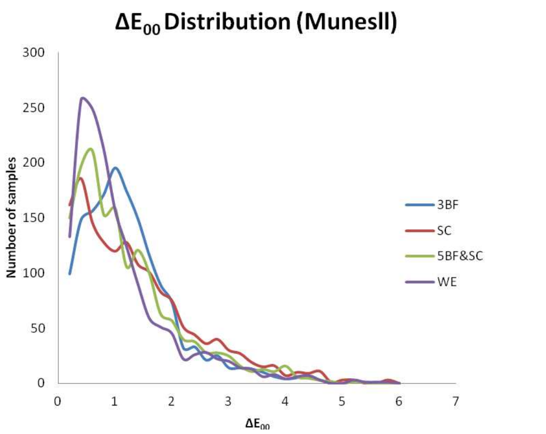
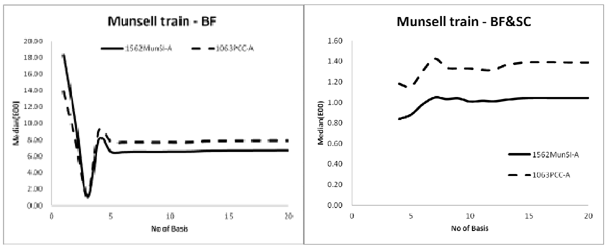


Figure 1: *Colour difference distribution of performance using Munsell train and Munsell test.*

To examine these methods, the effect of database is analysed using two different datasets which are used in different combinations as training and testing sets and the results are investigated. Each dataset was chosen as training as well as testing set in different sequences. First, Munsell was selected as training set, and PCC was chosen as testing sets. If the generated reflectance values exceed the range between 0 and 1, they are forced to equal to zero and one boundaries respectively. All recovered reflectance were evaluated in terms of CIEDE2000 colour differences (ΔE_{00}) under A/10˚ and F11/10˚ conditions. Figure 1 shows an example of the colour difference distribution of the four tradition methods It can be seem that the distributions of CIEDE2000 colour differences are not normal distribution. Thus, the median and maximum values of ΔE_{00} were employed the median and maximum values as criteria.

Effect of the number of basis functions and localisation set

For the BF and BF&SC methods, the number of basis functions should be first determined. Figures 2(a) and 2(b) show the performance under A/10˚ in terms of median ΔE_{00} values with Munsell training set against number of basis functions. It can be seen that the ΔE_{00} values became stable above 10 and 15 basis functions for both BF and BF&SC methods. The minimum of median ΔE_{00} with the BF method was obtained by 3 basis functions. For the BF&SC method, the best performance achieved with 4 to 6 basis functions. Therefore, the 3 and 5 basis functions were applied for the BF and BF&SC methods denoted as 3BF and 5BF&SC, respectively.



(a) (b)

Figure 2: *Effect of the number of basis function for (a) BF and (b) BF&SC methods in terms of median ΔE_{00} under illuminant A.*

Then, the localised training set was used to apply on the BF, BF&SC and WE methods. Figure 3 shows the median and maximum ΔE_{00} values plotted against the number of localised samples. For these three methods, both of the performance in terms of median and maximum ΔE_{00} became stable when 40 to 60 samples were used. Therefore, the 50 localised samples were chosen for improving the basic methods.

Tables 1 and 2 show the performance of four conventional methods, three basic methods with 50 localised training set denoted as 50Loc in parentheses, and that combined with a weighted matrix denoted as 50wLoc in parentheses. Both training and testing performances were evaluated. Best results in terms of median or maximum ΔE_{00} under the same training set modification method in each column is marked by a *. The SC method is not a learning-based method, so there is no need to define training or testing set. Without any training set modification, the 5BF&SC method gave lowest mean and maximumΔE_{00} units in most cases. The SC method with the advantage of no training set requiring gave the performance within 0.5 ΔE_{00} units from the best performance among these basic methods. For example, the best prediction for the Munsell set under illuminant A is 0.72 ΔE_{00} units with the WE method, and the SC method gave 1.05 ΔE_{00} units.

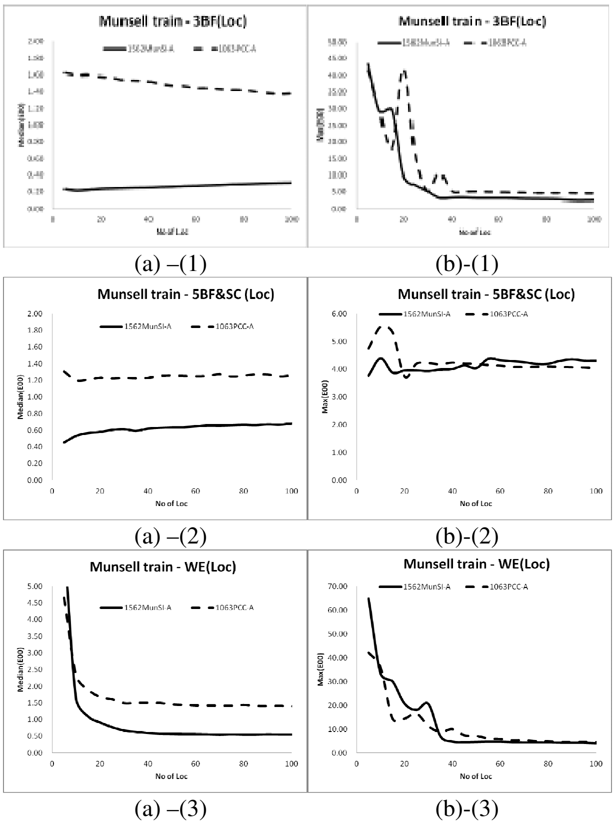


Figure 3: *Effect of the number of localised samples on maximum ΔE00 under illuminant A (a) median (ΔE_{00}) (b) maximum(ΔE00) – (1) BF (2) BF&SC (3) WE methods.)*

Table 1: Model Performance based on Munsell training set (median ΔE_{00} colour differences with maximum colour differences in parentheses), best performance is marked by an asterisk.

Training Set		1562MunSI			
Testing Set		1562MunSI		1063PCC	
Testing Ill.		A	F11	A	F11
Basic Method	SC	1.05 (5.71)	1.18 (7.97)	N/A	N/A
	3BF	1.01 (5.70)	1.27 (7.20)	1.23 (5.33)	1.10 (5.77)
	5BF&SC	0.88 (5.32)	*1.03 (8.43)	*1.16 (3.73)	1.03 (4.45)
	WE	*0.72 (5.74)	1.12 (7.25)	1.51 (5.15)	*0.86 (5.12)
With Localised Training data	3BF (50Loc)	*0.26 (3.09)	*0.40 (5.38)	1.47 (5.07)	*0.76 (7.48)
	5BF&SC (50Loc)	0.64 (4.04)	0.85 (7.23)	*1.26 (4.20)	0.86 (4.18)
	WE (50Loc)	0.57 (4.63)	0.67 (8.73)	1.47 (7.09)	0.99 (11.7)
With Localised Weighted Training data	3BF (50wLoc)	0.22 *(3.2)	0.33 (6.23)	1.52 (5.17)	*0.78 (7.60)
	5BF&SC (50wLoc)	0.60 (3.75)	0.79 (7.16)	*1.23 (4.17)	0.87 (4.08)
	WE (50wLoc)	*0.21 (3.26)	*0.29 (5.89)	1.63 (7.16)	0.89 (6.96)

With the modification of 50 localised training set, the training performances were improved on all basic methods but the testing performances were not always better than the basic methods. When applying the weighted matrix on the 50 localised training set, most of these methods were further improved especially for the WE method (training performance under illuminant A with the PCC set gave 0.79 and 0.08 median ΔE_{00} for WE(50Loc) and WE(50wLoc), respectively). The testing performance still does not always outperform than the basic methods, but it gave similar performance. For instance, the testing performance under illuminant A with the PCC set training and the Munsell set testing gave 1.29 and 1.31 median ΔE_{00} for WE and WE(50wLoc), respectively. Comparing all of these methods, the WE(50wLoc) gave the lowest median perceptual errors in training performance. For the performance in terms of maximum ΔE_{00} units, the 5BF&SC(50wLoc) slightly better than that of the WE(50wLoc) method. Therefore, considering the overall performance, the WE(50wLoc) method is recommended.

Note that when the PCC set used as training set for the 3BF(50Loc) and 3BF(50wLoc) methods, the maximum ΔE_{00} became extremely large. When the recovered spectrum is out of the boundary from 0 to 1, the values were clipped into the range and that caused extremely large colour difference. This also implied that three basis functions are not enough for the PCC set when using localised training modification.

Table 2: Model Performance based on PCC training set (median ΔE_{00} colour differences with maximum colour differences in parentheses), best performance is marked by an asterisk.

Training Set		1063PCC			
Testing Set		1063PCC		1562MunSI	
Testing Ill.		A	F11	A	F11
Basic Method	SC	1.39 (3.97)	0.90 (4.22)	N/A	N/A
	3BF	*0.82 (4.58)	*0.86 (6.04)	1.23 (5.84)	1.52 (9.76)
	5BF&SC	1.24 (3.71)	0.87 (4.10)	*1.08 (5.27)	*1.00 (7.98)
	WE	1.13 (4.78)	0.93 (4.68)	1.29 (5.90)	1.05 (8.74)
With Localised Training data	3BF (50Loc)	*0.23 (54.9)	*0.28 (69.3)	1.39 (45.1)	1.04 (43.6)
	5BF&SC (50Loc)	1.03 (4.97)	0.73 (5.85)	*1.04 (5.05)	1.02 (8.34)
	WE (50Loc)	0.79 (8.17)	0.73 (4.92)	1.32 (9.70)	*0.99 (10.1)
With Localised Weighted Training data	3BF (50wLoc)	0.12 (37.2)	0.13 (29.3)	1.37 (51.4)	1.05 (47.2)
	5BF&SC (50wLoc)	1.16 (4.01)	0.67 (4.69)	*0.99 (5.04)	*0.98 (8.22)
	WE (50wLoc)	*0.08 (3.42)	*0.09 (5.43)	1.31 (5.27)	0.99 (8.63)

Figure 4 shows an example of the reconstructed reflectance using the WE, WE(50Loc) and WE(50wLoc) methods. The solid line indicates the measured reflectance. The dash lines refer to the reflectance curved by these three methods. It can be seems that the WE(50wLoc) method had the closest curve to the measured curve, followed by the WE(50Loc) method. The WE method performed worst.

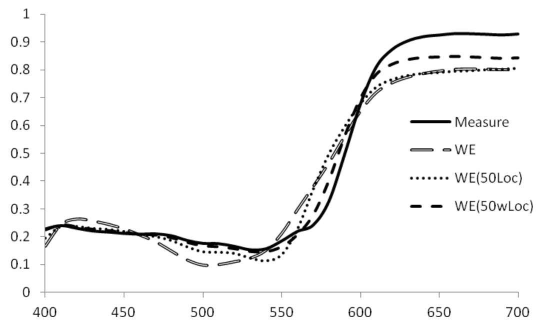


Figure 4: *An example of reconstructed reflectance using WE, WE(50Loc) and WE(50wLoc) methods*

Generally, the results showed a big improvement of the conventional method by using a localised training set. However, it was also found little improvement by introducing the weighted method. This is because the reconstructed reflectance based on the localised training samples is already very close to the target sample, the weighted matrix has limited improvement. It can be concluded that the largest improvement is the localised training set. The autocorrelation matrix used in the WE method could effectively improve the recovery performance when the properties of the training and testing samples are similar.

Conclusions

Four reflectance recovery methods were investigated together with some modifications of the training set or application of weighted matrix. Without any modification on training data, the 5BF&SC method gave the best performance in most cases. For the SC method, the training set was not required and the performance is not dissimilar with the conventional methods. With modification on training set, the Wiener estimation method with localised training data and a weighted matrix generally perform the best. Regarding to the BF method, more basis functions should be used for PCC set.

References

[1] Li, C.J. and M.R. Luo. The estimation of spectral reflectances using the smoothness constraint condition. *In: IS&T/SID 9th Color Imaging Conference*, 2001, pp.62-67.

[2] Sobagaki, H., K. Takahama and Y. Nayatani. Estimation of Spectral Reflectance Functions for Munsell Renotations. *In: 6th Congress of the International Colour Association*, 1989, pp.156-158.

[3] Cheung, V., S. Westland, C.J. Li, J. Hardeberg and D. Connab. Characterization of trichromatic color cameras by using a new multispectral imaging technique. *Journal of the Optical Society of America A: Optics and Image Science, and Vision*, 2005, **22**(7), pp.1231-1240.

[4] Babaei, V., S.H. Amirshahi and F. Agahian. Reflectance reconstruction by adapting Wiener restoration method: Using color difference values as weighting matrix. *In: 11th Congress of the International Colour Association*, 2009.

[5] Shen, H.L., P.Q. Cai, S.J. Shao and J.H. Xin. Reflectance reconstruction for multispectral imaging by adaptive Wiener estimation. *Optics Express*, 2007, **15**(23), pp.15545-15554.

[6] Nadal, M.E., C.C. Miller and H.S. Fairman. Statistical methods for analyzing color difference distributions. *Color Research and Application*, 2010, pp.n/a-n/a.

[7] Luo, M.R. The CIE 2000 colour difference formula: CIEDE2000. *In: 9th Congress of the International Colour Association*, 2001, pp.554-559.

[8] Vantrigt, C. Smoothest Reflectance Functions. II. Complete Results. *Journal of the Optical Society of America A: Optics and Image Science, and Vision*, 1990, **7**(12), pp.2208-2222.

[9] Vantrigt, C. Smoothest Reflectance Functions. I. Definition and Main Results. *Journal of the Optical Society of America A: Optics and Image Science, and Vision*, 1990, **7**(10), pp.1891-1904.

[10] Hardeberg, J.Y., F. Schmitt and H. Brettel. Multispectral color image capture using a liquid crystal tunable filter. *Optical Engineering*, 2002, **41**(10), pp.2532-2548.

[11] Imai, F.H. and R.S. Berns. A comparative analysis of spectral reflectance reconstruction in various spaces using a trichromatic camera system. *In: IS&T/SID 7th Color Imaging Conference*, 1999, pp.21-25.

[12] Hardeberg, J.Y. *Acquisition and reproduction of color images: Colorimetric and multispectral approaches*. PhD thesis, Ecole Nationale Superieure des Telecommunications, 1999.

[13] Jolliffe, I.T. *Principal Component Analysis*, New York: Springer, 1986.

[14] Maloney, L.T. and B.A. Wandell. Color Constancy - a Method for Recovering Surface Spectral Reflectance. *Journal of the Optical Society of America A: Optics and Image Science, and Vision*, 1986, **3**(1), pp.29-33.

[15] Imai, F.H. and R.S. Berns. Spectral estimation using trichromatic digital cameras. *In: Proceedings of the International Symposium on Multispectral Imaging and Color Reproduction for Digital Archives*, 1999, pp.42-49.

[16] Li, C. and M. Ronnier Luo. The estimation of spectral reflectances using the smoothness constraint condition. *In: IS&T/SID 9th Color Imaging Conference*, 2001, pp.62-67.

[17] Shi, M.H. and G. Healey. Using reflectance models for color scanner calibration. *Journal of the Optical Society of America A: Optics and Image Science, and Vision*, 2002, **19**(4), pp.645-656.

[18] Shen, H.L. and J.H. Xin. Spectral characterization of a color scanner by adaptive estimation. *Journal of the Optical Society of America A: Optics and Image Science, and Vision*, 2004, **21**(7), pp.1125-1130.

[19] Haneishi, H., T. Hasegawa, A. Hosoi, Y. Yokoyama, N. Tsumura and Y. Miyake. System design for accurately estimating the spectral reflectance of art paintings. *Applied Optics*, 2000, **39**(35), pp.6621-6632.

[20] Shen, H.L., J.H. Xin and S.J. Shao. Improved reflectance reconstruction for multispectral imaging by combining different techniques. *Optics Express*, 2007, **15**(9), pp.5531-5536.

[21] Nickerson, D. History of the Munsell Color System and its scientific application. *Journal of the Optical Society of America*, 1940, **30**(12), pp.575-586.

[22] Park, J. and K.M. Park. Professional Colour Communicator (PCC) - the Definitive Colour Selector. *Book of Papers - Aatcc 1994 International Conference & Exhibition*, 1994, pp.294-296.

Author Biography

Yi-Fan Chou graduated from the National Taiwan University of Science and Technology with a BSc degree in Electronic Engineering (2007). He then received an MSc in Colour and Imaging Science from the University of Leeds (2008). He is currently a joint Phd student between University of Leeds and National Taiwan University of Science and Technology. His work has focused on reflectance generation of multispectral imaging and camera characterization. He also involve in the development of new sample sets for evaluating colour rendering property.