Predicting reflectances of Neugebauer primaries by relying on separately measured ink transmittances

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Abstract

Modern printers allow printing on different paper types with many different coloured inks. Therefore, predicting the reflectances of paper printed with solid inks and ink superpositions (Neugebauer primaries) is of high importance. Given the separately measured transmittance of an ink and the reflectance of a white diffusing background, we try to predict the reflectance of the inked white diffusing background by relying on a modified Saunderson formula accounting for the differences in the measurement geometries when performing transmittance and reflectance measurements. Accurate reflectance predictions are obtained for the cyan and magenta ink patches as well as for their ink superpositions. Less accurate predictions are obtained for the yellow ink patches or for ink superpositions incorporating the yellow ink. We suspect that the yellow ink layer is at the origin of optical phenomena which behave differently in reflectance and transmittance modes, and therefore allow less accurate predictions.

Introduction

Existing spectral prediction models such as the Neugebauer and Yule-Nielsen models rely on the measured reflectance spectra of Neugebauer primaries [1]. The reflectance of the Neugebauer primaries is formed by the complex interaction between light, printed ink dots and paper. One important challenge in color reproduction is the prediction of Neugebauer primaries as a function of parameters defining separately the paper properties and the ink properties. This is especially important for printing with many colored inks, such as cyan, magenta, yellow, green, orange, blue and black inks.

An existing approach to estimate Neugebauer multi-ink superposition primaries relies on measurements of the individual solid ink patches, on the deduction of the ink absorption and on the additivity of the inks absorption coefficients in ink absorption space [2-5]. An alternative formulation relies on the additivity of the ink attenuations in the Yule-Nielsen space $\Psi(\lambda) = R^{1/n}(\lambda)$, where reflectances are raised to power of $1/n$, with $2<n<$• [6]. In these two approaches, the individual ink transmittance is deduced from the solid ink printed on a specific paper. Therefore, as underlined by Shaw, Sharma, Bala and Dalal in [7], the deduced ink transmittance is not independent of the paper substrate on which the ink is printed.

In the present contribution we would like to explore to which extent we can predict the reflectance of a solid ink patch printed on a white diffusing background, by direct deduction of the ink transmittance from measurements of the solid ink printed on a transparency and by separate measurement of the white diffusing background reflectance. In contrast to previous approaches, where transmittance measurements are deduced from reflectance measurements of ink printed on the target substrate [2-7], in the present setup, the transmission measurements are completely independent of the target substrate.

If we assume that inks are transparent and that they behave as a filter, then the Saunderson formula [8] accounting for the internal light reflections between the reflecting diffuse background and the ink–air interface should be valid. Knowing the intrinsic reflectance of paper and the transmittance of the ink, one should be able to deduce the reflectance of the inked paper. However a simple experiment shows that this simple approach does not allow to accurately predict the printed patch reflectance. One needs to account for the scattering behavior of the inks as well as for the difference in measurement geometry between transmittance measurements and reflectance measurements.

When using paper as the white diffusing substrate, further difficulties appear, such as the partial diffusion of the ink into the paper layer and in case of uncoated paper, the undefined optical interface between the paper layer and the air.

Therefore, in order to explore if we can predict the print reflectance from the reflectance of the white diffusing background and the independently measured transmittance of the ink, we create well controlled setups similar to the setup formed by ink deposited on a coated paper. We consider two experimental setups.

One setup consists of a diffuse white ceramic tile of known reflectance, and of a flat transparent sheet (transparency) either unprinted or printed with a solid ink. The unprinted side of the transparency is in optical contact with the diffuse white reflector by having sugared water (index of refraction ~1.5) between them. The unprinted transparency in optical contact with the ceramic tile forms the background diffusing layer.

The second setup consists of a printed or unprinted transparency in optical contact with a white plastic sheet, i.e. glued on the plastic. The unprinted transparency in optical contact with the plastic sheet forms the background diffusing layer.

The ink transmittance is obtained by dividing the captured light flux transmitted by the printed transparency by the captured light flux transmitted by the unprinted transparency. We show that the direct application of the Saunderson formula does not provide an accurate prediction. However, by assuming that the lengths of the light paths through the inks vary due to the different measuring geometries and that, due to the scattering behavior of the inks, the ink transmittance can be measured up to an unknown scaling factor, we are able to establish reliable reflectance predictions.

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The Saunderson Correction

The reflectance spectra of solid color prints strongly depend on the attributes of the inks and the substrate. For a transparency printed with a solid ink layer laid out in optical contact with a white strongly diffusing layer (Lambertian reflectance), phenomena influencing the reflectance of the setup are the surface (Fresnel) reflection at the interface between the air and transparency, light attenuation by the ink and by the transparency, the reflectance of the white diffusing substrate and the internal (Fresnel) reflection at the interface between the printed transparency and the air [9]. Therefore, multiple internal reflections occur between the substrate and the printed transparency air interface (for the derivation of the Saunderson formula, see Appendix B). The Saunderson formula (Eq. 1) predicts the observed reflectance at the air side of the setup.

$$R(\lambda) = K \cdot r_s + \frac{(1-r_s)\cdot r_g(\lambda)\cdot(1-r_i)\cdot t^2(\lambda)}{1-r_i\cdot r_g\cdot t^2(\lambda)} \tag{1}$$

Where $R(\lambda)$ is the predicted or observed reflectance spectrum, where r_s =0.092 and r_i =0.6, are respectively the specular and internal reflection coefficients for a (d:8°) measurement geometry [9],[10], and where $r_g(\lambda)$ is the intrinsic reflectance of the white diffusing background, i.e. the reflectance that would be measured in a medium having the same refraction index as the white diffusing background. When the specular reflection is excluded from the measurements, K=0.

Experiments

Solid ink cyan, magenta and yellow ink patches are prepared by printing the solid inks on a nonscattering transparency at 75lpi using an inkjet printer (Canon PIXMA Pro9500). In the first setup, a white ceramic tile is in optical contact with the printed transparency. In the second setup, the transparency is glued on a white plastic. In both setups, the unprinted transparency in optical contact with its white diffusing layer forms the white diffuse background.

The Color i7 spectrophotometer from Gretag-Macbeth and a custom optical transmittance measurement device built with components from Ocean Optics [11] were used to perform the transmittance measurements at a measurement geometry of (d:0°), i.e. diffuse illumination and transmitted light captured perpendicularly to the sample.

With the custom measurement setup (Figure 1), the transmittance of the ink $t(\lambda)$ is calculated by dividing the captured light flux transmitted by the printed transparency by the captured light flux transmitted by the unprinted transparency

$$t(\lambda) = \frac{\Phi(\lambda)_{ink+transparency}}{\Phi(\lambda)_{transparency}} \tag{2}$$

Since the Color i7 spectrophotometer gives directly transmittance measurements, the transmittance of the ink layer is the transmittance of the printed transparency divided by the transmittance of the unprinted transparency.

$$t(\lambda) = \frac{t(\lambda)_{ink+transparency}}{t(\lambda)_{transparency}} \tag{3}$$

The transmittances of the cyan ink layer obtained from measurements performed on the Color i7 spectrophotometer according to Eq. (3) and from the custom measurement device according to Eq. (2) are shown in Figure 2.

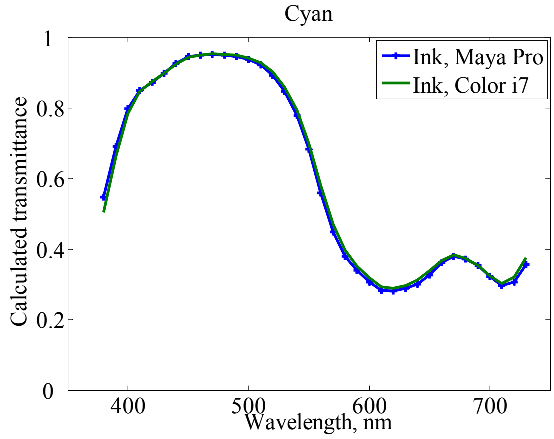

Figure 2. *Calculated transmittance of the cyan ink using (a) the custom setup with the Maya Pro spectrophotometer and (b) using the Color i7 spectrophotometer.*

Both measurement devices yield a nearly identical ink transmittance. Similar results were obtained when mesuring the transmittances of the magenta and yellow inks. Thus, further results rely on ink and ink superposition transmittance measurements performed with the custom setup.

The Color i7 spectrophotometer was used to measure the reflectance spectra with the measurement geometry (d:8 °). Its light source is an emulation of the D65 illuminant. The specular reflectance was excluded.

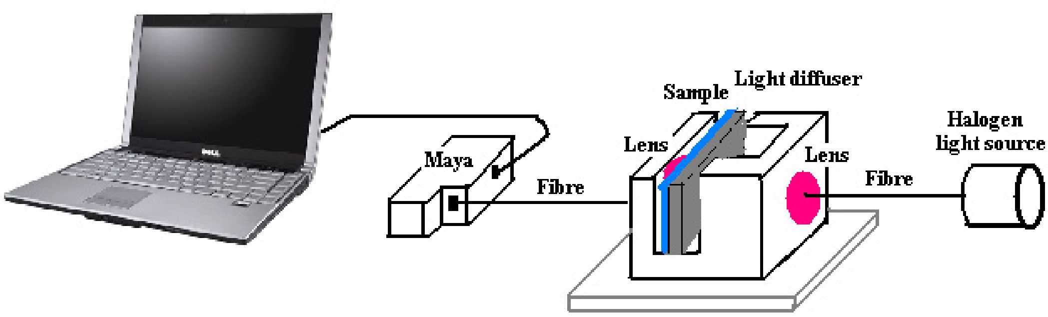

Figure 1. Custom transmittance measurement setup relying on a Maya Pro spectrophotometer from Ocean Optics [11].

Prediction of the Reflectance Spectra

Using Directly the Saunderson Formula

In the present work, we consider that the white diffusing background is formed by both the white diffusing substrate and the transparency in optical contact with it. Its reflectance $R_w(\lambda)$ is the reflectance $R(\lambda)$ of the Saunderson formula (1) that can be measured from its air side. The intrinsic reflectance of the white diffusing background is calculated by measuring $R_w(\lambda)$ and deducing according to equation (1) the intrinsic reflectance $r_g(\lambda)$. Equation (2) was used to calculate the ink transmittance. Figure 3 shows the measured reflectance spectra $R_w(\lambda)$ of the transparency laid out in optical contact with the white substrate. The calculated internal reflectance spectra $r_g(\lambda)$ are also shown.

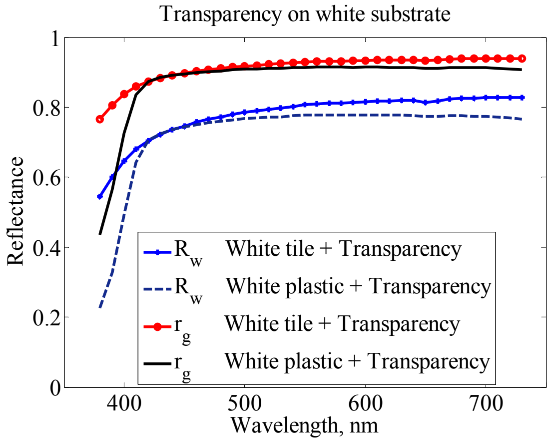

***Figure 3.** The measured reflectance $R_w(\lambda)$ and calculated internal reflectance $r_g(\lambda)$ of the diffusing background formed by the white substrate (white tile or white plastic) in optical contact with the transparency.*

Figure 4 shows the measured and predicted reflectances of the solid cyan printed on the transparency in optical contact with the white tile. A similar behavior was observed for the solid magenta and solid yellow inks printed on a transparency and put in optical contact with the white tile.

Figure 4 suggests that we can not use the Saunderson formula in a straightforward manner to predict the reflectance spectrum of solid cyan by relying directly on the ink layer transmittance deduced from transmittance measurements. The reflectance spectra predicted directly according to the original Saunderson formula are darker than the measured one. This can be explained by the fact that if the inks are slightly scattering and that during the measurements in transmission, less light reaches the sensor, the ink appears to be darker than it really is. This effect may become quite significant when the incident light reaching the ink layer is not Lambertian. Due to the interface between the air and the transparency at the side of the light source, even the nearly perfectly Lambertian light emitted by the Color i7 spectrophotometer becomes non-Lambertian within the transparency. With too darkly measured inks, the Saunderson prediction yields also too dark reflection spectra.

In contrast to the ink transmittance measurements where the incident light within the transparency is not Lambertian, in ink reflectance measurements, the incident Lambertian light reflected by the white diffusing substrate is nearly Lambertian and the weakly diffusing ink layer does therefore not induce an additional first order effect.

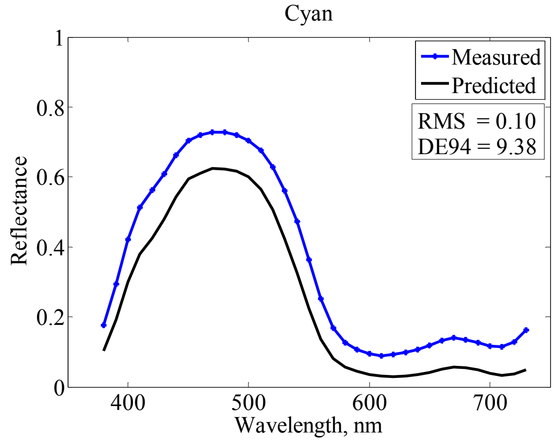

***Figure 4.** Prediction of the reflectance of cyan printed on a transparency in optical contact with the white tile according to the unmodified Saunderson formula.*

Accounting for Light Scattering and for Different Path Lengths within the Ink Layer

To account for the problem of measuring a too dark transmittance of the inks, we assume that the ink transmittance acquisition error is due to two distinct phenomena. The first phenomenon is due to the cone spreading of the light within the ink layer due to scattering and the second phenomenon is due to the difference in mean path length of light through the ink layer when performing transmittance measurements and performing reflectance measurements.

The spreading of light due to light scattering within the ink layer reduces the light flux that traverses the ink layer and is captured by a fiber connected to the spectrophotometer (Figure 1). Therefore, the captured light flux is the light flux that would be captured without light spreading divided by a scalar factor s larger than 1. Regarding the second phenomenon, according to Beer's law, a difference between two path lengths through the ink layer yields transmittances which differ by an exponent d.

We model these two phenomena by inserting into the Saunderson formula an ink transmittance t_i which is known up to a scaling factor s and to an exponent d_i. Equation (4) shows the modified Saunderson formula

$$R(\lambda) = K \cdot r_s + \frac{(1-r_s)\cdot r_g(\lambda)\cdot(1-r_i)\cdot\left(s\cdot t_i^{d_i}(\lambda)\right)^2}{1-r_i\cdot r_g\cdot\left(s\cdot t_i^{d_i}(\lambda)\right)^2} \tag{4}$$

where index i refers to the ink and $r_g(\lambda)$ refers to the intrinsic reflectance of the diffusing background formed by the unprinted transparency in optical contact with the diffusing substrate.

In order to find the scaling factor s and the exponent of the ink transmittance d_i, the *fminsearch* function offered by Matlab is used to minimize the sum of square differences between predicted reflection spectra components $R(\lambda)$ and measured reflectance components $R_m(\lambda)$.

In the case that a Neugebauer primary is formed by the superpositions of two inks, we have a global scaling factor and a separate exponent for each of the contributing inks. For the superposition of two inks i and j we use the following formula

$$R(\lambda)=K\cdot r_s+\frac{(1-r_s)\cdot r_g(\lambda)\cdot(1-r_i)\cdot\left(s\cdot t_i^{d_i}(\lambda)\cdot t_j^{d_j}(\lambda)\right)^2}{1-r_i\cdot r_g\cdot\left(s\cdot t_i^{d_i}(\lambda)\cdot t_j^{d_j}(\lambda)\right)^2} \tag{5}$$

For superpositions of 3 inks i, j and k, we have

$$R(\lambda)=K\cdot r_s+\frac{(1-r_s)\cdot r_g(\lambda)\cdot(1-r_i)\cdot\left(s\cdot t_i^{d_i}(\lambda)\cdot t_j^{d_j}(\lambda)\cdot t_k^{d_k}(\lambda)\right)^2}{1-r_i\cdot r_g\cdot\left(s\cdot t_i^{d_i}(\lambda)\cdot t_j^{d_j}(\lambda)\cdot t_k^{d_k}(\lambda)\right)^2} \tag{6}$$

Figure 5 shows the predicted and measured reflectance spectra of solid cyan, solid magenta and of solid yellow printed on a transparency in optical contact with the white tile by a layer of sugared water.

The fitted scaling factor and exponent of the ink transmittance along with the spectral and colorimetric errors of the predicted reflectance spectra of solid cyan, magenta and yellow inks are shown in Table 1 of Appendix A. We also show the results for the second setup made of a transparency glued on white plastic. In all considered cases, the scaling factors due to ink scattering are very similar. The exponents expressing differences in light path lengths between the transmission and reflection setups are also similar for the cyan and magenta ink patches. The exponent of the yellow ink transmittance is lower, indicating that the yellow ink is less transparent and that therefore the average light path in reflectance mode within the yellow ink layer is significantly shorter, compared with the cyan and magenta ink layers.

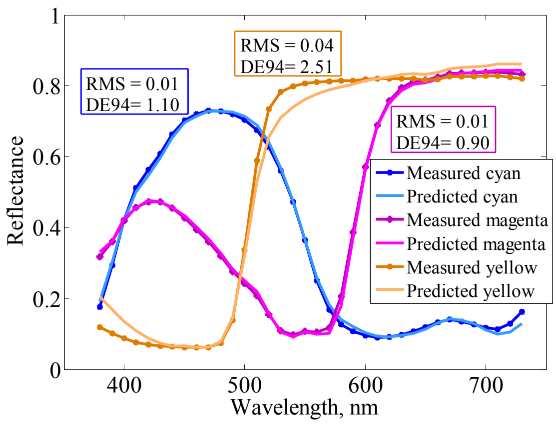

Figure 5. *The measured reflectance and the reflectance predicted according to Eq. (4) of solid cyan, solid magenta and solid yellow.*

According to Figure 5 and Table 1, fitting a scaling factor and an exponent for the transmittance inserted into the Saunderson formula results in accurately predicted reflectance spectra of solid cyan and magenta. The yellow reflectance predictions are less accurate. As can be seen from Fig. 5, the measured reflectance curve is different from the predicted one. There may be therefore further optical phenomena present within the yellow ink layer, which create differences between transmittance measurements and reflectance measurements.

Table 1 also shows the transmittance scaling factors and exponents for predicting the reflectance spectra in the case of superpositions of two or three primary solid inks, i.e. superpositions of cyan, magenta and yellow inks printed on the same transparency in optical contact with the white substrate. The prediction accuracies of the cyan-magenta (blue) and the cyan-yellow (green) ink superpositions are excellent. The magenta-yellow (red) and cyan-magenta-yellow (black) ink superpositions are less accurate.

Conclusions

One important challenge in color reproduction is the prediction of Neugebauer primaries as a function of parameters defining separately the underlying substrate (paper) and the ink properties. We have carried out a first step in this direction by measuring separately in transmission mode the transmittance of an ink layer on a transparency and in reflection mode the reflectance of the diffusing background formed by that unprinted transparency in optical contact with a white diffusing substrate. For substantially transparent inks such as the cyan and magenta inks, one may predict the reflectance of the assembly of the ink and the diffusing background according to a modified Saunderson formula, by solving for an unknown scalar ink transmittance scaling factor expressing the light spreading within the ink layer and for an unknown scalar transmittance exponent expressing a difference in average light path lengths through the ink layer, between transmittance and reflectance measurements.

The reflectances predicted according to this modified Saunderson formula accounting for the differences in measurement geometries are accurate for the cyan, the magenta and the superpositions of cyan and magenta ink layers. They are less accurate for the yellow ink layer or for ink superpositions incorporating the yellow ink. We suspect in the yellow ink layer the presence of further optical phenomena which behave differently in reflectance and transmittance modes, and therefore allow less accurate predictions.

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Author Biography

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APPENDIX A

Table 1: Prediction of the reflectance spectra of solid ink and ink superpositions: cyan, magenta, yellow, red, green, blue and black, printed on a non-scattering transparency in optical contact with a white tile or white plastic, according to Eqs. (4), (5) and (6).

Sample		S	d_j	*RMS*	ΔE_{94}
Printed transparency	**White substrate**				
Cyan	White tile	1.0177	0.6034	0.0108	1.0973
	White plastic	1.0235	0.6506	0.0146	2.0341
Magenta	White tile	1.0281	0.6950	0.0087	0.9051
	White plastic	1.0292	0.6423	0.0099	0.7199
Yellow	White tile	1.0290	0.4283	0.0361	2.5103
	White plastic	1.0423	0.4039	0.0396	2.7823
Red	White tile	1.0654	d_m= 0.7088 d_y = 0.4638	0.0188	3.8171
	White plastic	1.0745	d_m = 0.7423 d_y = 0.4033	0.0139	1.6985
Green	White tile	1.0005	d_c = 0.6418 d_y = 0.3901	0.0177	0.8949
	White plastic	1.0093	d_c = 0.6423 d_y = 0.3640	0.0161	0.7239
Blue	White tile	1.0568	d_c = 0.6185 d_m = 0.5916	0.0077	0.3923
	White plastic	1.0804	d_c = 0.6620 d_m = 0.6580	0.0095	0.4804
Black	White tile	0.8213	d_c = 0.3893 d_m = 0.4613 d_y = 0.2592	0.0072	1.6182
	White plastic	0.8375	d_c = 0.4071 d_m = 0.4837 d_y = 0.2624	0.0069	0.8235

APPENDIX B

Derivation of the Saunderson Correction

Let us consider collimated incident light at 45° hitting a medium composed of a flat uniform colorant layer of transmittance $t(\lambda)$ located over a white diffusing substrate of intrinsic reflectance $r_g(\lambda)$, see Figure 6. Both the colorant layer and the white diffusing substrate have a refraction index n=1.5.

Collimated incident light E_{in} from the air hits the colorant layer, part of it is specularly reflected according to the Fresnel reflection factor $r_s(\theta)$ and the other part $1-r_s(\theta)$ enters the medium. The part entering the medium traverses the colorant layer. When reaching the white diffusing substate, the light has been attenuated by the colorant layer by the factor $t(\lambda)$. The white diffusing substrate reflects over the whole hemisphere a ratio r_g of the light incident on it. The portion E_0 of the incident light E_{in} that is emitted from the substrate towards the colorant layer is $E_0 = (1-r_s)\ r_g\ t\ E_{in}$. It traverses the colorant layer, is partially reflected at the colorant layer – air interface, traverses again the colorant layer and reaches the substrate and is reflected from it for the second time. The attenuation of this single reflection cycle comprises the attenuation of traversing twice the colorant layer t^2, the internal Fresnel reflection at the colorant layer–air interface r_i , and the diffusion-reflection by the substrate $r_g(\lambda)$. At the medium-air interface (internal print interface), for a medium with an assumed index of refraction of n=1.5, the value of the internal Fresnel reflectance is r_i=0.596, see [9],[10].

After one full reflection cycle, the light is further attenuated by $t^2 r_i\ r_g(\lambda)$. At the k^{th} full reflection cycle, the attenuation of E_0 is $[t^2 r_i\ r_g(\lambda)]^k$. By summing these light attenuation components, we obtain the global attenuation yielding the portion of light E_S emerging from the substrate

$$E_S = \left(\sum_{k=0}^{\infty} \left(r_i \cdot r_g(\lambda) \cdot t(\lambda)^2 \right)^k \right) E_0 \tag{7}$$

which is a geometric series and can be expressed by

$$E_S = \frac{1}{1 - r_g(\lambda) r_i \cdot t(\lambda)^2} E_0 \tag{8}$$

Part $(1-r_i)$ of the resulting attenuated light E_s traverses the print-air interface and exits from the print after having been attenuated once again by the colorant layer. The exiting light captured by an integrated sphere is

$$\begin{aligned} E_{out} &= \frac{t(\lambda)(1-r_i)}{1 - r_g(\lambda) r_i \cdot t(\lambda)^2} \cdot E_0 \\ &= \frac{(1-r_s) \cdot t(\lambda)^2 \cdot r_g(\lambda) \cdot (1-r_i)}{1 - r_g(\lambda) r_i \cdot t(\lambda)^2} \cdot E_{in} \end{aligned} \tag{9}$$

If we include the fraction K of the captured specular reflectance component, the resulting reflectance is

$$R(\lambda) = \frac{E_{out}}{E_{in}} = K \cdot r_s + \frac{(1-r_s)(1-r_i) \cdot r_g(\lambda) \cdot t^2(\lambda)}{1 - r_i \cdot r_g \cdot t^2(\lambda)} \tag{10}$$

The Saunderson correction [8] specifies that for a reflecting medium of intrinsic reflectance ρ, the reflectance captured by measurement at the air side is

$$R(\lambda) = K \cdot r_s + \frac{(1-r_s)(1-r_i) \cdot \rho(\lambda)}{1 - r_i \cdot \rho(\lambda)} \tag{11}$$

Clearly, in our case, the intrinsic medium reflectance $\rho(\lambda) = r_g\ (\lambda)\ t\ (\lambda)^2$.

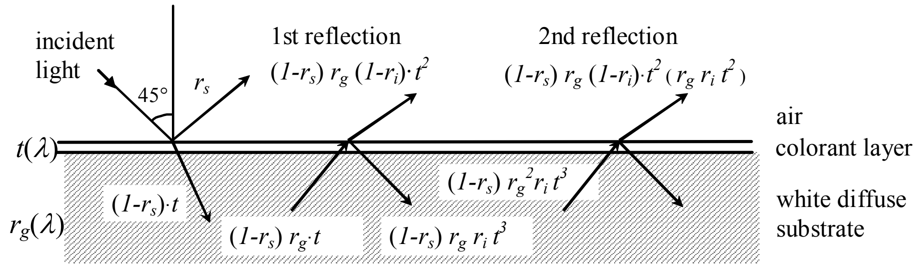

Figure 6. *Medium composed of a flat uniformly colorant layer of transmittance t(λ) located over a white diffusing substrate of intrinsic reflectance $r_g(\lambda)$*