Metamer Mismatch Volumes

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Abstract

A new algorithm for evaluating metamer mismatch volumes is introduced. Unlike previous methods, the proposed method places no restrictions on the set of possible object reflectance spectra. Such restrictions lead to approximate solutions for the mismatch volume. The new method precisely characterizes the volume in all circumstances.

Introduction

Two objects (surface reflectances) of the same colour (speaking informally) under one light can differ in colour under a second light. That is, they may be equal in CIE XYZ (for example) under the first light, but differ in CIE XYZ under the second. They are metameric matches under the first light, but fail to match, and hence are no longer metamers, under the second. If a colour under one light can become two colours under a second light, then it is natural to ask: What is the range of possible colours that the observed colour might become under the second light? For example, given an XYZ under CIE illuminant D65, what is the set of possible XYZ that could arise under CIE illuminant A? This set is commonly known as the metamer mismatch volume. In general, given the spectral power distributions of two illuminants and the colour of an object under one illuminant, the problem is to compute the metamer mismatch volume (i.e., the set of colours) that could possibly arise under the second illuminant.

Metamer mismatch volumes are important because they arise in colour correction, camera design (sensors leading to the smallest metamer mismatch volumes will be colormetrically the most accurate), and lighting design (lights that lead to the smallest mismatch volumes will have the best colour rendering). Previous work on mismatch volumes has been based on finding "natural" metameric spectra or metameric blacks [1, 9] using linear programming or Monte Carlo methods [10]. There have been promising results in colour correction using the volumes computed by these methods [9, 4, 2, 3]. However, of the methods proposed thus far, none directly describes the theoretical limits of the metamer mismatch volumes.

In this report we investigate the boundary of the metamer mismatch volume from the formal point of view and then provide an algorithm for computing metamer mismatch volumes for arbitrary, strictly positive illuminants without placing any restrictions on the reflectances.

Metamer Mismatch Volume Theory

Consider a set of *colour mechanisms* $\Phi = (\varphi_1, ..., \varphi_n)$, the response of each of which to a reflecting object with spectral reflectance function $x(\lambda)$ illuminated by a light with spectral power distribution $p(\lambda)$ is given by

$$\varphi_i(x) = \int_{\lambda_{\min}}^{\lambda_{\max}} x(\lambda)\, p(\lambda)\, s_i(\lambda)\, d\lambda \qquad (i = 1, ..., n), \quad (1)$$

where $[\lambda_{\min}, \lambda_{\max}]$ is the visible spectrum interval, and $s_i(\lambda)$ is the spectral sensitivity of the i-th colour mechanism. The vector $\Phi(x) = (\varphi_1(x), ..., \varphi_n(x))$ of the colour mechanism responses will be referred to as the *colour signal* produced by the colour mechanisms Φ in response to $x(\lambda)$ illuminated by $p(\lambda)$. In the case of trichromatic human colour vision $n = 3$, and $s_1(\lambda), s_2(\lambda)$, and $s_3(\lambda)$ are either *colour matching functions*, or *cone fundamentals*. Alternatively, $s_1(\lambda), s_2(\lambda)$, and $s_3(\lambda)$ can be treated as the sensors' spectral sensitivity functions of a digital camera or similar device.

Different objects may happen to produce equal colour signals. Such objects are called *metameric*. Specifically, two objects with spectral reflectance functions $x(\lambda)$ and $x'(\lambda)$ are called metameric under the illuminant $p(\lambda)$ if they produce equal colour signals, that is, $\Phi(x) = \Phi(x')$. Object metamerism depends on the illuminant. If the illuminant $p(\lambda)$ is replaced by a different illuminant $p'(\lambda)$ the hitherto metameric objects may cease to be metameric. In other words, the former metamers may no longer match under the new illuminant. This phenomenon—object metamers becoming non-metamers—will be called metamer mismatching.

The same type of metamer mismatch may happen if the spectral sensitivity of the colour mechanisms changes. An illuminant change (i.e., replacing $p(\lambda)$ with $p'(\lambda)$) is, formally, equivalent to changing the spectral sensitivity functions of the colour mechanisms. As a consequence, we will consider the general situation when a set of colour mechanisms $\varphi_1, ..., \varphi_n$ is replaced by a different set $\psi_1, ..., \psi_n$. The new set of colour mechanisms can be understood as the result of altering either the illuminant or the colour mechanisms' spectral sensitivities, or both.

Consider two sets of colour mechanisms: $\Phi = (\varphi_1, ..., \varphi_n)$ and $\Psi = (\psi_1, ..., \psi_n)$. Note that both the Φ and Ψ can be considered as linear maps (referred to as *colour maps*) of the form of: $\mathscr{X} \to \mathbf{R}^n$ where $\mathscr{X}$ is the set of all the spectral reflectance functions (i.e., such that $0 \le x(\lambda) \le 1$), and $\mathbf{R}^n$ is the arithmetic n-dimensional vector space. The sets of all colour signals, that is, $\Phi(\mathscr{X})$ and $\Psi(\mathscr{X})$, form convex volumes in $\mathbf{R}^n$ that are usually referred to as *object-colour solids*.

Given an object $x_0 \in \mathscr{X}$, the $\Phi-$pre-image $\Phi^{-1}(\Phi(x_0))$ (i.e., $\Phi^{-1}(\Phi(x_0)) = \{x \in \mathscr{X} | \Phi(x) = \Phi(x_0)\}$) of its colour signal $\Phi(x_0)$ is the set of all the objects metameric to x_0 (with respect to Φ), and is referred to as its metamer set. Generally, when this set of metameric objects $\Phi^{-1}(\Phi(x_0))$ is mapped by Ψ into the Ψ-colour solid, it will be spread into a non-singleton set. The resulting set is usually referred to as the metamer mismatch volume. Formally, the $\Psi-$image of the set of the Φ-metamers $\Psi(\Phi^{-1}(\Phi(x_0)))$ will be called *the metamer mismatch volume* induced by x_0.

Given two colour maps, $\Phi = (\varphi_1, ..., \varphi_n)$ and $\Psi = (\psi_1, ..., \psi_n)$, let us consider a map $\Upsilon : \mathscr{X} \to \mathbf{R}^{2n}$ such that $\Upsilon(x) = (\mathbf{z}; \mathbf{z}')$, where $\mathbf{z} = (\varphi_1(x), ..., \varphi_n(x))$ and $\mathbf{z}' = (\psi_1(x), ..., \psi_n(x))$. The corresponding object-colour solid $\Upsilon(\mathscr{X})$ is a convex subset in $\mathbf{R}^{2n}$. The Φ-object-colour solid, $\Phi(\mathscr{X})$, is the $z-$projection of $\Upsilon(\mathscr{X})$:

$$\Phi(\mathscr{X}) = \{\mathbf{z} \in \mathbf{R}^n : (\mathbf{z}; \mathbf{z}') \in \Upsilon(\mathscr{X}), \quad \mathbf{z}' \in \mathbf{R}^n\}.$$

Similarly, given an object $x_0 \in \mathscr{X}$ and its $\Phi-$colour signal $\mathbf{z}_0 = \Phi(x_0)$, the metamer mismatch volume $\Psi(\Phi^{-1}(\mathbf{z}_0))$ forms a cross-section of $\Upsilon(\mathscr{X})$; namely, $\{\mathbf{z}' \in \mathbf{R}^n : (\mathbf{z}_0;\mathbf{z}') \in \Upsilon(\mathscr{X})\}$.

To gain some intuition into the metamer sets, metamer mismatch volumes and why the metamer mismatch volume corresponds to a cross-section of the $\Upsilon(\mathscr{X})$ object-colour solid, consider the case of a monochromatic sensor system. In this 1-dimensional case, the colour maps become simply $\Phi = (\varphi_1)$ and $\Psi = (\psi_1)$, and $\Upsilon(\mathscr{X})$, their Cartesian product, becomes a convex region in 2-dimensions as shown in Figure 1. For Figure 1 the CIE 1931 $\bar{x}(\lambda)$ colour matching function has been used as the single underlying sensor class. Under illuminants $D65$ and A (spectral power distributions $p_{D65}(\lambda)$ and $p_A(\lambda)$) the corresponding colour mechanisms are then $\varphi_1 = (p_{D65}(\lambda)\bar{x}(\lambda))$ and $\psi_1 = (p_A(\lambda)\bar{x}(\lambda))$.

For a given colour signal z_0 obtained under $D65$, finding the metamer mismatch volume means determining the boundary of the set of possible colour signals z' arising under A whose corresponding reflectances would be metameric to z_0 under $D65$. The shaded region in Figure 1 shows $\Upsilon(\mathscr{X})$. Any point (z,z') inside $\Upsilon(\mathscr{X})$ represents the corresponding colour signals that would arise from a given object under illuminants $D65$ and A. As can be seen from the figure, given $z_0 = 35$, for example, all points (z_0,z') on the vertical line $z = z_0 = 35$ and lying within the shaded area arise from objects that are metameric under $D65$ and also result in colour signal z' under A. Hence the z' values from the vertical line segment lying within the shaded area make up the metamer mismatch volume for the colour signal $z_0 = 35$ under $D65$. In this example, the 'volume' degenerates to a line segment on the z' axis. The boundary of the volume is given by the z' at the intersections of the $z = 35$ line with the boundary of $\Upsilon(\mathscr{X})$ (i.e., $z' = 20.5$ and $z' = 58$).

The situation is analogous for a trichromatic colour device, but $\Upsilon(\mathscr{X})$ becomes 6-dimensional and the cross-section is defined by the intersection of a 3-dimensional affine subspace with the boundary of $\Upsilon(\mathscr{X})$. In the general n-dimensional case, evaluating the boundary of the metamer mismatch volume induced by the colour signal $\Phi(x_0)$ when switching from colour map Φ to colour map Ψ requires determining the cross-section of the boundary of the $2n$-dimensional object-colour solid $\Upsilon(\mathscr{X})$ defined by its intersection with the n-dimensional affine subspace containing $\Phi(x_0)$.

The object-colour solid is determined by its boundary—written as $\partial\Upsilon(\mathscr{X})$)—which is fully specified by those objects that map to its boundary. In the colour literature such objects are called *optimal* [10]. Schrödinger was, probably, the first to realize that the optimal spectral reflectance functions can take only two values: either 0 or 1 [8]. He claimed that for human colour vision the optimal spectral reflectance functions have the form of so-called elementary step functions. The reflectance functions

$$x_1(\lambda;\lambda_1) = \begin{cases} 0, & \text{if} \quad \lambda < \lambda_1; \\ 1, & \text{if} \quad \lambda \geq \lambda_1 \end{cases} \tag{2}$$

and

$$1 - x_1(\lambda;\lambda_1)$$

will be called the *elementary step functions of type 1*. Reflectance functions

$$x_m(\lambda;\lambda_1,...,\lambda_m) = \sum_{i=1}^{m} (-1)^{i-1} x_1(\lambda;\lambda_i) \tag{3}$$

and

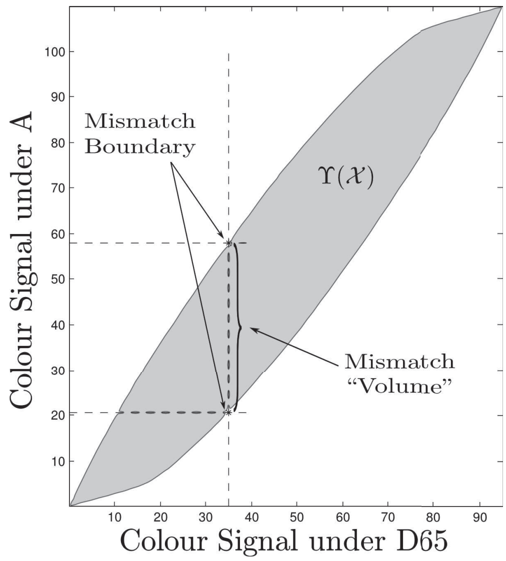

Figure 1: Illustration of metamer mismatch volume for a monochromatic colour device based on CIE $\bar{x}(\lambda)$. The shaded area indicates $\Upsilon(\mathscr{X})$, which is the set of all "colour" signal pairs arising under $D65$ and A from all possible object reflectances. The boundary of the metamer mismatch volume (two points in this example) for colour signal value 35 under $D65$ is obtained from the projection onto the vertical axis of the cross-section of $\Upsilon(\mathscr{X})$ defined by the intersection of the vertical line with the boundary of $\Upsilon(\mathscr{X})$. In this example, the colour signal $z_0 = 35$ under $D65$ could, under A, potentially take on any value in the metamer mismatch volume $z' \in [20.5, 58]$.

$$1 - x_m(\lambda;\lambda_1,...,\lambda_m)$$

where $\lambda_{\min} < \lambda_1 < \lambda_2 < ... < \lambda_m < \lambda_{\max}$, will be called the *elementary step functions of type m*, with $\lambda_1,...,\lambda_m$ being referred to as *transition wavelengths*. Schrödinger claimed that for human vision the optimal spectral reflectance functions were of type $m < 3$.

In the general case, the number of transition wavelengths may exceed the number of the colour mechanisms. Indeed, a theorem has been proved [7] showing that, for a colour map Φ with continuous spectral sensitivity functions $s_1(\lambda),...,s_n(\lambda)$, an elementary step function with transition wavelengths $\lambda_1,...,\lambda_m$ will be an optimal spectral reflectance function if $\lambda_1,...,\lambda_m$ are the only roots of the following equation

$$k_1 s_1(\lambda) + k_2 s_2(\lambda) + ... + k_n s_n(\lambda) = 0, \tag{4}$$

where $k_1, k_2, ..., k_n$ are arbitrary real numbers (at least one of which is not equal to zero).

Given another colour map Ψ with continuous spectral sensitivity functions $s'_1(\lambda), ..., s'_n(\lambda)$ and combining it with Φ to form the colour map Υ, the roots of the equation

$$k_1 s_1(\lambda) + ... + k_n s_n(\lambda) + k'_1 s'_1(\lambda) + ... + k'_n s'_n(\lambda) = 0 \tag{5}$$

will determine an optimal spectral reflectance function with respect to Υ. Let us designate it $x(\lambda;\mathbf{k},\mathbf{k}')$, where $\mathbf{k} = (k_1,...,k_n)$, and $\mathbf{k}' = (k'_1,...,k'_n)$.

Now, consider an arbitrary object x_0 mapping into the interior of the colour solid $\Phi(\mathscr{X})$, and let $\Phi(x_0) = \mathbf{z}_0$ be its colour signal. Then the boundary of the metamer mismatching volume in the colour solid $\Psi(\mathscr{X})$ will be implicitly defined by the following equation with respect to $\mathbf{k}$ and $\mathbf{k}'$:

$$\Phi\left(x\left(\lambda;\mathbf{k},\mathbf{k}'\right)\right)=\mathbf{z}_0. \tag{6}$$

As $\mathbf{z}_0$ is an interior point, $\mathbf{k}'$ cannot equal zero (if $\mathbf{k}'=0$, $x(\lambda;\mathbf{k},\mathbf{k}')$ will be an optimal spectral reflectance function with respect to Φ, thus, $\Phi(x(\lambda;\mathbf{k},\mathbf{k}'))$ will belong to the Φ-object-colour-solid boundary).

Let us consider a particular case, restricting ourselves to the CIE 1931 colour matching functions $(s_1(\lambda), s_2(\lambda), s_3(\lambda))$, and the CIE illuminants D65 $(p_{D65}(\lambda))$ and A $(p_A(\lambda))$[10]. Equation 5 then takes the form

$$\begin{aligned}(k_1 s_1(\lambda)+k_2 s_2(\lambda)+k_3 s_3(\lambda))\, p_{D65}(\lambda)\\ +\left(k_1' s_1(\lambda)+k_2' s_2(\lambda)+k_3' s_3(\lambda)\right) p_A(\lambda)=0.\end{aligned} \tag{7}$$

As shown elsewhere [6], for the CIE 1931 colour matching functions the optimal stimuli for the Φ- and Ψ-object-colour solids have the form of elementary step functions of type $m<3$ (in accord with Schrödinger's conjecture). From computational testing (i.e., random choices of the 5 transition wavelengths always led to solutions to equation (7)) it appears that the optimal stimuli for the Υ-object-colour solid have the form of elementary step functions of type $m<6$. Therefore, given a point $\mathbf{z}_0=(z_1,z_2,z_3)$ in the object-colour solid $\Phi(\mathscr{X})$, the boundary of the metamer mismatch volume in the colour solid $\Psi(\mathscr{X})$ will be implicitly defined by the following equations with respect to the transition wavelengths $\lambda_1,...,\lambda_5$:

$$\begin{aligned}\varphi_1\left(x_5\left(\lambda;\lambda_1,...,\lambda_5\right)\right)&=z_1,\\ \varphi_2\left(x_5\left(\lambda;\lambda_1,...,\lambda_5\right)\right)&=z_2,\\ \varphi_3\left(x_5\left(\lambda;\lambda_1,...,\lambda_5\right)\right)&=z_3.\end{aligned} \tag{8}$$

Given $\Psi(x_0)=\left(z_1',z_2',z_3'\right)$, let us introduce the polar coordinate system (ρ,β,γ) in the Ψ-subspace with the origin at $\Psi(x_0)$. Let $x_5(\lambda;\lambda_1,...,\lambda_5)$ satisfy Eq. 8. Then we have

$$\begin{aligned}\psi_1\left(x_5\left(\lambda;\lambda_1,...,\lambda_5\right)\right)-z_1'&=\rho\cos\beta\sin\gamma,\\ \psi_2\left(x_5\left(\lambda;\lambda_1,...,\lambda_5\right)\right)-z_2'&=\rho\sin\beta\sin\gamma,\\ \psi_3\left(x_5\left(\lambda;\lambda_1,...,\lambda_5\right)\right)-z_3'&=\rho\sin\gamma.\end{aligned} \tag{9}$$

Taken together, equations 8 and 9 define a two-dimensional manifold. Indeed, for each choice of β and γ, equations 8 and 9 can be uniquely resolved with respect to $\lambda_1,...,\lambda_5$, and ρ (provided that the corresponding Jacobi matrix is not singular). Therefore, equations 8 and 9 implicitly define a function $\rho(\beta,\gamma)$ which determines the metamer mismatch volume boundary induced by the point $\Phi(x_0)$. In other words, given β and γ, we have six equations in 6 unknowns. Whenever these equations have a solution, the solution provides the precise location of the metamer mismatch volume's boundary in the direction (β,γ).

Figure 2 illustrates the situation for a dichromatic sensor system.

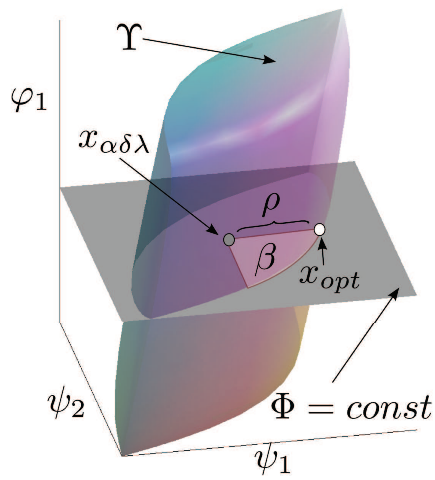

Figure 2: Illustration of the boundary of the metamer mismatch volume described by $\rho(\beta)$ relative to the origin $x_{\alpha\delta\lambda}$ for the case of a dichromatic system. Given two sets of colour mechanisms $\Phi=(\varphi_1,\varphi_2)$ and $\Psi=(\psi_1,\psi_2)$ (e.g., Ψ may simply be Φ under a different illumination), then $\Upsilon(x)=(\varphi_1(x),\varphi_2(x);\psi_1(x),\psi_2(x))$. In the figure, only 3 of the 4 dimensions of the object-colour solid, $\Upsilon(\mathscr{X})$, are shown. For a given $\Phi=(\varphi_1,\varphi_2)$, the boundary of its metamer mismatch volume is defined by the intersection of the plane it defines with the boundary Υ. To describe that boundary, an origin for the polar coordinate system is situated at $x_{\alpha\delta\lambda}$ from which the metamer mismatch volume is represented in terms of polar angle β and distance ρ to points $x_{opt}=x_3(\lambda;\lambda_1,...,\lambda_3)$ on the boundary of Υ. In this 4-dimensional case, the optimal functions on the boundary of Υ now are 3-transition functions. For a given β, the transition wavelengths of $\lambda,...,\lambda_3$ of x_{opt} are found by optimization under the constraint that it must lie on the plane defined by Φ.

Calculating Metamer Mismatch Volumes

Equations 8 and 9 define the metamer mismatch boundary. Any method of solving them will suffice. The following describes one approach that has been implemented in Matlab. To solve equations 8 and 9 for $\rho(\beta,\gamma)$, we need to choose the origin of the polar coordinate system so as to define β,γ, and ρ. The difficulty is that the origin must lie inside the metamer mismatch volume, but we do not yet know what that volume is. To solve this problem, any reflectance that is metameric to $\mathbf{z}_0=(z_1,z_2,z_3)$ under Φ will suffice; however, it is particularly convenient to use a rectangular reflectance function from Logvinenko's object-colour atlas [6, 5] because the elements of that atlas are invariant to the illuminant.

For any given point $\mathbf{z}_0=(z_1,z_2,z_3)$ in the object-colour solid $\Phi(\mathscr{X})$, its $\alpha\delta\lambda$ coordinates in the colour atlas specify a rectangular reflectance spectrum $x_{\alpha\delta\lambda}$ that is a linear combination of an elementary step function of type $m<3$ and $x_{0.5}(\lambda)=0.5$. This reflectance spectrum is by construction metameric to $\mathbf{z}_0$. The point $\Psi(x_{\alpha\delta\lambda})$ is non-optimal with respect to $\Upsilon(\mathscr{X})$) and is therefore guaranteed to lie within the metamer mismatch volume and can be used as the origin $\Psi(x_0)=\left(z_1',z_2',z_3'\right)$.

Determining $\rho(\beta,\gamma)$ proceeds in two steps. Given (β,γ) the first step is the more difficult one and involves finding the optimal 5-transition step function $x_{opt}=x_5(\lambda;\lambda_1,...,\lambda_5)$ metameric to $\mathbf{z}_0$ such that $\Psi(x_5(\lambda;\lambda_1,...,\lambda_5))$ lies in the direction defined by (β,γ). The second is then simply to calculate ρ directly using x_{opt} from the first step. As expressed in equations 8, any optimal 5-transition step function $x_{opt}=x_5(\lambda;\lambda_1,...,\lambda_5)$ metameric to $\mathbf{z}_0$ under the colour map Φ is guaranteed to lie on the surface of the mismatch volume.

To find a 5-transition spectrum $x_{opt}=x_5(\lambda;\lambda_1,...,\lambda_5)$ that is metameric to z_0 according to the mapping ϕ, requires minimizing the following objective function formed as the sum of two error measures,

$$E(x_{\text{opt}}) = E_{\text{XYZ}}(x_{\text{opt}}) + E_{\beta\gamma}(x_{\text{opt}}).$$

The first term corresponds to the constraints provided by equations 8 and is

$$E_{\text{XYZ}}(x_{\text{opt}}) = \left\| \phi(x_{\text{opt}}) - z_0 \right\|.$$

The second term ensures that the 5-transition spectrum lies in the desired direction under ψ and is defined by

$$E_{\beta\gamma}(x_{\text{opt}}) = \arccos\left(\frac{\hat{u} \cdot (\psi(x_{\text{opt}}) - z_0')}{\|\hat{u}\| \left\| \psi(x_{\text{opt}}) - z_0' \right\|} \right),$$

where $\hat{u} = (sin(\beta)cos(\gamma), sin(\beta)sin(\gamma), cos(\beta))$ is the unit vector in the direction given by (β, γ).

This problem can be solved using standard optimization tools available for Matlab. The variables are the transition wavelengths $\lambda_1, ..., \lambda_5$. Additional constraints are added to ensure the correct order of the transition wavelengths (i.e., $\lambda_1 \leqq \lambda_2 \leqq ... \leqq \lambda_5$) and to limit them to the range of the spectrum on which the colour mechanisms are defined. Random transition wavelengths are selected as starting points. Selecting random starting values from Logvinenko's reparameterized wavelength domain [6] works better than from the original wavelength specification since it leads to a more even distribution across the mismatch volume surface.

Once x_{opt} has been found, ρ can be directly calculated as

$$\rho = \left\| \Psi(x_{opt}) - \Psi(x) \right\|.$$

The above method means that the metamer mismatch boundary can be precisely computed as the distance $\rho\ (\beta, \gamma)$ from the chosen origin $\Psi(x_{\alpha\delta\lambda})$ to the boundary in any given direction as specified by the angles β and γ. To model the entire boundary, one possibility is to step through values of β and γ and thereby obtain a regular, albeit non-uniform, sampling of the boundary. However, this is not necessarily the best way to proceed, both because of the computation required for the optimization and the fact that the origin $\Psi(x_{\alpha\delta\lambda})$ is only guaranteed to lie within the mismatch volume, not necessarily near its center. An alternative to a regular sampling of the angles is to generate a large number of random points over $\partial\Upsilon(\mathscr{X})$. The optimization is then to minimize $\left\|\Phi(x_{opt}) - \Phi(x)\right\|$. This eliminates the angular term involved in $E_{\beta\gamma}(x_{\text{opt}})$ and significantly speeds up the calculation, but has the disadvantage that the resulting points are not necessarily uniformly distributed over the mismatch volume boundary.

Metamer mismatch volumes can be a useful tool for illustrating the effect of an illumination change. Figure 3 shows the mismatch volumes and their distribution in the object colour solid for 100 Munsell chips after an illumination change from D65 to A, using the CIE 1931 2-degree standard observer.

Conclusion

Evaluation of metamer mismatch volumes is an important long-standing problem in colour science. Although some methods [9, 4, 2, 3, 10, 1] have been proposed for finding a solution for various particular cases (e.g., for some special subsets of object reflectances), no general solution has been proposed previously. Here we have outlined an approach to this problem in its full generality. Specifically, we show how to evaluate the boundary of the metamer mismatch volume without imposing any restrictions on the objects considered. The theory has also been developed into an algorithm implemented in Matlab.

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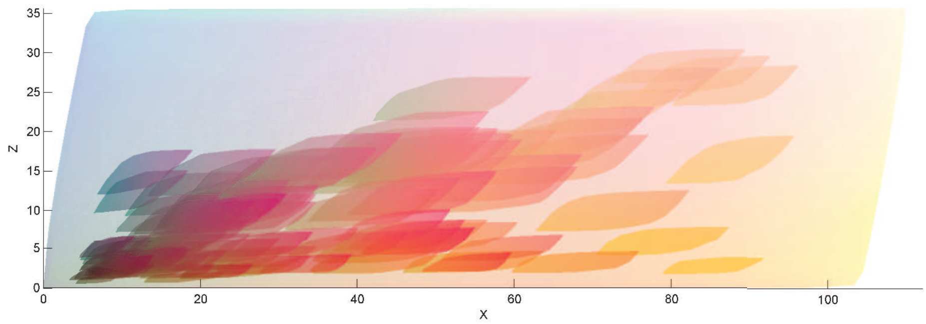
Author Biography

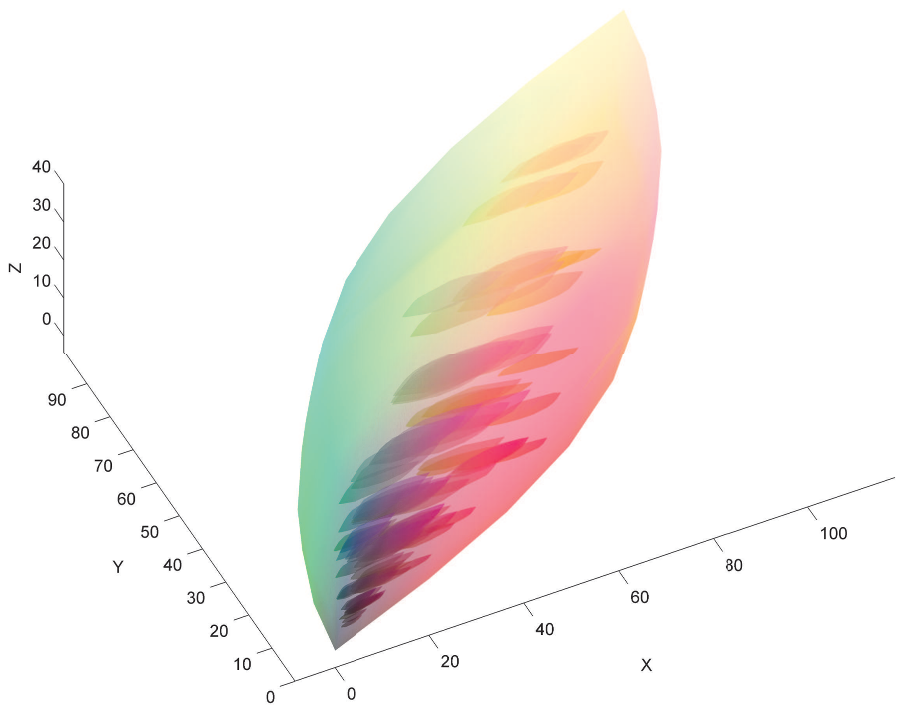
Alexander Logvinenko received his BS in psychology (1972) and in applied mathematics (1979), and his PhD in psychology (1974) from the Moscow State University, Russia. He worked in the Psychology Department of the Moscow State University (1975-1992), and at Queen's University of Belfast (1993-2004). Now he is Professor of Vision Sciences at the Glasgow Caledonian University. His research interests are in colour theory, colour vision, and psychophysics.

Christoph Godau received his BS in physics (2008) from the Ruhr-University Bochum, Germany and his MS in colour science (2010) from the University of Saint-Etienne, France and the University of Eastern Finland. He is currently a research engineer at the KU Leuven, Belgium, working on the inference of material properties from images.

Brian Funt is Professor of Computing Science at Simon Fraser University where he has been since 1980. He obtained his Ph.D. from the University of British Columbia in 1976. His research focus is on computational approaches to modelling and understanding color.



(a) The object-colour solid (transparent) with the mismatch volumes projected onto the XZ-plane.



(b) A rotated view.

Figure 3: The mismatch volumes and their distribution in the object-colour solid. The volumes appear to be similar in shape to the object colour solid. The largest volumes occur near gray, and they become smaller nearer the object-colour solid surface.