# Why We Don't Know How Many Colors There Are 

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#### Abstract

There have been many attempts to answer the question of how many distinct colors there are, with widely varying answers. Here we present an analysis of what it would take to arrive at a reliable answer and show how currently available models fail to make predictions under the wide range of conditions that needs to be considered. Gamut volumes are reported for a number of light sources and viewing modes and the conclusion is drawn that the only reliable data we have comes from psychophysical work. The color gamut of the LUTCHI data in CIECAM02 is therefore shown as an alternative to the gamut of all possible colors.


## Introduction

The question of how many colors there are has occupied enquiring minds for a long time and has been a tempting topic to apply color science methods and models to. Key papers here are those of Schrödinger [1] and Maric-France and Foster [2] who posited the Object Color Solid as encompassing all possible surface colors, Pointer [3], who measured a set of natural surfaces and whose findings have been the benchmark ever since, McCann [4] who emphasized the importance of color spaces in gamut computation, Morovic et al. [5] who illustrated how color gamuts change with viewing conditions, Inui et al. [6] who showed how the SOCS database - a new set of measured ('real') samples - compares to Pointer's data, Heckaman et al. [7] who demonstrated the existence of new colors that come about when illuminant and adapted white are decoupled and Linhares et al. [8], whose estimate of the color volume occupied by natural scenes and the OCS in CIELAB, again showed similar results to Pointer's findings.

Instead of being after a number ('How does 16.8 million sound?'), the aim of this paper is to present a framework for answering the question and to show that the question is either unanswerable or that its answer is the expression of predictions made by models under conditions for which they were never intended and under which they have no empirical basis or known predictive powers. Nonetheless the limits of color are relevant both due to their intrinsic scientific interestingness and due to their value in engineering decision processes as a means of judging the approach of diminishing returns.

The remainder of this paper will present a framework for how to count all possible colors 'by hand', introduce a computational and modeling based solution, analyze its predictions and conclude with thoughts on the limitations of gamut computation and appearance prediction.

## What does 'all possible colors' mean?

In order to address the question of how many colors there are, it is necessary to define what is meant by the term color. The CIE system of 1931 provided a method for specifying a color using a trichromatic system that takes into account the spectral properties of light reflected or emitted by an object and the spectral response of the human visual system. A typical RGB display device accepts in excess of 16 million unique color-channel inputs $\left(\left(2^{8}\right)^{3}-8\right.$ bits per each of 3 channels), potentially each with a unique trichromatic specification. However, some studies have suggested that our visual system
can only distinguish about 2 to 10 million colors (e.g. [9-11]). The set of $\sim 16$ million unique stimuli (according to the CIE system) that an RGB imaging device can generate defines the tristimulus gamut of the device. The lower estimates for the number of colors suggests that some of these colors are visually indistinguishable (and this could depend upon factors such as the gamma of the display).

The suggestion that color is essentially the property of an object does not explain the many so-called, known color illusions. For example, it is well known a gray patch displayed on a black background will look brighter than a physically identical gray patch on a white background. This 'illusion' occurs because color is a perceptual response. Our visual system does not operate like a device that records and reproduces internally a replica of what it is presented with. Rather, the purpose of the visual system is to extract information from the environment and to create an internal representation. The 'illusion' of the identical gray patches described above occurs precisely because the visual system computes spatial contrast in what it views, which helps it to extract invariant information in its environment [12].

The phenomenon of spatial contrast indicates that color is not something that can be represented by a one-to-one mapping. Rather, color depends upon context. However, spatial contrast is only one way that color varies. The process of how the visual system perceives color is complex; not least, there is an opposite effect to spatial contrast in which color tends to be more similar to the background. It is known as White's effect [13] or, generally, assimilation. Furthermore, color also depends upon temporal contrast or after-image effects [14], spatial frequency [15], stimulus size including the factors of viewing angle and viewing distance [16], etc. It should now become clear that although the number of distinguishable colors is significantly lower than the number of physically different stimuli, the perceptual color gamut may be much larger than the gamut of physically-distinct stimuli. For example, one might imagine that the most red that can be obtained on a display would be where the red primary is maximally and uniquely activated (e.g. RGB $=\left[\begin{array}{lll}255 & 0 & 0\end{array}\right]$ ). However, it may be possible to generate an even more colorful red by displaying the 'full primary' red on a green background, or after having viewed a scene under a greenish light source.

## Counting all possible colors 'by hand'

To consider counting all possible colors, we could imagine someone laying out infinitely many colored cards, having spectral properties that represent all variation in the visible range, on a uniformly gray surface. Next, the observer would group together those that are indistinguishable, and then counting the resulting groups. Unfortunately, this exercise would only tell us about how many colors there are on the grey background and when viewed from a certain distance, in a certain sequence, under a certain light source, etc. Changing the configuration (e.g. background) or the viewer's adaptation state (e.g. by looking at a different color beforehand) would change some or all of the color experiences that this set of cards give rise to. Therefore, given the complexity of the context, the
prospect of counting the total number of indistinguishable colors is practically impossible.

One might interpret this impossibility as being due to the large number of factors involved. For example, Fairchild [17] argued that the generally accepted estimate of 10 million unique distinguishable colors are for a single viewing condition and for a particular observer. The change of viewing conditions and observers will lead to different / new distinguishable colors and, therefore, the number of possible colors is infinity. However, this argument may be too extreme, given the tendency of the visual system to saturate.

This can be illustrated using the following thought experiment: Given three colors along a line in color space: $\mathrm{A}, \mathrm{B}$ and C , let A be just distinguishable from B and B just distinguishable from C under some viewing conditions. If the viewing conditions change, $\mathrm{B}^{\prime}$ (the color of the stimulus having color B under the original viewing conditions) may either end up remaining indistinguishable from $\mathrm{A}^{\prime}$ and $\mathrm{C}^{\prime}$ or end up matching one or both of them. This new set of between 1 and 3 colors may in turn be either distinct from the original colors A, B and C or match some of them (Figure 1). Therefore, 1000 different viewing conditions do not necessarily correspond to 1000 new / unique / distinguishable colors. How many new colors they give rise to is the question that needs to be answered though to know how many distinct color appearances can be had.


Figure 1: Impact of viewing condition change on number of distinguishable colors.

## Computationally predicting all possible colors

As shown above, there are a myriad ways in which an ecosystem can vary that result in varying color experiences. Since even a coarse sampling of all combinations of varying these degrees of freedom psychophysically is vastly impractical, the approach taken here is to computationally predict some of them. Since the most well-tested and predictively accurate model of color appearance available today is CIECAM02 [18], the focus here will be on the viewing ecosystem parameters that it bases its predictions on, which will already limit the comprehensiveness of the resulting, total gamut volume.

Since CIECAM02 predicts color appearance attributes from stimulus colorimetry, adapted white, degree of adaptation, background luminance, luminance of the adapting field and surround type, they will be varied with the aim of expressing their effect on predicted gamut. Figure 2 shows these model inputs and means for varying each one of them.


Figure 2: CIECAMO2 inputs and means of sampling them.

The sets of appearance attributes that will result from varying CIECAM02 inputs need to have a gamut boundary wrapped around them, for which purpose the convex hull method will be used here. While this is not an entirely robust choice in CIECAM02, it would be fully justified in CIEXYZ, since there the convex combination of any pair of stimuli can be obtained by additive, optical mixing. Such a convex gamut may then need to be transformed to CIECAM02, but since it may be insufficient to just re-express the hull (due to the nonlinearity of the CIECAM02-CIEXYZ relationship), a sampling of the interior too would need to be involved and the resulting CIECAM02 coordinates would need to be described using nonconvex techniques. Bearing in mind that we can at best hope for an approximation of the total gamut of all color experiences, the convexity shortcut will be used in spite of the above limitations.

With the two pieces of sampling CIECAM02 inputs and describing the gamuts of the resulting appearance attributes in place, the effect of the various model parameters can be explored.

## Results

First, the gamut of all possible Lambertian surfaces under D50 can be computed by independently varying spectral reflectance across the visible spectrum (here 16 bands were used), lighting them with the D50 light source viewing them under standard viewing conditions (ISO 3663 - average surround, $20 \%$ background, $\sim 60 \mathrm{~cd} / \mathrm{m}^{2}$ luminance of adapting field). The resulting, "D50 vanilla," color gamut has a volume of $3.8 \times 10^{6} \mathrm{Jab}$ units ( 3.8 MJab ) where J is predicted lightness and a and b are orthogonal coordinates corresponding to chroma (C) and hue (h) predictions. This gamut volume relates to the number of just noticeably different colors it encloses and while it does not equal it, since it says how many (partial) cubes it contains, as opposed to how many unit spheres it can be packed with, it will be used here as an approximation. That the two approaches give similar results has been shown before [19].

Changing the surround type to dim reduces gamut volume to 3.5 MJab while viewing the stimuli in a dark surround
results in 3.0 MJab. Worth noting is also that the union of these three gamuts (that differ only by surround type) matches the gamut of the average condition (within $0.35 \%$ ). In other words, viewing the stimuli in dim and dark surrounds gives rise to subsets of the color experiences accessible under in an average surround.

The next aspect of the ecosystem that can be varied is the light source under with surfaces are viewed. Here, changing from D50 to F11 leads to a gamut with 4.2 MJab volume - i.e., the range of colors under F11 is close to $10 \%$ greater than under D50. Here, however, the F11 gamut is not a superset of D50 and considering the union of these two gamuts gives a volume of 4.4 MJab, which exceeds the F11 gamut alone by $6 \%$. Adding the gamut under illuminant A (which by itself is 3.5 MJab under the standard conditions), leads to a total gamut of 4.5 MJab.

Extending this exploration beyond standard and recommended illuminants to freely varying their spectral power across the visible spectrum (in 5 bands in this case, giving 242 synthetic light source spectra) leads to the following view of available color gamut as a function of illuminant chromaticity (Figure 3).

What can be seen straight-away are some outliers for which CIECAM02 dramatically predicts color gamuts with $10^{11}$ volumes in Jab space - i.e., 100000 times the gamut volume of all possible surfaces under D50. Here we are still only considering the viewing of all possible reflective, diffuse surfaces under a single illuminant at a time to which an observer is fully adapted. The result of a five orders of magnitude increase just does not agree with experience and is a case of the CIECAM model, which is derived based on psychophysical data from under a small set of conditions being applied to viewing conditions under which it has no data on which to base its predictions and under which it has never been tested. Instead of the $10^{11}$ gamut volume being about color gamuts, it highlights the limited applicability of CIECAM02. The same considerations apply to any other color appearance model though and we arrive back to the point where psychophysical quantification of all possible colors was declared impractical earlier on. Instead of fundamentally solving the challenge of predicting the number of all possible colors, CAMs just shift the challenge to the basis on which they themselves are built. The difficulty of viewing all possible visual ecosystems therefore remains even when attempts are made to answer this questions computationally, since we come up against the realization that our models are constrained in applicability and all we can hope for is to effectively interpolate among the psychophysical data they are based on (and to attempt extrapolation in their close neighborhood).



Figure 3: Gamut volume in Jab as a function of synthetic light source chromaticity. (a) Light source SPD, (b) gamut volume histogram, (c) light source chromaticity and (d) gamut volumes across chromaticity.

(a)


Figure 4: Gamut volume in Jab as a function of measured light source chromaticity: (a) light source SPD, (b) gamut volume histogram, (c) light source chromaticity (crosses) and (d) gamut volumes across chromaticity.

With the above in mind, the initial computation can be changed to use a set of 173 measured light sources instead of the synthetic ones, which leads to a total gamut of 6.6 MJabs (i.e. taking the color gamuts of the freely varying surface reflectances under each of the light sources shown in Figure 4 and computing their union). In other words, the surfaces, which under D50 have a gamut with 3.8 MJab volume, result in close to double that range of colors when viewed under a variety of light sources, one light source at a time. One interesting result to observe is that the gamut volumes depend on chromaticity and not on whether the light sources are spectrally synthetic or represent physically-realizable light emitters.

Zooming in on the gamut volumes for measured light sources, there is a clear relationship between correlated color temperature and gamut volume, with higher CCT (i.e., 'bluer' light sources) giving rise to a greater number of colors (up to 6 MJab) than lower CCT ones (down to 3 MJab ) by a factor of two (Figure 5).

Viewing the D50 versus 173 measured light source color gamuts in CIECAM02 (Figure 6) shows that the latter tends to the shape of an almost perfect cylinder, while the former has a shape that relates to the D50 illuminant's spectral makeup.


Figure 5: Gamut volumes as function of chromaticity (same data as Figure 3c).


Figure 6: Gamut volumes as function of chromaticity (same data as Figure 3c).

Applying CIECAM02 to cases where the adopted white is of lower luminance than the perfect diffuser also leads to much increased color gamut volumes as has been showed before [7] (e.g., pushing the idea to reducing the luminance of the adopted white to $5 \%$ yields a gamut volume of 900 MJab under D50 as opposed to $3.8-\mathrm{a}>200 \mathrm{x}$ increase). While there is an effect of adaptation state on stimulus perception and being presented with stimuli with luminances above the adopted white does lead to new color experiences, whether this prediction (which is not based on psychophysical data) is reliable is questionable. It should also be noted that CIECAM02 does not include many complexities of colour vision such as contrast effects.

Using CAM (any CAM) to say something about all possible colors therefore results in reliably talking about the color gamuts of color appearances they are derived from. In the case of CIECAM02, it is the LUTCHI data [20], whose color gamut has a volume of 1.7 MJab and is shown in Figure 7. Anything else is mistaking the mathematical constructs used for making psychophysics-based predictions for the nature of stimulus-appearance relationship in general.


Figure 7: Color gamut of LUTCHI data set in CIECAMO2.

## Conclusions

All we can reliably say based on the data available to date is that there are at least around 1.7 million colors, since the 5828 stimulus color attribute magnitude estimations span a gamut of that volume. To go beyond this type of number would require either having a model of color appearance that closely mimics the human visual system and that can therefore make predictions of its responses for arbitrary inputs, or to extend the psychophysical basis available to color appearance models today. In the absence of the above, the constraints on answering the question of how many colors there are, are the same as wanting to answer it 'by hand.'

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