Surface Reflectance Models Based on Characteristic Functions

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Abstract

Surface reflectance functions (SRFs) and spectral power distributions (SPDs) of illuminants are typically modeled as elements in an N-dimensional linear function subspaces. Each SRF and SPD is represented by an N-vector and the mapping between SRF and SPD functions and an N-dimensional vector assigns N-dimensional "color" codes representing surface and light information. The N basis functions are chosen so that SRFs and SPDs can be accurately reconstructed from their N-dimensional vector codes. Typical rendering applications assume that the resulting mapping is an isomorphism where vector operations of addition, scalar multiplication and component-wise multiplication on the N-vectors can be used to model physical operations such as superposition of lights, light-surface interactions and inter-reflection. When N is small, this implicit isomorphism can fail even though individual SPDs and SRFs can still be accurately reconstructed by the codes. The vector operations do not mirror the physical. However, if the choice of basis functions is restricted to characteristic functions (*that take on only the values 0 and 1) then the resulting map between SPDs/SRFs and N-vectors is an isomorphism that preserves the physical operations needed in rendering. The restriction to bases composed of characteristic functions can only reduce the goodness of fit of the linear function subspace to actual surfaces and lights. We will investigate how to select characteristic function bases of any dimension N (number of basis functions) and evaluate how accurately a large set of Munsell color chips can approximated as a function of dimension.*

Introduction

With the development of multimedia technology increasingly realistic representations of actual and virtual scenes become possible. In a typical application we specify surface and light sources in a scene and then model light-surface interactions. The resulting image is only as accurate as the information about the spectral properties of light and surface used to describe the contents of the scene. Most rendering applications use simple RGB 3-channel codes to represent the spectral properties of lights (SPDs) and surface reflectance. They model interaction by addition and component-wise multiplication of these 3-vectors. These mathematical operations do not correspond to actual light surface interactions. In any case, it is not clear exactly what such an RGB code represents about a surface or light and there are likely advantages of using higher-dimensional representations.

To reproduce color with hi-fidelity, recent color systems are going 4-channel LED, 5-channel DLP, 8-channel printer and so on. The resulting color representations are potentially more accurate in representing colors once some rule is specified for how code maps to display.

Typical approaches to modeling spectral information includes developing subspace models for lighting [1], [2], surface reflectance [3], [4], [5], [6] and using these models to model human perception [7], [8]. A typical "color constancy" algorithm is a method that models lights and surfaces by subspace models and attempts to recover the basis coordinates of the surfaces independent of those of the illumination [6].

Most conventional methods of modeling used in this literature involve criteria for selection of basis functions and assigning weights to surfaces and lights [2], [3], [9]. These models can reproduce spectral information to any desired accuracy by increasing the number of basis functions used (the dimension N of the model) and there is no need to stop at N=3. A central focus of this research concerns how accurately such linear models capture surface reflectance or illumination as a function of the number of basis elements. Previous results show that we can approximate surface reflectance very accurately using a computation method with eight or more basis functions [3].

Once the eight basis functions are chosen we can in effect replaced each physical surface and light by its model representation and that representation can be specified by specifying 8 numbers, its coordinates with respect to the basis function. That is, the relevant spectral information about a surface or natural illumination can be captured by a vector of eight numbers, a modest increase over the three numbers used in typical rendering applications. Moreover, these eight numbers have a known relation to the physical surface represented; an accurate reconstruction of the surface reflectance given the eight numbers is possible. If the demands on rendering do not require so accurate a representation we can represent surfaces by lower-dimensional models and still capture useful information about surfaces and lights.

However, although we can represent light and surface by N-dimensional vectors the typical rendering operations of addition and multiplication of vectors do not correctly model corresponding physical operations. If light $(\varepsilon_1,\cdots,\varepsilon_n)$ is absorbed and emitted by surface $(\sigma_1,\cdots,\sigma_n)$, the emitted light does not in general have the spectral power distribution corresponding to $(\varepsilon_1\sigma_1,\cdots,\varepsilon_n\sigma_n)$, at least for most choices of basis elements.

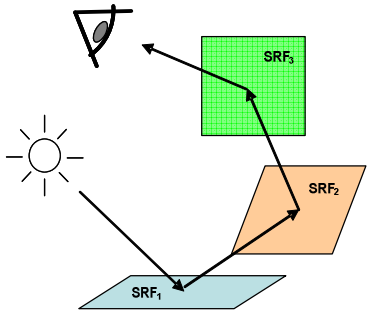


Fig. 1. Example of the effect of a secondary illuminant with three surface reflectance factors.

Alternatively put, even though a particular illuminant and a particular surface are accurately represented by a common linear model, the secondary illumination that results when the first illuminant is absorbed and reflected by the surface need not have any close representation in the model. Figure 1 shows an example of secondary illuminant. Even if a basis of a particular illuminant and SRF_1 is the same fortunately, light reflected on SRF_3 can not model as $(\varepsilon_1\sigma_1,\cdots,\varepsilon_n\sigma_n)$ when a basis function of SRF_2 or SRF_3 is different from that of SRF_1.

If, however, the choice of basis functions are restricted to *characteristic functions* (defined below) then light-surface interaction is correctly represented by component-wise multiplication of N-vectors representing light and surface. The isomorphism between the N-dimensional function space and the N-dimensional real function space preserves addition, scalar multiplication and component-wise multiplication.

A characteristic function is a function that takes on only the values 0 or 1. It is typically used in mathematics to represent a subset of the domain of the function which in this article will always be the electromagnetic spectrum. We confine attention a constrained set of basis elements each of which is the characteristic function for an interval (λ_1,λ_2) in the visible spectrum. The characteristic function is 1 for wavelengths $\lambda\in(\lambda_1,\lambda_2)$ and otherwise 0. The characteristic functions in a basis must be orthogonal precisely when their intervals are non-overlapping. We use the same basis to represent both lights and surfaces. It is easy to show that typical rendering operations of vector addition and multiplication then correspond to physical superposition of lights or surfaces and light-surface interactions. With characteristic function bases, it is easy to show that, if light $(\varepsilon_1,\cdots,\varepsilon_n)$ is absorbed and emitted by surface $(\sigma_1,\cdots,\sigma_n)$, the emitted light does have exactly the spectral power distribution corresponding to $(\varepsilon_1\sigma_1,\cdots,\varepsilon_n\sigma_n)$. Light-surface interaction is mimicked by component-wise multiplication of color codes. Scalar multiplication and addition are also preserved as they would be for any isomorphism between vector spaces.

The key questions are, (1) how do we assign weights (color codes) to lights and surfaces with respect to a characteristic function basis and (2) how accurately can an N-dimensional characteristic function basis capture naturally-occurring lights and surface *and* their interactions? We first propose a Min & Max rule to determine wavelength for three basis functions. We focus on evaluation of characteristic function models for a large set of color reference surfaces, the Munsell Color Chips [10].

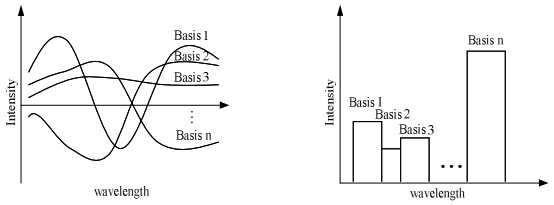
Surface Reflectance Based on Characteristic Functions

In general, color response ρ is described as integral of an ambient illuminant $E(\lambda)$, the surface reflectance of the object $S(\lambda)$, and photoreceptors $R_k(\lambda),\ k=1,2,3$. They are as follows:

$$\rho_k=\int_{\min}^{\max}S(\lambda)E(\lambda)R(\lambda)_k\,d\lambda\ \ (k=1,2,\cdots,m). \tag{1}$$

This equation can be denoted in the vector form,

$$\mathrm{P_k}=\mathrm{SER_k}(k=1,2,\cdots,m) \tag{2}$$



(a) (b)

Fig. 2. Examples of basis function with *n* parameters; (a) conventional basis function and (b) characteristic function.

Here, if we know the color stimulus obtained under any illuminant, we could estimate the surface reflectance of the object by the inverse matrix, as computational color constancy. Previous studies tried to model without an error using a linear [3], [4], [6] or non-linear model [9].

However, conventional basis functions can be biased by themselves because they are extracted the population. And also, it faces to the difficulty problem when implementing to surface-light interaction in virtual reality [11]. So, we propose a novel basis function intended to reproduce surface-light interaction In general, the surface reflectance with a basis function is follow,

$$\mathrm{S}=\sum_{i=1}\mathrm{a_i u_i}(i=1,2,\cdots,n) \tag{3}$$

where $\mathrm{u_i}$ is the principal component (or basis function) a function of wavelength. Examples of conventional basis functions and proposed characteristic functions are shown in Figure 2. Conventional basis functions have a negative or positive intensity and cover full wavelength in each basis while characteristic model does cover not the entire wavelength and basis function has a variable wavelength I_i as follow,

$$\mathrm{u_i}=\begin{cases}1 & \lambda\in I_i\\ 0 & \lambda\notin I_i\end{cases} \tag{4}$$

Therefore, we can represent arbitrary function as,

$$f(x)=\alpha\,\mathrm{u_1}+\beta\,\mathrm{u_2}+\cdots+\gamma\,\mathrm{u_n} \tag{5}$$

This is because the each coordinate is orthogonal and non-overlapping in wavelength. If there are two surface reflectances,

$$f_1(x)=\alpha_1\,\mathrm{u_1}+\beta_1\,\mathrm{u_2}+\cdots+\gamma_1\,\mathrm{u_n} \tag{6}$$

and.

$$f_2(x)=\alpha_2\,\mathrm{u_1}+\beta_2\,\mathrm{u_2}+\cdots+\gamma_2\,\mathrm{u_n} \tag{7}$$

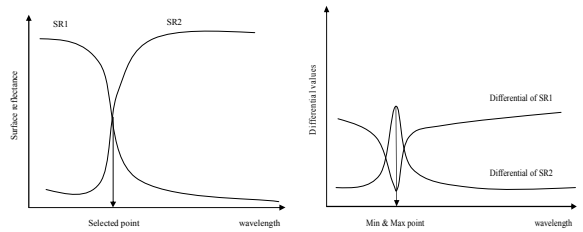
We can add and multiply each as follow,

$$f_1(x)+f_2(x)=(\alpha_1+\alpha_2)\,\mathrm{u_1}+(\beta_1+\beta_2)\,\mathrm{u_2}+\cdots+(\gamma_1+\gamma_2)\,\mathrm{u_n} \tag{8}$$

and.

$$f_1(x)\times f_2(x)=(\alpha_1\times\alpha_2)\,\mathrm{u_1}+(\beta_1\times\beta_2)\,\mathrm{u_2}+\cdots+(\gamma_1\times\gamma_2)\,\mathrm{u_n} \tag{9}$$

From these factors, we can easily calculate the scalar value without spectra if we have basis functions with an orthogonal and non-overlapping in wavelength. To select the optimal basis functions, first of all, we have to determine which selection of intervals is optimal. Here, we face to some difficulties. It is impossible to survey for entire data because there are so many spectra in a real world. Fortunately, we have much information for surface reflectance (SR) and measured lights. In special, we might suppose that Munsell Color Chips are the sub-population from spectra of a real world. From now, we will make the characteristic function using Munsell Color Chips even if real spectra and Munsell chips have differences in some ways. We have the assumption that surface reflectance is continuous and differentiable. It is plausible that the surface reflectance functions of Munsell chips satisfy above assumptions.



(a) (b)

Fig. 3. Examples of surface reflectance and its differential: (a) surface reflectance with single rapid change and (b) its differential.

Min & Max rule for characteristic function

Intuitively, places where SSRs change rapidly or where they achieve minima or maxima provide useful information about many physical surfaces. For example, if there is only a single rapid change about 480 nm in arbitrary surface reflectance, its color might be a kind of blue or yellow depending on the direction of change. Two surface reflectances and their differential values are shown in Figure 3.

These SSRs are roughly complementary in color even though they both change rapidly at the same point. If the derivative has a minimum at the selected wavelength is a minimum v, we can assume it is a kind of blue color. If the selected wavelength is a maximum of the derivative, it might be a kind of yellow color. If there is, of course, no or little change in wavelength, it would be a gray, white, or black.

What if there are two rapid changes in wavelength? In this case, we are able to expect to its color using the position of two wavelengths with minimum and maximum values. This is a basic concept of a Min and Max rule. If there is an arbitrary surface reflectance with both minimum value in 480 nm and maximum value in 560 nm, we can assume that it might be magenta color. If it also changes the position with minimum value to the position with maximum value, it might be green color. Now we can obtain both minimum and maximum values from a differential result of surface reflectance and that will be a characteristic function model with N=3 dimension.

Figure 4 shows a histogram of two wavelengths with the minimum and maximum values for each surface reflectance. The maximum frequency is located at the point with about 480 nm and 550 nm. However, the color difference should be also used to consider for human perception. To calculate color difference, we used a D65 illuminant and CIE 1931 standard colorimetric observer (2°).

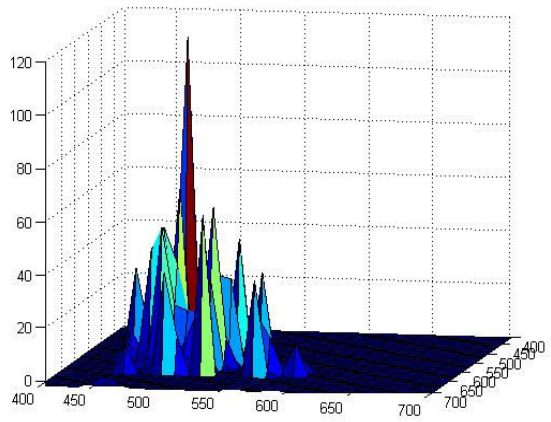


Fig. 4. Histogram of two wavelengths.

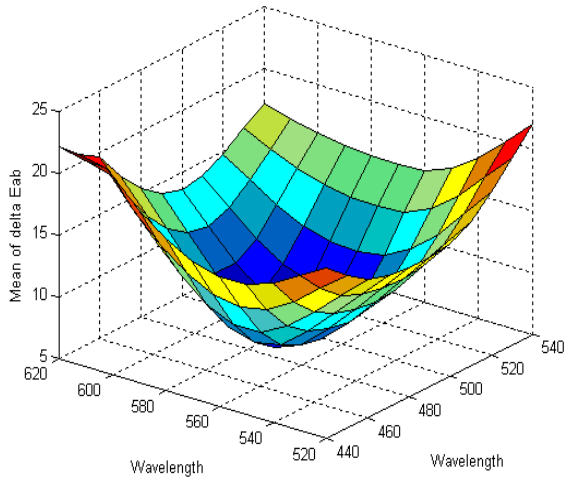


Fig. 5. Distribution of color difference according to first two wavelengths in Munsell Color Chips.

Figure 5 shows distribution of color difference according to the first two wavelengths in each of the Munsell Color Chips. Those wavelengths are each 490 and 570 nm. Intuitively, the SSRs cut at these points can be best approximated by step functions generated by a characteristic function basis defined on the intervals which begin and end at these wavelength. We will use this heuristic max-min method to derive characteristic functions models and evaluate their fit to the Munsell Surfaces Set.

Expanded characteristic function

Suppose we have selected wavelengths that determine for our choice of characteristic functions characteristic function for the case N=3. We can evaluate how well the resulting 3-dimensional function space captures the Munsell Set. But now suppose that we wish to find out how much better we can do with higher values of N. For N=3, we choose a new wavelength to expand a characteristic function model, by subdividing one of the existing intervals. This subdivision algorithm is fast because the optimum is determined from previous results in the every step. We are only adding a single new wavelength.

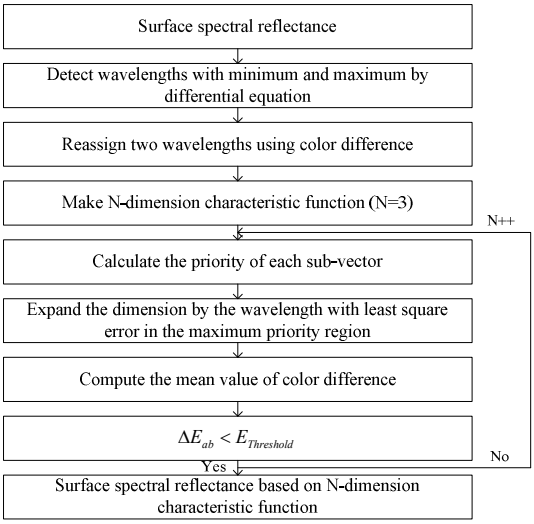


Fig. 6. Flowchart of the proposed algorithm.

It is however an example of a greedy algorithm that can only succeed if the best characteristic function basis for N=4 shares three cutoff wavelengths with that for N=3. In general, this will not be the case. Figure 6 shows the flowchart of proposed algorithm. We detected the two points through differential of the surface reflectance and color difference for characteristic function with N=3 dimension. To determine the next region, we considered the variation of a sub-vector. The priority, to quantize the variation, is calculated the properties in a sub-vector as follows; the singular value, slope of a principal component, amplitude, and standard deviation. Next, we find the wavelength with least square error in the maximum priority region when it was divided as characteristic function with each mean value. If we are not satisfied with that result after calculating the color difference, the error can be reduced while increasing N. Finally, we will be able to obtain the characteristic function for each surface reflectance.

Now, how can we assign an N-dimensional characteristic function when N is over 3? First of all, we have to consider factors such as variation. If there is no variation or no change like Figure 7(f), there are no difference results of before and after separating. Also, if a sub-vector is symmetric like Figure 7(b), the result is similar to the mean of a sub-vector. We might assume that Figure 7(a) and 7(c) are needed to separate among them preferentially. Now we can calculate the priority amount based on its variation from these facts. The first thing is the variation of population. We can consider the variation as computing the standard deviation of a sub-vector. Figure 7(a) is superior to Figure 7(c) in comparison of their standard deviation. The second thing starts tendency of principal component. The principal component indicates the representative of total population. Here, a singular value and slope of a principal component represents the tendency of the total population. Although Figure 7(c) and 7(f) are the same shape and length, its slope is an important factor when we need to discriminate them. Third thing focuses on the scale of a sub-vector. Although a sub-vector has a rapid slope and considerable consistency of principal component but it has a low scale as shown in Figure 7(d), its priority will be low. The difference between the minimum and maximum amplitude of surface reflectance can be used to measure this factor. Finally, we defined priority p as follow;

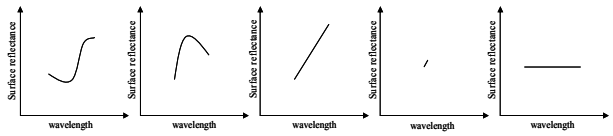


Fig. 7. Examples of sub-vector in surface reflectance

$$p = \|w_1\| \times \sigma \times \|\tan(\theta_{w_1})\| \times \|\max(x) - \min(x)\| \quad (10)$$

where x is a sub-vector, $w_1 = \arg\max_{\|w\|=1} E\left\{\left(w^T x\right)^2\right\}$ is the is the largest singular value, $\sigma = \sqrt{E\left((x - E(x))^2\right)}$ is the standard deviation, $\tan(w_1)$ is the slope of a principal component, and $\|\max(x) - \min(x)\|$ is a difference of the minimum and maximum amplitude in each sub-vector.

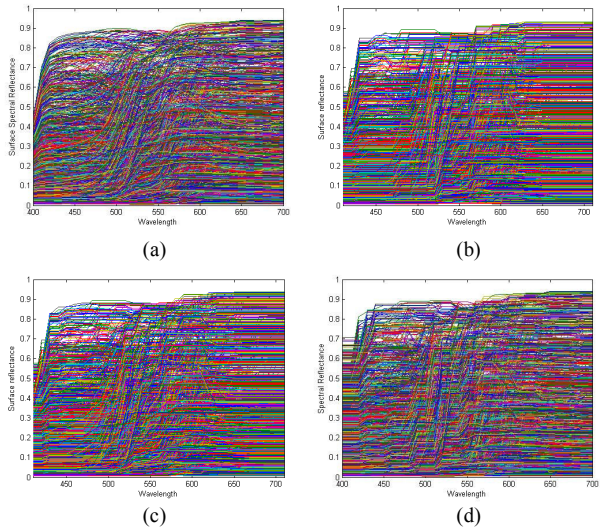


Fig. 8. Surface reflectance based on N-dimension characteristic function; (a) original, (b) N=6, (c) N=9, and (d) N=12.

Before determining basis functions, we examined the optimal N-dimension characteristic function in each surface reflectance. Fig. 8 shows examples of surface reflectance based on N-dimension function. Fig. 8(a) is original surface, 8(b) is 6-dimension, 8(c) is 9-dimension, and 8(d) based on 12-dimension characteristic function. As the dimension is increasing, the results of dimension characteristic function are similar to the original surface reflectance. However, if basis functions are varied for each surface reflectance, surface-light interaction would not have been implemented accurately. That is because we do not use these results in our algorithm.

Simulations

This paper proposed to use four factors denoted as follows: ST, SV, SL and AM when the priority of each sub-vector is determined by expanding 3-basis function to an N-dimensional characteristic function model. ST is here a standard deviation across an interval, SV is the largest singular value, SL is an angle of principal component, and AM is the amplitude of minimum and

maximum values. We could select as the optimal wavelength with the least square mean error ($LMSE$) in the sub-vector selected by these factors. The simulations consist of two methods. One is to observe the effect of a factor and the other is to observe the effect of multiple factors except each factor in all. From these two results, we can confirm the importance of each factor when selecting the wavelength. And also, we considered two measures to evaluate the performance of each factor as follows: Mean of ΔEab($MEab$) between the original and the estimated based on characteristic. Figure 9 shows the $MEab$ according to $LMSE$ points by one factor. The method with singular value is the best, while the method with slope of principal component is the worst in $MEab$. And the method with the amplitude is the best, while the method with slope of principal component is the worst in MSE between original and the estimated based on characteristic function. The slope with method is even worse than bisection method. The histogram of color difference according to characteristic function with N-interval is shown in Figure 9. As N is increasing, the mean value of ΔEab is smaller and its gravity shifts to the left side. The smaller errors than one were counted as one in histogram in Figure 10.

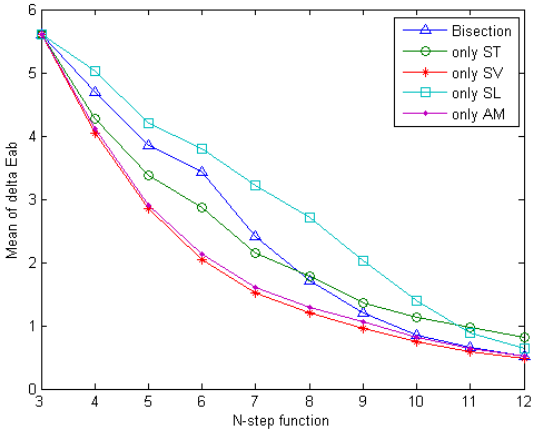


Fig. 9. Mean of ΔEab according to $LMSE$ points by each factor.

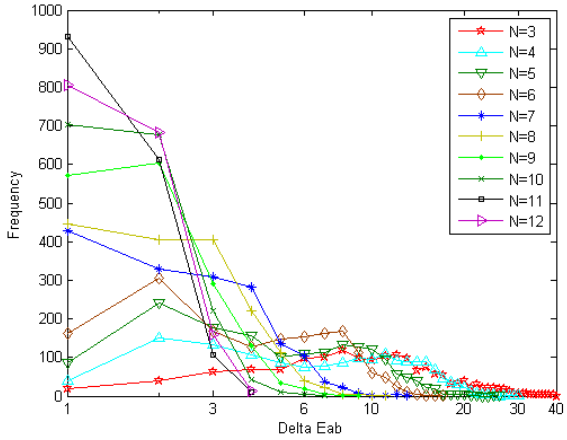
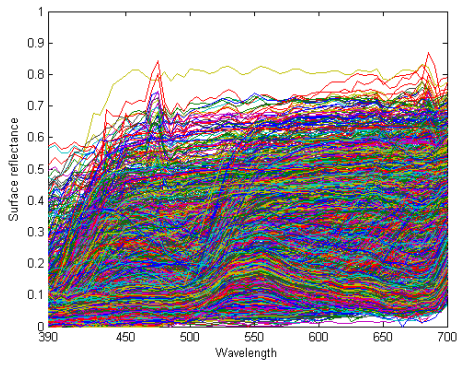
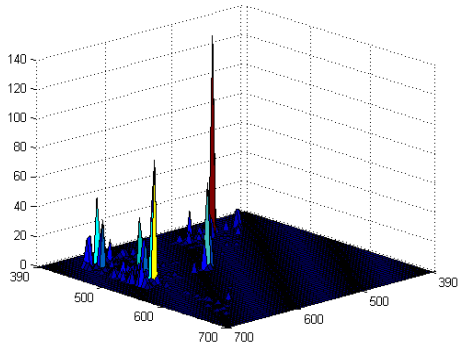


Fig. 10. Histogram of color difference according to N-characteristic function.



(a)



(b)

Fig. 11. Application of real world data (a) Surface reflectance of CHITTKA and (b) first two wavelengths.

We applied the proposed method to the data of surface reflectance [12] including flowers and leaves in a real world. We will be able to evaluate the proposed method as applying this surface reflectance. Real data are not also rougher than that of Munsell color chips but include much more the noise in Fig. 11(a). However, first two wavelengths are indicated in the special wavelength as shown in Fig 11(b). This might be because these data have a similar feature such as flowers, while we could confirm they distributed like the Gaussian shape in Munsell color chips. In special, we can find the minimum first two wavelengths though calculating color difference in real surfaces and lights [12].

Conclusions

This paper proposed a novel set of basis functions for linear models of surface reflectance. The vector of weights in a linear model of surfaces or lights is effectively a representation of particular surfaces or light. If we constrain the choice of basis functions to be orthogonal characteristic functions then ordinary addition and multiplication of vectors mirrors the physical superposition of lights and surfaces and light-surface interaction. Normal rendering computations are then physically correct. We developed a heuristic algorithm to select basis elements based on four factors that characterize the local shape and variation of SSRs or SPDs: SV, ST, SL, and AM. We evaluated the algorithm on the Munsell Color Chips and selected daylights. We are able to reproduce color accurately in surface-light interaction by fixing

optimal wavelengths to determine basis functions. Moreover, we can readily scale the precision of the representation with ease. The representation for N=6 for example has the three characteristic functions for N=3 as its first three characteristic functions. This nested structure allows us to carry out rendering using very accurate representations of surface and light and project the result to the N=3 case that matches a display device. In future work, we plan human psychophysical experiments evaluating the proposed representations.

Acknowledgement

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