Evaluating Color Difference Formulae by Riemannian Metric

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Abstract

For precision color matching, visual sensitivity to small color difference is an essential factor. Small color differences can be measured by the just noticeable difference (JND) ellipses. The points on the ellipse represent colours that are just noticably different from the colour of the centre point. Mathematically, such an ellipse can be described by a positive definite quadratic differential form, which is also known as the Riemannian metric. In this paper, we propose a method which makes use of the Riemannian metric and Jacobean transformations to transform JND ellipses between different colour spaces. As an example, we compute the JND ellipses of the CIELAB and CIELUV color difference formulae in the xy chromaticity diagram. We also propose a measure for comparing the similarity of a pair of ellipses and use that measure to compare the CIELAB and CIELUV ellipses to two previously established experimental sets of ellipses. The proposed measure takes into account the size, shape and orientation. The technique works by calculating the ratio of the area of the intersection and the area of the union of a pair of ellipses. The method developed can in principle be applied for comparing the performance of any color difference formula and experimentally obtained sets of colour discrimination ellipses.

Introduction

Color spaces and color difference metrics have been active fields of research for many decades and it is still going on. Among the many aspects, one important objective is to reduce the gap between the visual perception of the color difference and the mathematical model describing it. Since the establishment of The International Commission on Illumination (CIE), many colour difference formulae have been developed to measure the visual color difference, but no single formula can be considered a perfect one for all applications due to, among other things, the curvilinear nature of the color space as pointed out by many previous researchers [2, 3, 4]. In the CIELAB and CIELUV systems, the color space is considered as a flat space and the color difference in such a space is simply the Euclidean distance between points. In a Euclidean space, the distance between points are straight lines and the advantage of such a space is simplicity for calculating the color difference in practice. The disadvantage of such a space is that color difference calculation or color distance does not agree sufficiently with the perceptually observed color difference. For this reason, colour difference calculations using CIELAB and CIELUV formulae between standards and their matches have been a disputed issue with respect to the visual perception of the color difference [8, 10, 11]. Hence, it is highly desirable to know how well CIELAB and CIELUV colour difference formulae map the visual perception of the color difference.

For precision color matching, visual sensitivity to small color difference is the essential factor. The first systematic studies of visual color matching precision in the different parts of the tristimulus space were done by MacAdam[1], MacAdam and Brown[4], but also by other researchers such as Wyszecki and Fielder [6], Guild and Wright [9]. MacAdam pointed out that small color difference can be measured by the just noticeable differences (JND) through the discrimination ellipse which ultimately manifests an observer's precision of matching the chromaticity of the test color [1, 2]. These findings suggest that the colour space is Riemannian where the small distance between two points is described by a positive definite quadratic differential form, also known as the Riemannian metric. From this positive definite metric, the discrimination ellipse is uniquely determined and vice versa. Hence, considering the color space as a Riemannian space the difference between two colors are described by the line element which describe the colour perception properties of an observer from the measured discrimination thresholds [2, 7, 14, 18].

Many current color science researchers suggest that the color matching ellipses and the Riemannian metric still hold significant role in the color perception or visual color difference and can be applied in many practical cases where it is required to discriminate small or medium color differences[11, 13, 14, 15, 16]. In these contexts, it is useful to study the performance of CIELAB and CIELUV color difference formulae based on small color distances. In other words, MacAdam's approach to compute the just noticeable difference (JND) or the discrimination ellipse on the chromaticity diagram would be the reliable evaluating tool to study the performance of different colour difference formulae.

In this paper, the authors present a method of local linear transform of the CIELAB and CIELUV colour difference metrics into the chromaticity diagram using the principle of the Riemannian metric and Jacobean transformations. This is then used to visualize the color differences predicted by the ΔE_{ab} and ΔE_{uv} colour difference metrics. In other words, the above mentioned formulae from their respective color spaces are transformed into the xyY space by the Riemaninan metric. Then, the corresponding JND or the color matching ellipses are plotted into the chromaticity diagram. The principal axes (semi major axis and semi minor axis) of the ellipse are calculated from the coefficients of the metric tensor, g_{ik}. The ellipse corresponds to the chroma and hue differences and can be considered as a tool for representing an observer's ability to determine perceptual color difference.

To test our method, We have used the visual colour matching experimental data done by MacAdam [1] and Wyszecki and Fielder [6]. MacAdam's data set was prepared by the experiment performed by a single observer. Wyszecki and Fielder's data set was prepared by the color matching matching experiments done by three observers having extensive experience in visual colorimetry. In this paper, we have used average of three sets color matching data. Both these data sets are based on the xyY colour space. CIELAB and CIELUV ellipses are computed using these data sets. Then, computed CIELAB and CIELUV ellipses are compared with ellipses obtained from the experimental data by two approaches. The first approach is to compare each pair of computed and observed ellipses by the size, the shape and the orientation respectively. The second approach is to calculate a single value comparison index of each set of ellipse by calculating the ratio of the intersection area to the union area of the ellipses. The obtained results shows that this is a useful method for comparing the performance of different color difference formulae.

Riemannian Metric and the Ellipse Equation

In a Riemannian space, there exists a positive definite symmetric metric tensor called the Riemannian Metric. In general, the metric tensor g_{ik} is a function that tells us how to compute the infinitesimal distance between any two points in a given space. So, considering the 2D color space as the Riemannian space, an ellipse whose length is equal to the arc length of a curve between two points is expressed by a differential quadratic form:

$$ds^2 = g_{11} \cdot dx^2 + 2 \cdot g_{12} \cdot dx \cdot dy + g_{22} \cdot dy^2 \tag{1}$$

The matrix form of Equation (1) is

$$ds^2 = \begin{bmatrix} dx & dy \end{bmatrix} \cdot \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \cdot \begin{bmatrix} dx \\ dy \end{bmatrix} \tag{2}$$

where, ds is the distance between two points, dx is the difference of x coordinates, dy is the difference of y coordinates and g_{11}, g_{12} and g_{22} are the coefficients of the metric tensor g_{ik}. Here, the coefficient g_{12} is equal to the the coefficient g_{21}. Mathematically, it is written as :

$$g_{ik} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \tag{3}$$

The metric g_{ik} gives intrinsic properties of the color of a geometric surface. Alternatively, it represents the chromaticity difference of any two colors measured along the geodesic of the surface [2]. The coefficients of g_{ik} also determine an ellipse in terms of its parameters a, b and θ defined as the semi major axis, the semi minor axis and the angle of inclination in a geometric plane respectively and vice versa. To determine the value of the the coefficients g_{ik} in terms of the parameters of an ellipse, let us consider the standard equation of an ellipse having center at origin in a geometric plane in the matrix form as follows:

$$1 = \begin{bmatrix} x & y \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = X^T \cdot D \cdot X \tag{4}$$

where X is a 2×1 vector and equals to $[xy]^T$, the 2×2 diagonal matrix $D = Diag(\frac{1}{a^2}, \frac{1}{b^2})$ and the superscript T denotes the transpose operation. The ellipse can be rotated in different orientation by a 2×2 rotation matrix R expressed as :

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{5}$$

The general transformation is $Y = RX$ with inverse $X = R^T Y$. Substituting this into Equation (4), we have :

$$Y^T \cdot R \cdot D \cdot R^T \cdot Y = 1 \tag{6}$$

where, Y equals $[x^{'} y^{'}]^T$, new axes after the rotation. Similarly, the transformation matrix M_t equals $R \cdot D \cdot R^T$. In the expanded form, Equation (6)is

$$1 = \begin{bmatrix} x^{'} & y^{'} \end{bmatrix} \begin{bmatrix} \frac{1}{a^2}\cos^2\theta + \frac{1}{b^2}\sin^2\theta & \cos\theta\sin\theta\,(\frac{1}{a^2} - \frac{1}{b^2}) \\ \cos\theta\sin\theta\,(\frac{1}{a^2} - \frac{1}{b^2}) & \frac{1}{a^2}\sin^2\theta + \frac{1}{b^2}\cos^2\theta \end{bmatrix} \begin{bmatrix} x^{'} \\ y^{'} \end{bmatrix} \tag{7}$$

If we consider the value of ds in Equation (1) is constant and compare with it Equation(7), the coefficients of g_{ik} can be related to the parameters of an ellipse as follows:

$$\begin{aligned} g_{11} &= \frac{1}{a^2}\cos^2\theta + \frac{1}{b^2}\sin^2\theta \\ g_{12} &= \cos\theta\sin\theta\,(\frac{1}{a^2} - \frac{1}{b^2}) \\ g_{22} &= \frac{1}{a^2}\sin^2\theta + \frac{1}{b^2}\cos^2\theta \end{aligned} \tag{8}$$

The angle formed by the major axis with the positive x-axis is given by

$$\tan(2\theta) = \frac{2g_{12}}{(g_{11} - g_{22})} \tag{9}$$

The value of θ is $\leq$ 90° when $g_{12} \leq 0$ otherwise θ is $\geq$ 90°. Similarly, the inverse of Equation(7) is written as:

$$\begin{aligned} \frac{1}{a^2} &= g_{22} + g_{12}\cot\theta \\ \frac{1}{b^2} &= g_{11} - g_{12}\cot\theta \end{aligned} \tag{10}$$

Alternatively, the semi major axis (a) and the semi minor axis (b) of an ellipse can also be determined by the eigenvector and eigenvalue of the metric g_{ik}. If λ_1 and λ_2 are eigenvalues of the metric g_{ik}, the semi major axis (a) and the semi minor axis (b) are equal to $\frac{1}{\sqrt{\lambda_1}}$ and $\frac{1}{\sqrt{\lambda_2}}$ respectively. [16]

Color Space Transformation

In order to compare the CIELAB and CIELUV colour difference formulae to the visual perception of the color difference, we compute the JND threshold ellipses of the CIELAB and CIELUV color difference formulae. Since the experimentally observed ellipses are based on the xyY space, it is necessary to map color vectors of CIELAB and CIELUV colour spaces to the xyY space by a Jacobean transformation. The mapping is done in two steps: first mapping of colour vectors of the CIELAB and the CIELUV spaces into the XYZ tristimulus space and then to the xyY space.

CIELAB is defined as given below:

$$\begin{aligned} L^* &= 116\left(\frac{Y}{Y_r}\right)^{\frac{1}{3}} - 16 \\ a^* &= 500\left[\left(\frac{X}{X_r}\right)^{\frac{1}{3}} - \left(\frac{Y}{Y_r}\right)^{\frac{1}{3}}\right] \\ b^* &= 200\left[\left(\frac{Y}{Y_r}\right)^{\frac{1}{3}} - \left(\frac{Z}{Z_r}\right)^{\frac{1}{3}}\right] \end{aligned} \tag{11}$$

where L^*, a^* and b^* corresponds to the Lightness, the redness-greenness and the yellowness-blueness scales in the CIELAB color space. Similarly, X, Y, Z and X_r, Y_r, Z_r are the tristimulas values of the color vectors and reference white respectively. The color difference in the CIELAB colour space is

$$\Delta E^*_{ab} = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2} \tag{12}$$

If we Take line element distance to measure the color difference at a point in the color space, the Equation (12) becomes differential. In terms of the metric form, we can write

$$(dE^*_{ab})^2 = \begin{bmatrix} dL^* & da^* & db^* \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dL^* \\ da^* \\ db^* \end{bmatrix} \tag{13}$$

Now, to transfer or map color vectors L^*, a^*, b^* into X, Y, Z tristimulas color space, we use the Jacobean transformation where the variables of the two color spaces are related by the continuous partial derivatives. By the chain rule, we have

$$\begin{bmatrix} dL^* \\ da^* \\ db^* \end{bmatrix} = \begin{bmatrix} \frac{\partial L}{\partial X} & \frac{\partial L}{\partial Y} & \frac{\partial L}{\partial Z} \\ \frac{\partial a}{\partial X} & \frac{\partial a}{\partial Y} & \frac{\partial a}{\partial Z} \\ \frac{\partial b}{\partial X} & \frac{\partial b}{\partial Y} & \frac{\partial b}{\partial Z} \end{bmatrix} \begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} \tag{14}$$

Again, from the equation 13 and 14, we have

$$(dE^*_{ab})^2 = \left[dXdYdZ\right] \frac{\partial(L,a^*,b^*)}{\partial(X,Y,Z)}^T I \frac{\partial(L,a^*,b^*)}{\partial(X,Y,Z)} \begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} \tag{15}$$

where I is a 3 by 3 identity matrix and $\frac{\partial(L,a^*,b^*)}{\partial(X,Y,Z)}$ is a Jacobian matrix,

$$\begin{bmatrix} \frac{\partial L}{\partial X} & \frac{\partial L}{\partial Y} & \frac{\partial L}{\partial Z} \\ \frac{\partial a}{\partial X} & \frac{\partial a}{\partial Y} & \frac{\partial a}{\partial Z} \\ \frac{\partial b}{\partial X} & \frac{\partial b}{\partial Y} & \frac{\partial b}{\partial Z} \end{bmatrix} = \begin{bmatrix} 0 & \frac{116}{3} Y^{\frac{-2}{3}} & 0 \\ \frac{500}{3} X^{\frac{-2}{3}} & \frac{-500}{3} Y^{\frac{-2}{3}} & 0 \\ 0 & \frac{200}{3} Y^{\frac{-2}{3}} & \frac{-200}{3} Z^{\frac{-2}{3}} \end{bmatrix} \tag{16}$$

Again, the relationship between X, Y and Z tristimulus colour vectors and x, y and Y colour vectors are

$$\begin{aligned} X &= \frac{xY}{y} \\ Y &= Y \\ Z &= \frac{(1-x-y)Y}{y} \end{aligned} \tag{17}$$

Similarly, transformation from X, Y, Z tristimulus colour space into x, y, Y colour space is done by another Jacobian matrix $\frac{\partial(X,Y,Z)}{\partial(x,y,Y)}$ and expressed as :

$$\begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} = \begin{bmatrix} \frac{\partial X}{\partial x} & \frac{\partial X}{\partial y} & \frac{\partial X}{\partial Y} \\ \frac{\partial Y}{\partial x} & \frac{\partial Y}{\partial y} & \frac{\partial Y}{\partial Y} \\ \frac{\partial Z}{\partial x} & \frac{\partial Z}{\partial y} & \frac{\partial Z}{\partial Y} \end{bmatrix} \begin{bmatrix} dx \\ dy \\ dY \end{bmatrix} \tag{18}$$

where, $\frac{\partial(X,Y,Z)}{\partial(x,y,Y)}$ are

$$\begin{bmatrix} \frac{\partial X}{\partial x} & \frac{\partial X}{\partial y} & \frac{\partial X}{\partial Y} \\ \frac{\partial Y}{\partial x} & \frac{\partial Y}{\partial y} & \frac{\partial Y}{\partial Y} \\ \frac{\partial Z}{\partial x} & \frac{\partial Z}{\partial y} & \frac{\partial Z}{\partial Y} \end{bmatrix} = \begin{bmatrix} \frac{Y}{y} & \frac{-xY}{y^2} & \frac{x}{y} \\ 0 & 0 & 1 \\ \frac{-Y}{y} & \frac{(x-1)Y}{y^2} & \frac{1-x-y}{y} \end{bmatrix} \tag{19}$$

Finally, the transformation from of L^*, a^*, b^* into x, y, Y with two Jacobian matrices is

$$(dE^*_{ab})^2 = \left[dxdydY\right] \frac{\partial(X,Y,Z)}{\partial(x,y,Y)}^T \frac{\partial(L,a^*,b^*)}{\partial(X,Y,Z)}^T I \frac{\partial(L,a^*,b^*)}{\partial(X,Y,Z)} \frac{\partial(X,Y,Z)}{\partial(x,y,Y)} \begin{bmatrix} dx \\ dy \\ dY \end{bmatrix} \tag{20}$$

Here, the whole transformation matrix is $\frac{\partial(X,Y,Z)}{\partial(x,y,Y)}^T \frac{\partial(L,a^*,b^*)}{\partial(X,Y,Z)}^T I \frac{\partial(L,a^*,b^*)}{\partial(X,Y,Z)} \frac{\partial(X,Y,Z)}{\partial(x,y,Y)}$ and represents the metric tensor of three dimensional color space. The coefficients of the first two columns and rows of the 3D metric tensor gives us the JND threshold ellipse in the chromaticity diagram.

By the same approach as described in Equations (12–20), we can map CIELUV colour space into xyY colour space with the following standard formulae:

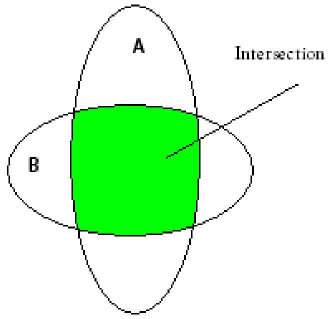


Figure 1: Illustration of the union and the intersection area of two Ellipses.

$$\begin{aligned} L^* &= 116 \left(\frac{Y}{Y_r}\right)^{\frac{1}{3}} - 16 \\ u^* &= 13L \left[\left(\frac{4X}{X+15Y+3Z}\right) - \left(\frac{4X_r}{X_r+15Y_r+3Z_r}\right)\right] \\ v^* &= 13L \left[\left(\frac{9Y}{X+15Y+3Z}\right) - \left(\frac{9Y_r}{X_r+15Y_r+3Z_r}\right)\right] \end{aligned} \tag{21}$$

Method of Comparison

Using the principles of union–intersection and ratio testing, we present the method to compare two ellipses with respect to their shape and orientation. Figure (1) shows two ellipses A and B. The shaded area is the intersection area between them and the total area of A and B is known as the union area. From the statistical point of view, the acceptance region is the intersection area and the rejection region is the union area. The ratio of these intersection and union area gives us a nonnegative value less than or equal to one. Large value of the ratio gives strong evidence that the two ellipses are closely matched.

Result and Discussion

We have applied our method on visual experimental data sets known as Macadam and three observer. Let us begin from MacAdam's data. In Figure(2), the subfigure (2(a)) shows MacAdam's color matching ellipses in the CIE chromaticity diagram according to his visual experiment data. The next two subfigures (2(b)) and (2(c)) are the computed CIELAB and CIELUV color matching ellipses obtained by the Jacobean transformation of the Riemenian metric as described in the section two. The color centers for these computed ellipses, at which color matches are according to the MacAdam's data. To do comparison with experimentally obtained MacAdam's ellipses, ellipses are computed in xyY color space where the Y component (brightness) is in the range [0.01.0] and plotted in the xy chromaticity diagram. Here, ellipses are computed at $L^* = 70$, and to achieve this lightness value the Y component is approximated at 0.4 scale. However, in our method the Y component of xyY can be scaled at any value in the range between 0.0 to 1.0 which allows to compare visual color difference at any lighting conditions. Table 1 gives the calculated area (size), the ratio of semi major (a) and semi minor (b) axes (shape) and the orientation in angle of MacAdam, the CIELAB and CIELUV ellipses. Comparing with area of MacAdam's ellipses, it can be seen that the CIELAB and CIELUV ellipses have general trends of agreement among different set of ellipses. For example, the blue is the smallest, the green largest and that red, blue and yellow are more elongated

Table 1: Calculated Area (Size), Ratio of major (a) and minor (b) axes (Shape) and Angle of orientation of MacAdam's and computed CIELAB and CIELUV ellipses.

Area of Ellipses $\times log10$			Ratio of major(a) and minor(b) axes			Orientation in Angle(Degree)		
MacAdam	CIELAB	CIELUV	MacAdam	CIELAB	CIELUV	MacAdam	CIELAB	CIELUV
-4.03	-3.96	-3.97	2.43	5.18	1.81	62.5	12.75	11.36
-3.42	-3.42	-3.45	4.00	3.12	1.54	77	23.17	16.95
-3.41	-3.38	-3.42	5.00	3.63	1.68	55.5	18.64	23.11
-2.16	-2.3	-2.24	4.17	2.10	1.83	105	85.74	89.08
-2.53	-2.61	-2.56	2.35	2.70	1.50	112.5	78.42	89.89
-2.38	-2.44	-2.4	2.52	1.76	1.61	100	75.1	80.45
-2.50	-2.53	-2.51	2.50	1.62	1.51	92	60.92	69.52
-2.64	-2.72	-2.7	2.00	2.31	1.21	110	67.53	78.84
-2.72	-2.68	-2.74	2.67	1.68	1.47	70	34	61.81
-2.78	-2.72	-2.76	3.67	1.09	1.83	104	44.53	22.22
-3.20	-3.11	-3.24	2.21	2.20	1.23	72	40.43	40.75
-3.06	-3.02	-3.16	3.44	2.12	1.31	58	39	50.77
-3.19	-3.01	-3.06	2.56	1.84	1.50	65.5	34.14	53.39
-2.72	-2.67	-2.92	2.38	1.40	1.76	51	5.16	49.13
-2.85	-2.76	-2.94	2.29	1.25	2.05	20	3.55	43.21
-2.97	-2.84	-3.03	2.00	1.43	2.30	28.5	18.74	41.2
-3.00	-2.80	-3.19	2.64	1.86	2.13	29.5	14.82	35.15
-3.04	-2.92	-3.40	2.00	2.50	2.44	13	4.32	36.38
-2.97	-2.96	-3.30	2.00	1.75	2.78	60	30.11	43.98
-3.19	-3.02	-3.18	2.56	2.04	1.64	47	22.01	37.65
-3.10	-3.09	-3.33	2.50	2.47	1.87	34.5	16.27	33.26
-3.08	-3.00	-3.49	2.95	3.07	2.20	57.5	30	36.56
-3.38	-3.21	-3.32	4.36	2.38	1.48	54	20.54	29.24
-3.26	-3.26	-3.31	4.83	3.18	1.70	86.74	86.69	94.83
-2.97	-3.04	-3.07	3.79	3.74	1.99	40	15.93	29.39

than others.

In Table 1, the comparative data of shape (the ratio of semi major (a) and semi minor (b) axes) and the orientation between MacAdam, CIELAB and CIELUV formulae show that there are some disagreement with experimentally observed ellipses and computed ellipses using the Riemenian metric. We can see in Table 1 that CIELAB ellipses have higher values of a/b ratio than CIELUV ellipses. Thus, they are more closer in shape than CIELUV ellipses with respect to the observed MacAdam's ellipses but neither ellipses fully comply with the original ones. Similarly, with regard to the orientation, most of the ellipses are inclined downwards compared to the MacAdam ellipses. The computed ellipses are more circular than the MacAdam ellipses.

Similarly, comparing between computed CIELAB and CIELUV ellipses with respect to the shape, it is found (Table 1) that a/b ratio is significantly higher in most of the CIELAB ellipses than CIELUV ellipses of same color centers. This result indicates that CIELUV ellipses are more circular than CIELAB ellipses.

We also computed the ratio of the area of intersection and the area of union between MacAdam's ellipses and CIELAB and CIELUV ellipses. Such ratio gives the correlation between computed and original ellipses in terms of size, shape and orientation which is a more informative way for inter comparing the different sets of ellipses by a single value or number. For example, if the computed ellipse and the observed ellipse are same in terms of size, shape and orientation, the ratio of the area of intersection and the area of union is 1. This assures the full compatibility between a pair of ellipses in terms of size, shape and orientation. Table 2 gives the numerical values of such comparison of CIELAB and CIELUV formulae with respect to MacAdam. We have also done sign test for these ratio values of CIELAB and CIELUV ellipses. The results actually shows that CIELUV is performing better than CIELAB at a level of significanc $p = 0.015$.

Our second data set is the three observers color-matching ellipses data. Here, the ellipse parameters are taken as the average of three sets of color-matching ellipses made by three observers. Figure (3(a)) shows the three observers color-matching ellipses in the chromaticity diagram. Similarly, Figures (3(b)) and (3(c)) gives computed ellipses of CIELAB and CIELUV having the same color centers of three observers color-matching ellipses respectively. Like for the MacAdam's ellipses, the ellipses are plotted at constant lightness , and we can see the similarity between experimentally observed and computed ellipses from our method described above. Table 3 shows comparative data of area, shape and orientation between three observer ellipses and computed CIELAB and CIELUV ellipses. The result shows a similar behaviour as for the MacAdam data. The last table 4 gives the single value comparison index for each set of ellipses by comparing the ratio of the area of intersection and the area of union. In our second data set too, the sign test of the ratio of the area of intersection and the area of union shows that CIELUV is better than CIELAB with $p = 0.0125$.

Conclusion

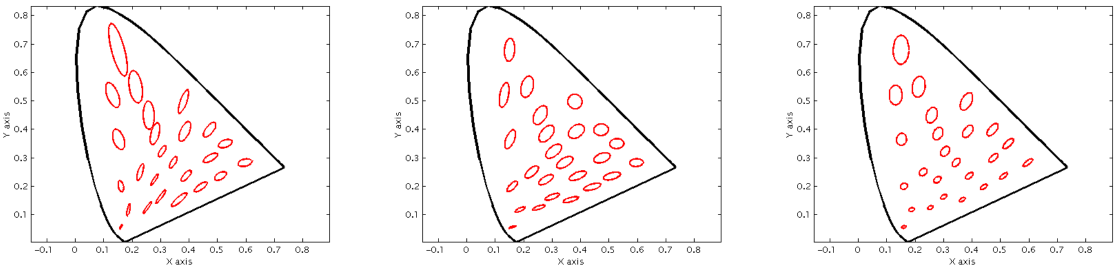
We have developed a method to compare the behaviour of colour difference metrics to experimentally observed JND ellipses. The method uses Jacobean to transform the Riemannian metric tensor to the same colour space as the experimental data.

The presented method can compute JND ellipses of CIELAB and CIELUV formulae in the chromaticity diagram and thus gives the opportunity to evaluate how well they match the visual colour difference obtained in experiments.

Such a pairs of ellipses are compared in terms of size, shape and orientation to see the compatibility between computed and observed ellipses. In our method, JND ellipses of CIE color difference formulae can be plotted at any value of lightness (L^*) in the CIE xy diagram. This feature allows to compare the visual colour difference in order to achieve the best possible match.

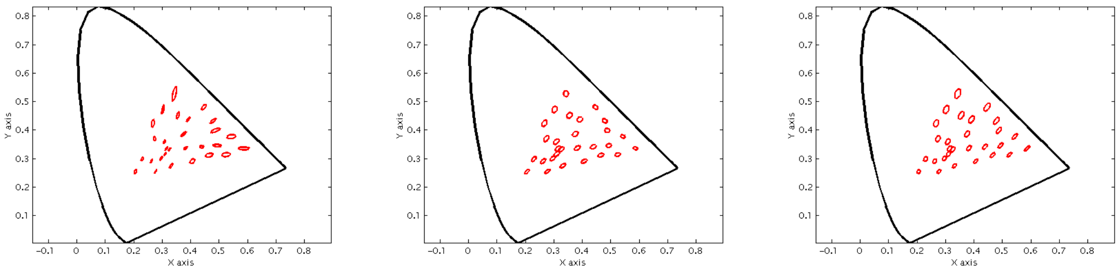
A pair of ellipses can be compared by using the ratio of the area of intersection and the area of union. This gives the single value index which represents three parameters of ellipse to compare in terms of shape, size and orientation respectively.

The comparison between the computed CIELAB and CIELUV ellipses with different visual data sets reveal out that



(a) MacAdam Ellipses . (b) CIELAB Ellipses having same color centers as MacAdam . (c) CIELUV Ellipses having same color centers as MacAdam.

Figure 2: MacAdam's original and Computed CIELAB and CIELUV ellipses in the CIE31 Chromaticity diagram(Enlarged 10 times)



(a) Wyszecki Color-matching Ellipses (mean of three sets of data) . (b) CIELAB Ellipses having same color centers as Wyszecki Color-matching Ellipses. (c) CIELUV Ellipses having same color centers as Wyszecki Color-matching Ellipses.

Figure 3: Wyszecki Color-matching ellipses and Computed CIELAB and CIELUV ellipses in the CIE31 Chromaticity diagram (Enlarged 5 times)

Table 3: Calculated Area(size), Ratio of major(a) and minor(b) axes(Shape) and Angle of orientation of 3 observers and computed CIELAB and CIELUV ellipses.

Area of Ellipses $\times log10$			Ratio of major(a) and minor(b) axes			Angle of orientation (Degree)		
3 observers	CIELAB	CIELUV	3 observers	CIELAB	CIELUV	3 observers	CIELAB	CIELUV
-4.348	-4.922	-4.948	2.95	1.34	2.08	35	61.1	51.2
-4.557	-4.781	-4.907	2.72	1.18	1.8	48	30.6	56.7
-4.227	-4.903	-5.156	3.57	1.49	2.16	179	11.2	44.2
-4.706	-4.801	-5.049	1.74	1.69	1.52	63	41.9	55.2
-4.278	-4.921	-4.837	2.14	1.49	1.99	50	50.6	56.3
-4.532	-4.855	-5.144	1.63	1.61	1.94	5	21.6	46.1
-4.649	-4.834	-5.132	2.47	1.7	1.74	34	29.1	48.2
-4.697	-4.919	-5.165	1.6	1.98	1.29	59	46.5	51.6
-4.525	-4.925	-5.273	2.63	2.09	1.6	54	29.5	43.3
-4.935	-4.981	-5.304	3.7	2.16	1.44	65	33.4	41.6
-4.479	-5.028	-5.272	1.83	2.1	1.24	73	44.2	41.4
-4.056	-5.086	-5.223	3.57	1.53	2.61	179	71.9	40.2
-4.302	-4.751	-4.865	3	1.23	1.72	72	45.6	60.5
-4.833	-4.915	-5.198	3.25	1.98	1.38	70	40.3	48.2
-4.838	-4.858	-5.155	2.36	1.86	1.52	60	36.6	49.6
-4.852	-4.833	-5.12	2.29	1.78	1.56	50	36.3	50.8
-4.697	-4.886	-5.188	2.5	1.94	1.48	57	36.7	48.2
-4.787	-4.83	-5.109	3.08	1.78	1.53	59	38.2	51.8
-3.865	-4.765	-4.715	4.52	1.11	1.78	76	16	66.6
-4.313	-4.721	-4.807	4.78	1.37	1.62	77	59.1	66.6
-4.117	-4.904	-5.026	3.33	1.27	2.11	21	80.2	48.4
-4.01	-5.015	-5.099	2.27	1.46	2.35	8	66.8	44.5
-4.223	-4.744	-4.892	2.43	1.63	1.49	82	56.5	66.2
-4.416	-4.788	-5.012	3.05	1.45	1.73	36	33.7	53.4
-4.043	-4.954	-5.254	2.11	1.66	2.33	8	10	40.7
-4.492	-4.794	-5.011	1.64	1.73	1.44	103	48.7	59.6
-4.108	-4.9	-5.233	2.15	1.78	2.07	14	18.3	42.5
-4.251	-4.901	-5.264	1.46	1.99	1.86	40	23.6	42.7

Table 2: Ratio of area intersection and Union of Ellipses with respect to MacAdam Ellipses

CIELAB	CIELUV
0.29	0.47
0.25	0.33
0.28	0.35
0.34	0.5
0.48	0.65
0.57	0.6
0.58	0.54
0.49	0.55
0.59	0.6
0.41	0.61
0.4	0.52
0.44	0.47
0.35	0.66
0.59	0.62
0.55	0.81
0.68	0.63
0.64	0.64
0.69	0.44
0.83	0.47
0.4	0.68
0.52	0.59
0.59	0.39
0.32	0.46
0.31	0.38
0.39	0.24

Table 4: Ratio of area intersection and Union of Ellipses with respect to 3 observer ellipses

CIELAB	CIELUV
0.4	0.58
0.26	0.36
0.56	0.38
0.2	0.35
0.47	0.58
0.33	0.54
0.26	0.52
0.27	0.47
0.4	0.69
0.16	0.34
0.55	0.72
0.51	0.32
0.4	0.53
0.19	0.36
0.17	0.32
0.16	0.3
0.25	0.49
0.17	0.32
0.4	0.52
0.34	0.41
0.51	0.49
0.56	0.43
0.51	0.65
0.37	0.55
0.77	0.38
0.32	0.52
0.88	0.46
0.7	0.62

neither formulae is good enough for the perfect visual color matching. However, the ratio test method shows that CIELUV performs better than CIELAB to predict visual colour difference for both of the data sets tested. The general trend of color matching ellipses of CIE color difference formulae (CIELAB and CIELUV) are along the direction to experimentally obtained ellipses.

Finally, by our method we can transform any colour space to other colour space and vice versa, preserving the property of their original colour space and can be extended to our future work to study advanced colour difference formula like ΔE_{00}.

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Number 1, 2006

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