The Compositional Markovian Reflectance Model of Halftone Prints on Diffusing Substrate

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Abstract

In the development of the technology of halftone imaging there has been significant interest in physically modeling the halftone microstructure. An important aspect of the microstructure is the scattering of light within the paper upon which the halftone image is printed. And the scattering of light within paper is the key factors which affect the color reproduction. A new spectral reflection prediction model was proposed in this paper that modified the classical Williams-Clapper model using markov chains, By considering the fact that the incident light entered into a given medium surface and exited from another medium, such as entered from colorant patch and exited from paper sheet. And the complex multiple reflection-transmission process occurring between different layers was described by Markov chain. Considering the transverse scatting influence of light propagated in the paper sheet, we introduced experience probability theory, then we calculated four fundamental transfer matrixes under different situation; finally, a new spectral reflectance model of halftone images based on the optical dot gain was established.

Instruction

High-fidelity color reproduction is quite important in digital imaging systems for printing, to offer accurate spectral predictions, the models need to take into account, at least to some extend, the phenomena determining the interactions of inks and paper and of light and halftone prints. Many different phenomena influence the reflection spectrum of a color halftone patch printed on a diffusely reflecting substrate (e.g. paper). These phenomena comprise the surface (Fresnel) reflection at the interface between the air and the paper, light scattering and reflection within the substrate (i.e. paper bulk), and the internal (Fresnel) reflections at the interface between the paper and the air. The lateral scattering of light within the paper substrate and the internal reflections at the interface between the paper and the air are responsible for what is generally called the optical dot gain, Because of light scatter, a photon may exit the paper at a point different from the point at which it entered the paper. And the optical dot gain can be characterized by lateral scattering probabilities, which is the probability that a photon entering the paper through a particularly inked region exits the paper through a similar or different type inked region.

For predicting the spectral reflectance of object, it is necessary to describe the behavior of light at each interface and layers, such as reflection, refraction, scattering, diffraction, which generally depended on the directionality and the polarization of light. And in practice, reflection properties on the surface of these inhomogeneous objects are measured as a hybrid of diffuse and specula reflection components. Since Williams and Clapper[1] proposed the spectral reflection model that considered directional incident light at 45°and radiance detector placed at 0°, Simonot[4,5] extended the Williams-Clapper model to the case where the coating is replaced by a stack of partly absorbing transparent layers having distinct refractive indices and flat interfaces. Simonot and Trachsler .B[6] established the random scatting theory using Markov chains; Arneydescribed the probability of the Yule-Nielsen Effect[11]; Hébert. M extended the Williams-Clapper model to Reflectance and transmittance model for recto-verso halftone prints[8]. Then Hébert. M[9, 10]develop a method for characterizing papers independently of the measuring geometry and draw the bases of a reflectance and transmittance prediction model for recto-verso halftone prints.

In this paper, we assumed that the incident light is incoherent and unpolarized, and the structure of this paper as follows: We first recalled basic notions of geometrical optics and conception of Markov chains in section 2, then several fundamental transfer matrix and the relationship between the emerging probabilities of light propagated in substrate were calculated, finally, the halftone spectral reflection model combined Demichel equation was build in section 3, conclusion was drawn in section 4.

Light Progation and Markov Chains

Account for the properties of light propagated in the interface between different material and the properties of substrate, The layers and the interfaces forming a multilayer specimen are responsible for the reflection and the transmission of light, and multilayer specimens may be considered simply as a superposition of homogeneous unit optical elements, i.e. layers and interfaces between layers, between which the transfers of light can be described thanks to models relying on geometrical optics. Hébert. M [9,10] introduced the biface concept and classified bifaces according to the angle-dependence of their reflectance and transmittance, and expressed it as matrix-like notation, called this pseudo-matrix as fundamental transfer matrix of the biface. A biface represents a single layer or a single interface, whose reflection and transmission properties are azimuthally isotropic, it correspond to perfectly flat interfaces, perfectly nonscattering layers (transparent layers) and perfect diffusers, such as Lambertian bifaces, Because bifaces could receive light on their two sides, they are the junction of two faces, each one being characterized by its reflectance and its transmittance for a given angular distribution of light. The fundamental transfer matrix corresponding Markov chain of biface can be represented as

$$\begin{bmatrix} T(\theta) & R(\theta) \\ R'(\theta) & T'(\theta) \end{bmatrix} \quad (1)$$

Where $R(\theta), T(\theta)$ is directional reflectance and transmittance of upper-face, and $R'(\theta), T'(\theta)$ is directional reflectance and transmittance of lower-face. The behavior of a physical system which describes the different states the system may occupy and indicates how the system moves from one state to another state was specify at the PHD thesis[9], as shown in Fig 1. and the global transfer matrix of the considered multiface, and called "Compositional equation" , correspond to the definition of operation " O ",was expressed as

$$\begin{bmatrix} T_U & R_U \\ R_V & T_V \end{bmatrix} = \begin{bmatrix} p_u & s_u \\ r_u & x_u \end{bmatrix} \circ \begin{bmatrix} x_v & r_v \\ s_v & p_v \end{bmatrix} = \begin{bmatrix} \frac{p_u x_v}{1 - r_u r_v} & s_u + \frac{p_u x_u r_v}{1 - r_u r_v} \\ s_v + \frac{p_v x_v r_u}{1 - r_u r_v} & \frac{p_v x_u}{1 - r_u r_v} \end{bmatrix} \quad (2)$$

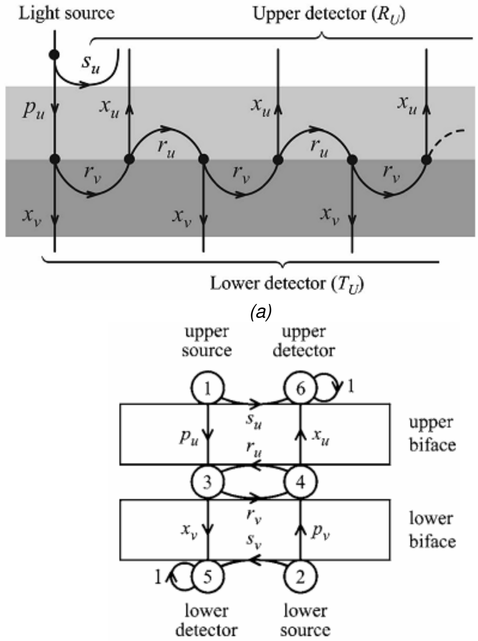


Figure 1. *(a) Multiple reflection–transmission of light within two superposed layers. (b) Markov chain within a quadriface*

The Fundmental Multiface And Probability Distribution

Transverse scatting and probability of light

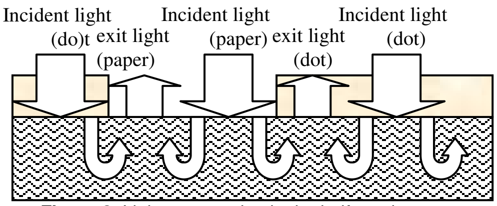


Figure 2. *Light propagation in the halftone image*

In the early prediction models of color halftone prints, the optical dot gain mainly caused by the transverse scatting of light and it occurs due to scatting within the paper sheet and to internal reflectance at the print-air interface. Yule and Nielsen [12] modified the classic model considered the optical dot gain, and it has played a significant role for building color management systems. Ruckdeschel and Hauser[14] analyzed the Yule-Nielson model by modeling the lateral propagation of light within the paper sheet by a point spread function $H(x, y)$. They proposed the following 2-D convolution integral for the reflection spectrum R at position (x, y) :

$$R(x, y) = R_p \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H(x - x', y - y') dx' dy' \quad (3)$$

Light enters at the positions (x', y'), located in the neighborhood of (x, y), was attenuated by the paper absorption $H(x - x', y - y')$ according to the paper reflectance R_p, the integral simply sums up the contributions of all light components incident at positions (x', y'), which exit at position (x, y).

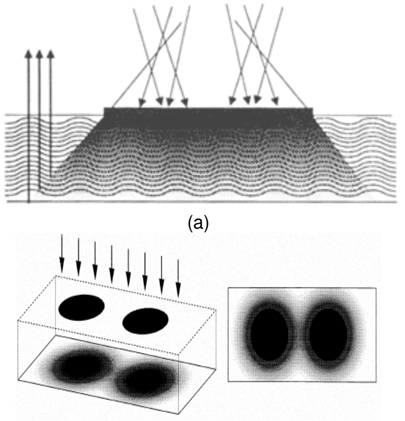


Figure 3. *the description of optical dot gain*

In order to describe the transverse scatting in the paper sheet and conveniently calculate the spectral reflectance model, here we defined the experience probability function[11] $P_i(a)$ as the partition of light incident and exit from dot area, and $P_p(a)$ as the partition of light entered the paper sheet and exit form dot area through the transverse scatting in the paper sheet, $P_i(a)$ $P_p(a)$ are the function of the point spread function, so the $1 - P_p(a)$ is the partition of light entered and exit from paper sheet, and $1 - P_i(a)$ is the partition of light entered the dot area and exit from paper sheet. Considered the image medium penetration and spreading on the substrate, such as ink printed on the paper, we could get $P_p(a) \neq P_i(a)$, and $P_p(a), P_i(a)$ is the function of dot coverage α.

Multiface transfer matrixes

The light propagation in the halftone image could decomposed into four different situations, such as solid ink patch transfer matrix, solid ink patch pseudo-transfer matrix, substrate transfer matrix and substrate pseudo-transfer matrix, these transfer matrices are dedicated to describe the reflectance and transmittances in the cases needed for the ink-ink, paper-ink, paper-paper and ink-paper lateral light scattering probabilities. its illumination and detector set same to the ink patch transfer matrix. In this letter, the number 0, 1, 2 stand for the medium of air, ink, and paper sheet.

The probabilities of light propagation in ink patch itself

The probabilities of light propagating from ink to ink can be expressed as substrate pseudo-transfer matrix, which corresponded to the light that incident from solid ink patch and exit paper sheet, and the probability is $T_U(1 - P_i(a))$, the transfer matrix is as follows equation 4.

$$\begin{bmatrix} T_U & R_U \\ R_V & T_V \end{bmatrix} = \left\{ \begin{bmatrix} T_{01} & R_{01} \\ R_{10} & T_{10} \end{bmatrix} \circ \begin{bmatrix} t^{1/\cos\theta} & 0 \\ 0 & t^{1/\cos\theta} \end{bmatrix} \circ \begin{bmatrix} T_{12} & R_{12} \\ R_{21} & T_{21} \end{bmatrix} \right\}_L \circ \begin{bmatrix} \tau_g & \rho_g \\ \rho_g & \tau_g \end{bmatrix} = \begin{bmatrix} T_{012} & R_{012} \\ r_{210} & t_{210} \end{bmatrix} \circ \begin{bmatrix} \tau_g & \rho_g \\ \rho_g & \tau_g \end{bmatrix} = \begin{bmatrix} \dfrac{T_{012}\tau_g}{1-r_{210}\rho_g} & R_{012} + \dfrac{T_{012}t_{210}\rho_g}{1-r_{210}\rho_g} \\ \rho_g + \dfrac{r_{210}\tau_g^2}{1-r_{210}\rho_g} & \dfrac{t_{210}\tau_g}{1-r_{210}\rho_g} \end{bmatrix} \quad (4)$$

Where R_{01}, T_{01} is the Fresnel reflection and transmittance of ink layer-air interface from air to ink layer separately, in a similar way, R_{10}, T_{10} is from ink layer to air; and $t^{1/\cos\theta}$ is the absorption index of ink layer, R_{12}, T_{12} is the Fresnel reflectance and transmittance of ink-substrate interface from ink layer to substrate, so as to R_{21}, T_{21} ; τ_g, ρ_g is the reflectance and transmittance of substrate; and t_{210}, r_{210} is the global Lambertian reflectance and transmittance of the combination of air-ink interface, ink layers, and ink-substrate interface, where

$$T_{012} = \frac{T_{01}T_{12}t^{1/\cos\theta}}{1-R_{10}R_{12}t^{2/\cos\theta}},\ R_{012} = R_{10} + \frac{T_{10}R_{12}t^{2/\cos\theta}}{1-R_{10}R_{12}t^{2/\cos\theta}}$$

$$t_{210} = \int_0^{\frac{\pi}{2}} T_{210}\sin 2\theta d\theta = \int_0^{\frac{\pi}{2}} \frac{T_{01}T_{12}t^{1/\cos\theta}}{1-R_{10}R_{12}t^{2/\cos\theta}}\sin 2\theta d\theta$$

$$r_{210} = \int_0^{\frac{\pi}{2}} R_{210}\sin 2\theta d\theta = \int_0^{\frac{\pi}{2}} [R_{12} + \frac{T_{12}R_{01}t^{2/\cos\theta}}{1-R_{10}R_{12}t^{2/\cos\theta}}]\sin 2\theta d\theta$$

the symbol L stand for the lower illumination is "Lambertian light" (right subscript)

The probabilities of light propagation from paper to ink

The solid ink patch pseudo-transfer matrix, which illumination and detector lies in different plane to transfer matrix, that the illumination lay on the lower while the detector lies on the upper. This matrix corresponded to the part of light incident from paper sheet and exit from ink patch, and the probability is $T_M P_P(a)$,where T_M is the global transmittance of light propagation from air to paper sheet. The solid ink patch pseudo-transfer matrix is expressed at equation 5, and the upper global transmittance, lower global reflectance and lower global transmittance wasn't calculated while it wasn't influenced the final reflectance.

$$\begin{bmatrix} T_M' & . \\ . & . \end{bmatrix} = \begin{bmatrix} \tau_g & \rho_g \\ \rho_g & \tau_g \end{bmatrix} \circ^L \left\{ \begin{bmatrix} T_{12} & R_{12} \\ R_{21} & T_{21} \end{bmatrix} \circ \begin{bmatrix} t^{1/\cos\theta} & 0 \\ 0 & t^{1/\cos\theta} \end{bmatrix} \circ \begin{bmatrix} T_{01} & R_{01} \\ R_{10} & T_{10} \end{bmatrix} \right\} = \begin{bmatrix} \tau_g & \rho_g \\ \rho_g & \tau_g \end{bmatrix} \circ \begin{bmatrix} t_{210} & r_{210} \\ R_{012} & T_{012} \end{bmatrix} = \begin{bmatrix} \dfrac{t_{210}\tau_g}{1-\rho_g r_{210}} & . \\ . & . \end{bmatrix}$$

(5)

The probabilities of light propagation in paper sheet itself

The probabilities of light propagating in papers only can be expressed as the substrate transfer matrix, which composed of a Lambertian substrate and a flat interface with air, this matrix corresponded to the part of light incident from paper sheet and exit from paper too. then the transfer matrix can be expressed as:

$$\begin{bmatrix} T_M & R_M \\ R_N & T_N \end{bmatrix} = \begin{bmatrix} T_{02} & R_{02} \\ R_{20} & T_{20} \end{bmatrix}_L \circ \begin{bmatrix} \tau_g & \rho_g \\ \rho_g & \tau_g \end{bmatrix} = \begin{bmatrix} \dfrac{T_{02}\tau_g}{1-r_{20}\rho_g} & R_{02} + \dfrac{T_{02}t_{20}\rho_g}{1-r_{20}\rho_g} \\ \rho_g + \dfrac{r_{20}\tau_g^2}{1-r_{20}\rho_g} & \dfrac{t_{20}\tau_g}{1-r_{20}\rho_g} \end{bmatrix} \quad (6)$$

Where T_{02}, R_{02} is the Fresnel reflection and transmittance of paper-air interface from air to paper separately, t_{20}, r_{20} is the global Lambertian reflectance and transmittance of the paper-air interface, and $t_{20} = \int_0^{\frac{\pi}{2}} T_{20}\sin 2\theta d\theta$, $r_{20} = \int_0^{\frac{\pi}{2}} R_{20}\sin 2\theta d\theta$.

The probabilities of light propagation from ink to paper

The substrate pseudo-transfer matrix, which corresponded to the light that incident from solid ink patch and exit paper sheet, and the probability is $T_U(1-P_i(a))$, in this section, the upper global transmitance,lower global reflectance and lower global transmittance was't calculated while it was't influenced the final reflcetance. the transfer matrix is as follows:

$$\begin{bmatrix} T_U' & . \\ . & . \end{bmatrix} = \begin{bmatrix} \tau_g & \rho_g \\ \rho_g & \tau_g \end{bmatrix} \circ^L \begin{bmatrix} T_{20} & R_{20} \\ R_{02} & T_{02} \end{bmatrix} = \begin{bmatrix} \dfrac{t_{20}\tau_g}{1-\rho_g r_{20}} & . \\ . & . \end{bmatrix} \quad (7)$$

Where T_{20}, R_{20} is the Fresnel reflection and transmittance of paper-air interface from paper to air separately, t_{20}, r_{20} is the global Lambertian reflectance and transmittance of the air-paper interface.

The Relationship between the Experience probabilities

Thanks to the transfer matrix built above, the composition spectral reflectance model was easier to reconstruct. The exit light and the reflectance from paper sheet and solid ink patch can be easy calculated. The light and the reflectance exit from solid ink patch as follows:

$$I_i = aP_i(a)I_0R_U + (1-a)P_p(a)T_M I_0 T_M' \quad (8)$$

$$R_i = \frac{I_i}{aI_0} = P_i(a)R_U + \frac{(1-a)}{a}P_p(a)T_M T_M' \quad (9)$$

Where α is the dot surface coverage, also, the light and the reflectance exit from paper sheet as follows

$$I_P = [1-P_i(a)]I_0 T_U a T_U' + [1-P_p(a)](1-a)I_0 R_M \quad (10)$$

$$R_P = \frac{I_p}{(1-a)I_0} = \frac{a}{1-a}[1-P_i(a)]T_U T_U' + [1-P_p(a)]R_M \quad (11)$$

Here we need to describe the relationship of the probability $P_i(a)$ and $P_p(a)$. When no ink is printed in the paper sheet, and the incident light is total reflected, then we can get as follows: $\rho_g = t = T_{01} = 1$, $R_{01} = 0$, $r_{10} = 0, t_{10} = 1$, and the relationship between $P_i(a)$ and $P_p(a)$ is $R_i = R_p$,combined with the equation 9 and equation 10, we can get:

$$P_i(a) = 1 - P_p(a)(\frac{1-a}{a}) \quad (12)$$

Corresponded to equation 12, the equation 9 and equation become

$$R_P = T_U T_U' P_p(a) + [1-P_p(a)]R_M \quad (13)$$

$$R_i = P_i(a)R_U + (1-P_i(a))T_M T_M' \quad (14)$$

Considering of the optical dot gain caused by the transverse scattering in the paper sheet, the probability can be approximated as,

$$P_i(a) = a^w,\quad P_P(a) = 1-(1-a)^w \tag{15}$$

Where $w = 1-\exp(-\delta.l_k.f)$, f is the screen frequency, l_k is the optical path within the paper sheet, δ is the experiment parameter, and combined the above equation [12,13], the equation 13 and equation14 are,

$$R_p = T_U T_U^{'}[1-(1-a)^w] + (1-a^w)R_M \tag{16}$$

$$R_i = a^w R_U + (1-a^w)T_M T_M^{'} \tag{17}$$

The Spectral Reflectance Model

When printing with multiple, at least partly transparent inks, the superposition of two or more inks yields new colorants. Therefore, with the inks cyan, magenta and yellow, one may obtain by printing superposed dots the colorants cyan, magenta, yellow, red ,green, blue and black, within a single halftone ink layer, the probability that light hits at a position an inked dot is equal to the surface coverage of that ink in the region within which the ink position is located. Accordingly, in the case of surface coverage of C, M, and Y three inks, the so-called Demichel equations yield the respective surface coverage of the colorants as the function of the surface inks. So we can obtain the spectral reflectance of only one ink, combined with the equation 16 and equation 17, we can get

$$\begin{aligned} R &= R_P + R_i = \left(R_{p\_p} + R_{i\_p}\right) + \left(R_{i\_i} + R_{p\_i}\right) \\ &= T_U T_U^{'}[1-(1-a)^w] + (1-a^w)R_M + a^w R_U + (1-a^w)T_M T_M^{'} \end{aligned} \tag{18}$$

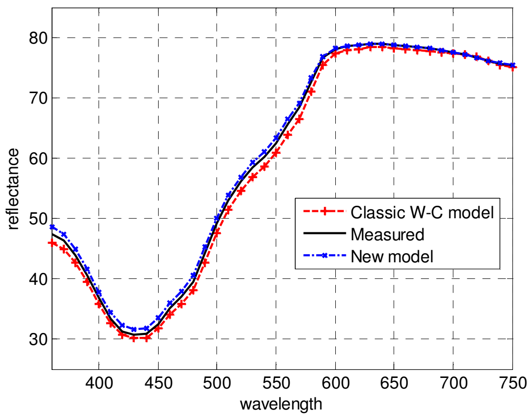
Combined with the four case of light propagation proposed above, we can get the final spectral reflectance of the one ink halftone image, the equation 18 can be expressed as

$$R = R_i + R_p = [1-(1-a)^w]\frac{T_{012}\tau_g}{1-r_{210}\rho_g}\frac{t_{20}\tau_g}{1-\rho_g r_{20}} + (1-a^w)(R_{02} + \frac{T_{02}t_{20}\rho_g}{1-r_{20}\rho_g}) + a^w(R_{012} + \frac{T_{012}t_{210}\rho_g}{1-r_{210}\rho_g}) + (1-a^w)\frac{T_{02}\tau_g}{1-r_{20}\rho_g}\frac{t_{210}\tau_g}{1-\rho_g r_{210}} \tag{19}$$

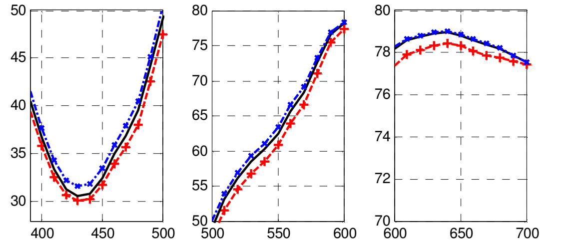
For a pair of colors, the color difference formula measures the difference, and color rendering index, Here, for the purpose of examining the similarity of two spectral, Δ *Elab*, Δ *Ecmc* , *mRMS* are used in this paper, and the *mRMS* expressed as

$$mRMS = \sqrt{\frac{\sum_{\lambda=360}^{750}\sqrt{w(\lambda)}\Delta\beta(\lambda)^2}{n}} \tag{20}$$

Where $w(\lambda)$ is the weighting factor, and $w(\lambda) = 1/R(\lambda)_{measured}$, $\Delta\beta(\lambda)$ is the spectral error between predicted and measurement, and n is the number of wavelength intervals. We carried out experiments using the new predicting reflectance model and classic model shown in figure 4. For the new model, we applied our methods to values which were more accurately in spectral reflectance prediction. The experimental results compared to the measured reflectance are shown in table 1, and the evaluation parameters are color difference Δ ***Elab,*** Δ ***Ecmc,*** and ***mRMS*** with the measured spectral reflectance, note that color different between the new model and measured is smaller than the classic one. The spectral errors is showed in figure 5, the spectral errors of the new model is slightly bigger than the classic one, that may be caused by the contribution of light scattering form the paper sheet we considered.



(a)



(b)

Figure 4. (a) the spectral reflectance of classic Williams-Claper model, new model and measurement of 100% coverage.(b) the partial enlarged details

Table 1. the accuracy between different models

Type	Δ Elab	Δ Ecmc	mRMS
Classic model	1.1111	2.8151	0.4239
New model	0.8471	1.1309	0.2871

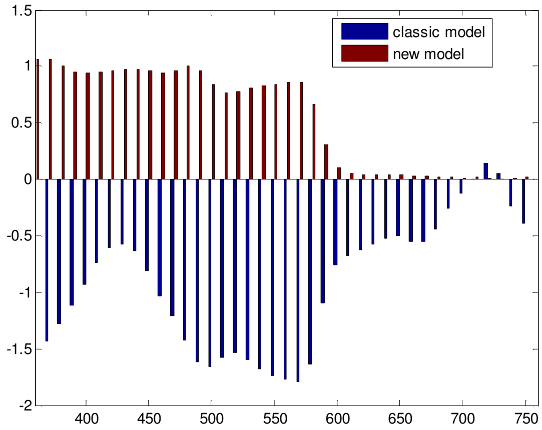


Figure 5. *The spectral errors between predict models and measurement*

Conclusion

We proposed a new spectral reflection prediction model that represent a considerable progress compared with the classical Williams-Clapper model by taking into account the fact that proportionally incident light through a given colorant surface onto other paper sheet surface and exit, that is the Yule-Nielsen effect caused by the transverse scatting in the paper bulk. Thanks to the Markov chains, we get a better approach to understanding the complex interactions between light, ink and paper, and we build four easier multiface transfer matrixes, combined with the experience probability, we obtain the new reflection model of halftone image considering the optical dot gain. Further developments relying on these spectral prediction models may possibly allow keeping printer parameters such as ink thickness or dot gain constant over long printing periods.

References

[1] Williams F. C, Clapper F. R, Multiple Internal Reflections in Photographic Color Prints, J. Opt. Soc. Am. 43,595-599(1953)

[2] M. Hebert, R.D. Hersch., Classical Print Reflection Models: A Radiometric Approach, Jour. Imaging. Sci. and Technol., 48,363-374(2004)

[3] Lionel Simonot, Mathieu Hébert, Extension of the Williams–Clapper model to stacked nondiffusing colored coatings with different refractive indices, J. Opt. Soc. Am. 23, 1432-1441(2006)

[4] M. Elias, L. Simonot, Bidirectional reflectance of a diffuse background covered by a partly absorbing layer, Opt. Communications. 191, 1–7(2001)

[5] M. Elias and L. Simonot, Bi-directional reflectance of a varnished painting part 1: influence of the refractive indices without using the approximations of Saunderson correction—exact computation,"Opt. Communication. 231, 17–24 (2004)

[6] Simon K. and Trachsler B. A random walk approach for light scattering in material. Discrete Mathematics and Theoretical Computer Science AC, 289–300 (2003)

[7] J. D. Shore and J. P. Spoonhower, Reflection density in photographic color prints: generalizations of the Williams–Clapper transform, J. Imaging Sci. Technol. 45, 484–488 (2001).

[8] Mathieu Hébert and Roger David Hersch, Reflectance and transmittance model for recto–verso halftone prints, J. Opt. Soc. Am., 23, 2415-2432(2006)

[9] M. Hebert, R.D. Hersch. Compositional reflectance and transmittance model for multilayer specimens, J. Opt .Soc. Am. A, 24(9):2628-2644 (2005)

[10] M. Hebert, Compositional model for predicting multilayer reflectance and transmittances in color reproduction, Ph.D. Thesis No. 3576 (Ecole Polytechnique Fédérale deLausanne), (2006).

[11] J. S. Arney, A Probability Description of the Yule-Nielsen Effect I, Journal of Imaging Science and Technology, 41, 633-636 (1997)

[12] J. S. Arney, "Modeling the Yule-Niesen effect on color halftone," Journal of Imaging Science and Technology, 42,335-340(1998)

[13] Thomas Bugnon, Model-Based Deduction of CMYK Surface Coverages from Visible and Infrared Spectral Measurements of Halftone Prints, Color Imaging XII: Processing, Hardcopy, and Applications;,(2007)

[14] F. R. Ruckdeschel and O. G. Hauser, "Yule-Nielsen effect in printing: a physical analysis," Appl. Opt. 17(21), 3376–3383(1978).

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