A compact singularity function to predict WCS Color Names and unique hues

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Abstract

Understanding how colour is used by the human vision system is a widely studied research field. The field, though quite advanced, still faces important unanswered questions. One of them is the explanation of the unique hues and the assignment of color names. This problem addresses the fact of different perceptual status for different colors.

Recently, Philipona and O'Regan have proposed a biological model that allows to extract the reflection properties of any surface independently of the lighting conditions. These invariant properties are the basis to compute a singularity index that predicts the asymmetries presented in unique hues and basic color categories psychophysical data, therefore is giving a further step in their explanation.

In this paper we build on their formulation and propose a new singularity index. This new formulation equally accounts for the location of the 4 peaks of the World colour survey and has two main advantages. First, it is a simple elegant numerical measure (the Philipona measurement is a rather cumbersome formula). Second, we develop a colour-based explanation for the measure.

Introduction

Opponent space has been defined as a confrontation of non-mixable colours. That is, it is impossible to perceive a reddish green, neither a yellowish blue. These four colours: red, green, yellow and blue are considered 'cardinal', and their hues are considered unique hues. However, there is not a widely accepted theory explaining this uniqueness. Since, opponent theory does not adequately predict the hues perceived perceptually unique [16],[8].

Whether if opponent theory is underlying it or not, what is widely assumed is the asymmetry in human perception of different color surfaces. Specific color properties hold a different status in the perception, such as, red, green, yellow and blue, and possibly purple, orange or pink for specific cultures. Explanations for this fact could be essentially found in the neuronal representation of color in the human visual system [10] [11], or could be given by cultural or linguistic facts [6], but is an open issue.

How this asymmetric perception can be achieved in the human visual system has been studied in a recent work by Philipona and O'Regan [11]. In this work they explore the hypothesis of a representation that copes with the reflection properties of surfaces independently of the lighting conditions of the observation. They build a linear biological model by finding a linear constraint between the tricromatich representation about the illuminant and the tricromatich representation about the reflected light. This is a biological approach towards what physicists define as reflectance: the relationship between the spectrum of light illuminating a surface and the spectrum of light reflected by the surface. Practically, this is equivalent to the relation between the RGBs of a surface under different lights with an achromatic surface viewed under the same light set. For each surface, this linear model finds a matrix containing the reflectance properties which are illuminant invariant. They propose the eigenvalues of this matrix as a triplet representing the inherent reflectance properties of the surface. We will denote these coefficients as (r_1^s, r_2^s, r_3^s) where s represents the surface that is being represented with this triple of coefficients.

These reflectance coefficients are used to compute a singularity index that will quantify the special case or the degree of asymmetry of the corresponding surface. This index is built in such a way that it allows to predict the psychophysical data of the unique hues or the color names of the World Color Survey [1]. The formulation for this Singularity Index is based on ordering the coefficients, $r_1^s > r_2^s > r_3^s$, and they are related in this way

$$\beta^s = \left(\frac{r_1^s}{r_2^s}, \frac{r_2^s}{r_3^s} \right) \tag{1}$$

finally, the singularity index is given by maximizing a normalized version of them

$$SI = \max \left(\frac{\beta_1^s}{\max(\beta_1^s)}, \frac{\beta_2^s}{\max(\beta_2^s)} \right). \tag{2}$$

Although this index is predicting the asymmetries, the formulation is not compact and is defined in a very ad-hoc way to predict the asymmetric properties of the color categories. Moreover, it is not related with any known property of colour. In this paper we focus on these two points. We propose a new singularity function, completely compact, and related with well-known colour measures, such achromaticity. We will show that this our formulation also predicts the unique hues and matches the World color survey data as well as the previous formulation.

The paper is divided as follows. In the next section, we will explain the details of the mathematical background, where we base our approach. Later on, we develop our singularity function and we show the results of our predictions versus the psychophysical data of the mentioned sets.

Mathematical Background

The linear biological model introduced in [11] is built on the assumption that human vision system it is able to extract the reflection properties of the world surfaces independently of the lighting conditions of the observation. It brings to a canonical representation of the reflectance.

This model is based on the computation of the CIE R,G,B coordinates ro represent physical properties of the light reflected by a surface achieving the observer eye which lose part of the colour information due to the photopigments absortion. This is referred as the accessible information by the authors [11].

This model will find a matrix containing the surface reflectance properties for each surface. From these matrix, we are able to extract a colour triple (reflectance) that is the colour of the surface independent from the illuminant. Once they obtain this triple, they developed a formulation that explains the location of WCS color names and unique hues.

To build the data they select a wide number of illuminants and reflectances. Moreover, they select the photopigments. For photopigments they used the 10-deg Stiles and Burch Color Matching Functions (CMFs) [14] (they checked that using Stockman and Sharpe [15] cone fundamentals the results do not present any noticiable modification). For the set of illuminants (from now on set E) they used the 99 daylight spcetra from Romero [13] et al, a Gaussian sample of 200 spectra constructed from the basis functions S0, S1, S2 derived by Judd et al [5] , and the 239 daylight spectra from Chiao et al [3]. Finally, the reflectances used are the set of 1600 Munsell glossy chips from Joensuu [9]

Firstly, we define v^s as the accessible information about the reflected light for a given surface s

$$v^s{}_i = \int_w R_i(\lambda)S(\lambda)E(\lambda)d\lambda, i = 1,2,3 \tag{3}$$

where λ is a set of wavelengths,$E(\lambda)$ the spectral power distribution of the light in each wavelength, $R_i(\lambda)$ the absorption of photopigments presents in L,M and S photoreceptors respectively and $S(\lambda)$ the reflectance of a surface.

Secondly, we define u as the accessible information about the incident illuminant

$$u = \int_w R_i(\lambda)E(\lambda)d\lambda, i = 1,2,3 \tag{4}$$

from these two equations we can solve by linear regression

$$v^s = A^s u \tag{5}$$

for a set of illuminants E. This equation uses only the information about light that is (physically) accessible to an organism given the photoreceptors it posseses. This means, that matrix A^s is containing the surface reflectance properties inside it.

Mathematically we will solve the matrix A^s by linear regression, and as A^s is a 3-by-3 matrix, it will be diagonalized

$$A^s = (U^s)^{-1}V^sU^s \tag{6}$$

where V^s is a diagonal matrix containing the eigenvalues of A^s and U^s containing the respective eigenvectors. Philipona and O'regan in their paper show that they form a basis, and then, these eigenvalues are a colour triple relating the surface reflectance and a white reflectance.

After that, Philipona and O'Regan also develop a formulation that by using this colour triple show the relation between these eigenvalues and the four main color Names and the four unique hues. This formulation is the one explained in the Introduction where (r_1^s, r_2^s, r_3^s). are the eigenvalues for a particular surface in decreasing order. Then, they define the equation 1 that will give high numbers if one or two of the values are close to zero. Finally, they define the singular index SI as shown in equation 2. From now on, we will use (r_1, r_2, r_3) instead of (r_1^s, r_2^s, r_3^s).

In this paper we will use the framework explained for obtaining the color triple, but we will use this colour triple in order to improve the formulation defined by Philipona and O'Regan since their formula is complex. Normalization is needed and there is no specific colour information. Then, our idea in the next section is to find a less complex formula also relating the results to some well-known color measures.

Singularity Function

In this section we propose a new singularity index that pursuits a simpler and more compact formulation with specific properties. First property will be to have a measure that should be independent of the order of the values, that means, the triple (r_1, r_2, r_3) being the eigenvalues of a matrix A^S of a surface S, can be given in any order since the formulation will extract the relative information of each component over the other two. A second property we want to fulfill is to normalize independently of which is the maximum value of the components. Our proposal is to boost the importance of a particular coefficient over the other two by a mathematical function. To this end, we propose to use a cubic function normalized by the product of the components, this is to compute the terms

$$I_1 = \frac{r_1^3}{r_1 \cdot r_2 \cdot r_3} \tag{7}$$

$$I_2 = \frac{r_2^3}{r_1 \cdot r_2 \cdot r_3} \tag{8}$$

$$I_3 = \frac{r_3^3}{r_1 \cdot r_2 \cdot r_3} \tag{9}$$

Once, the components has been normalized and boosted, they can be simply combined by a sum. In this case, if the surface has a singularity it will be reflected in at least one of the these three components, and it will eventually appear in the addition, hence our Compact Singularity Index (CSI) is given by

$$CSI = I_1 + I_2 + I_3 = \frac{r_1^3 + r_2^3 + r_3^3}{r_1 \cdot r_2 \cdot r_3} \tag{10}$$

Let us now continue explaining different properties that can be derived. Firstly, let us explain the formulation from a color basis point of view. In the previous section we showed that the triple (r_1, r_2, r_3) of the reflection properties of a surface where derived as the eigenvalues of a matrix. Then we can consider the orthogonal basis formed by the corresponding eigenvectors $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ as the basis of a 3D color space where the reflection properties can be considered as the color of a surface. In this color space, achromatic surfaces will have three equal reflection coefficient and will cope the diagonal axis of the space (this fact relates this new space to an RGB space). Then, in this space our formulation will represent a chromaticness measure that can be computed as the determinant of the following matrix

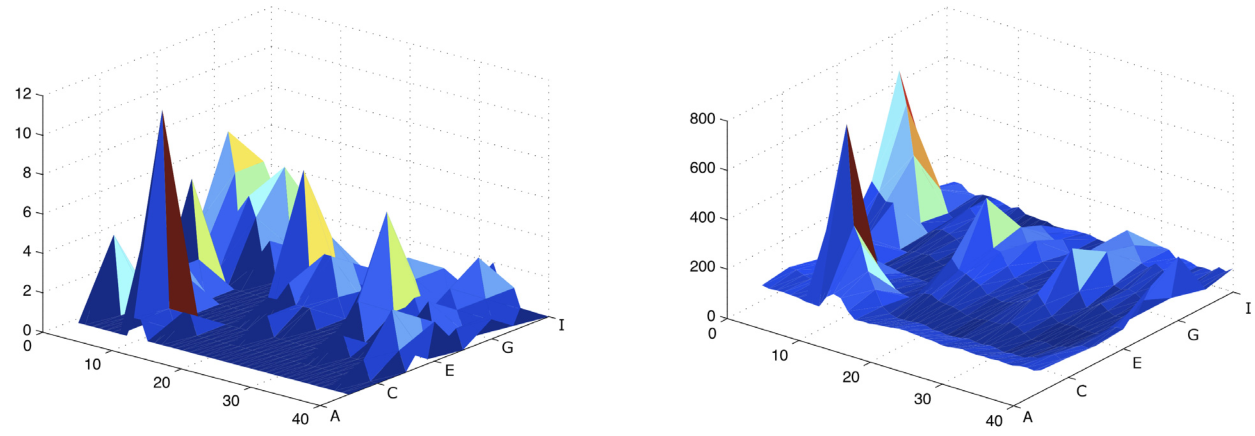
$$M = \begin{pmatrix} r_1 & r_2 & r_3 \\ r_2 & r_3 & r_1 \\ r_3 & r_1 & r_2 \end{pmatrix} \tag{11}$$

that is given by

$$det(M) = r_1^3 + r_2^3 + r_3^3 - 3 \cdot r_1 \cdot r_2 \cdot r_3 \tag{12}$$

whose normalisation brings to the compact singularity function

$$\begin{aligned} \frac{det(M)}{r_1 \cdot r_2 \cdot r_3} &= \frac{r_1^3 + r_2^3 + r_3^3 - 3r_1r_2r_3}{r_1 \cdot r_2 \cdot r_3} && (13) \\ &= \frac{r_1^3 + r_2^3 + r_3^3}{r_1 \cdot r_2 \cdot r_3} - 3 && (14) \\ &\propto \frac{r_1^3 + r_2^3 + r_3^3}{r_1 \cdot r_2 \cdot r_3} && (15) \\ &= CSI && (16) \\ & && (17) \end{aligned}$$



a b

Figure 1. *a) Berlin and Kay psychophysical data for evolved languages b)WCS psychophysical data for unwritten languages*

another interesting property is its independence to intensity if it is considered as a color representation, this is

$$\frac{(s\cdot r_1)^3+(s\cdot r_2)^3+(s\cdot r_3)^3}{(s\cdot r_1)\cdot(s\cdot r_2)\cdot(s\cdot r_3)}=$$
$$=\frac{s^3(r_1^3+r_2^3+r_3^3)}{s^3\cdot(r_1\cdot r_2\cdot r_3)}=\frac{r_1^3+r_2^3+r_3^3}{r_1\cdot r_2\cdot r_3} \tag{18}$$

Finally, we introduce another interesting property of this formulation, since it can be seen as an approximation of the perceptual space given by

$$r_1=\rho_1^{\frac{1}{3}}, r_2=\rho_2^{\frac{1}{3}}, r_3=\rho_3^{\frac{1}{3}} \tag{19}$$

Hence, by replacing equation 19 in equation 10 we found

$$CSI=\frac{r_1^3+r_2^3+r_3^3}{r_1\cdot r_2\cdot r_3}=\frac{\rho_1+\rho_2+\rho_3}{\rho_1^{\frac{1}{3}}\rho_2^{\frac{1}{3}}\rho_3^{\frac{1}{3}}}=$$
$$=\frac{\rho_1+\rho_2+\rho_3}{(\rho_1\cdot\rho_2\cdot\rho_3)^{\frac{1}{3}}}\propto\frac{aritmean}{geomean} \tag{20}$$

where *aritmean* refers to the arithmetic mean and *geomean* refers to the geometric mean in a perceptual space.

Results

In this section we show the results in two experiments that use two different sets of data. First experiment will show how the CSI predicts the World Color Survey CS data [1] (WCS), that can be resumed as the prediction of the 4 universally unique colours. In the second experiment we will deal with the problem of finding the unique hues.

Experiment 1

WCS data was collected in order to extend the elementary theory of colour names developed by Berlin and Kay in 1969 [2]. In this early book they proposed an schema of how colour names correlates with the degree of evolution of different languages, converging to the most evolved ones as those having 11 basic terms. They provided psychophysical data for 20 written languages. With the goal of generalizing the results of this early experiment WCS data compile a similar experiment but with a wider range of languages and samples. Conclusions are not exactly the same. Six basic colours arise in this experiment: red, green, blue, yellow, black and white instead of the 11 proposed earlier. Their universality is still a controversial topic being supported in [6],[7], while contradicted in others [4],[12].

To recap, while Berlin and Kay original psychophysical data is collected from speakers of 20 written languages (where all the subjects spoke also English) and it finds 11 colour categories (8 of them chromatic: red, green, blue, yellow, pink, purple, orange and brown), WCS data is collected from 24 native speakers of 110 unwritten languages and it concluded that 6 colors arised (4 of them chromatic: red, green, blue, yellow). These last four colours are considered as the universal colours due to they appear in all the languages. See Figure 1 where we show both Berlin & Kay chromatic data 1.a, and WCS chromatic data 1.b.

Then, we will use our compact singularity index to fit the chromatic WCS data. We will then, for each chip in the dataset, use its reflectance to construct the matrix A_s and the reflection components (r_1, r_2, r_3). Once we obtain these values we will compute the compact singularity index for the surface. In figure 2 we can compare both singularities indexes (Philipona and O'Regan (SI) and our (CSI)) versus the WCS data. Figure 2.a represent the contour of the WCS data, where clearly the four colours appear. Figure 2.b is the contour produced by the singularity index developed by Philipona and O'Regan. Figure 2.c represents the contour produced by our compact singularity index. Here we can observe that the local maxima is close to the WCS data. Moreover, comparing figures 2.a 2.b and 2.c we can conclude that our formulation fits really well the blue and the yellow (better than Philipona and O'Regan) while in the red colour our CSI index obtains two local maxima (one perfectly located while the other is a few displaced), but when considering the influence region for both these maxima, the red region fits well with the WCS data. In both cases the green region is also well fitted.

The comparison of these results can be observed in figure 2.d where we plot an overlapping of the contours of the WCS data (Figure 2.a) and the level curves representing SI index (Figure 2.b). And in figure 2.e we plot the contours of the WCS data

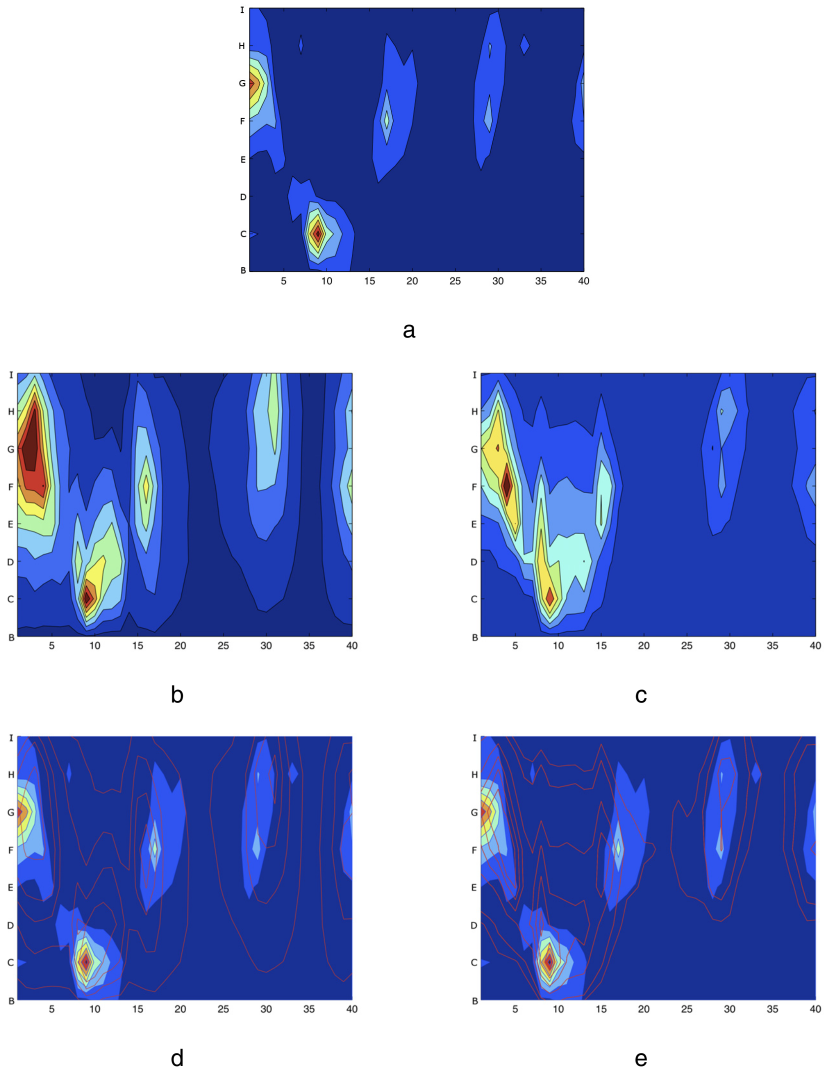


Figure 2. a) World Color Survey Data contour plot b) Philipona and O'Regan Singulariry Index plot c) Our CSI index plot d) Combination of (a) and the level curves of (b)(in red) e) Combination of (a) and the level curves of (c) (in red)

(Figure 2.a) and the level curves of our *CSI* index (Figure 2.c).

Experiment 2

Unique hues are still an open problem. There is not an accepted theory explaining the arise of this four unique hues [16]. Until now, neither the trichromatic theory nor the first opponent stages have dealt with an explanation of them. However, Philipona and O'Regan's biological model approximates efficiently these unique hues locations. Following their idea, we will also try to fit these unique hues by using our CSI index.

In order to use our formulation to fit unique hues we will make a similar assumption as is done in previous work. This means trying to simulate experiments where observers classically face 'aperture colours'. The main problem is while in these experiments the stimuli is created through the use of lights of controlled spectra composition projected directly into the eye, in our case the index works with surface properties. Then, we will use the assumption that the stimuli produced by these experiments is equivalent to the stimuli produced by the observation of a surface reflectance under the most common illuminant, *D*65.

Moreover, following again Philipona and O'Regan' paper, we will simplify the representation of the reflectances by using sums of only three basis functions, and we will plot the results of our Singularity Function in the CIE 1931 chromatic coordinates [17].

We have used as reflectances all the set of chips in the Munsell book. Our results are plotted in figure 3. In particular, in 3.a we can observe that again the four local maxima of our function are located on the position of the four unique hues. Moreover in figure 3.b we plot the contour of the surface in 3.a to better classify our local maxima.

Conclusion

Different approaches have previously tried to explain the perceptual asymmetries of colour, in particular, unique hues have been revealed as a key point on this research. However, the problem of unique hues is still open to debate. In this paper we have gone further in the idea developed by Philipona and O'Regan in [11] using their biological model to develop a new formulation regarding color properties (chromaticity). We have proved that our new compact singularity function (*CSI*) fits very well both, World Colour Survey data and Unique Hues data.

Moreover, the advantages of the new Compact Singularity Index (*CSI*) are twofold. Firstly, *CSI* formulation is completely compact, while previous formulation [11] is cumbersome. Secondly, *CSI* is related to a well-known colour measure about chromaticity.

However, considerable amount of work still needs to be done in this area. Firstly, Philipona and O'Regan biological model deals with some complex eigenvalues that are truncated. These complex eigenvalues leads to some numeric errors. Secondly, the fitting of data should be improved by going further into the *CSI* index and relating it to other colour properties.

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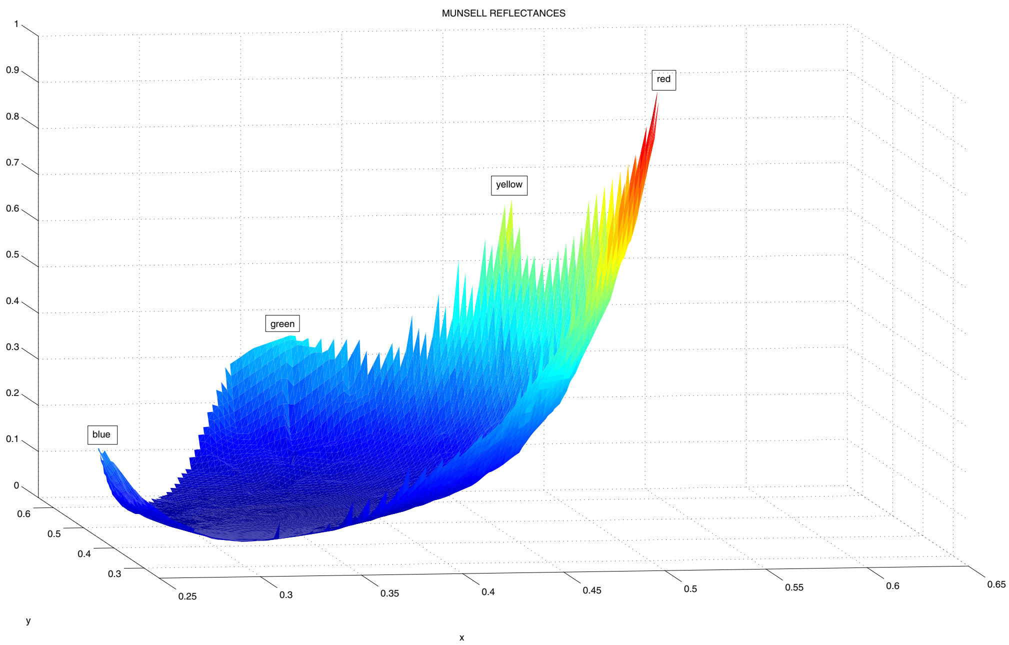
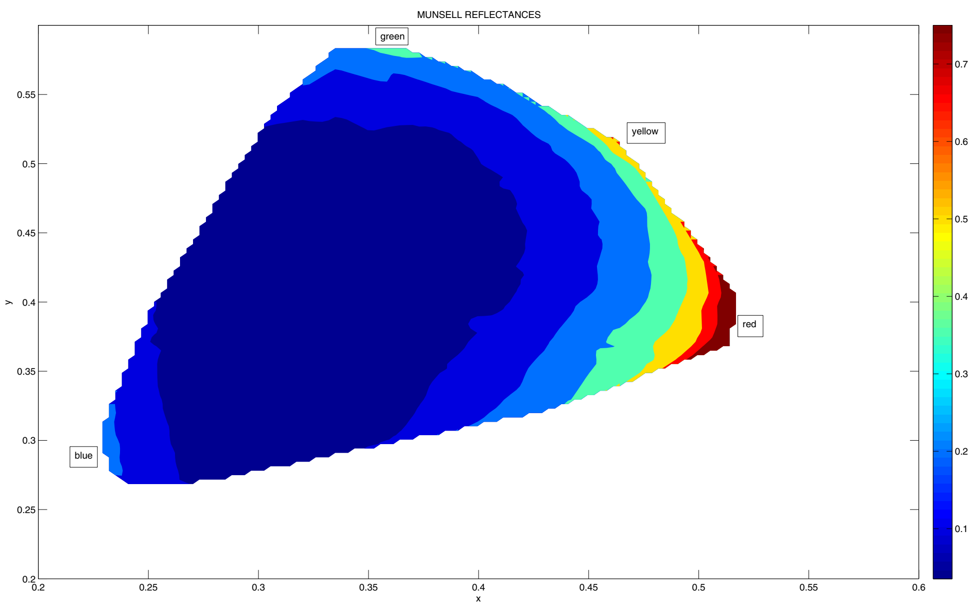


a



b

Figure 3. *a) Unique hues founded by our formula represented in the CIE xy Space b) Contour plot of our unique hues in the CIE xy space*