

Image Color Mapping and Clustering in Luma/Chroma Fundamental Color Space

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Abstract

Human vision extracts the visible spectral component C^* , called fundamental, from n -dimensional spectrum C . The projection from C to C^* is described by the matrix R in FCS (Fundamental Color Space). FCS is spanned by a matrix F with a selected triplet in R . The matrix R is decomposed into “achromatic” R_A and “chromatic” R_C by choosing matrix F .

This paper presents a Luma/Chroma opponent-color space that is created from spectral decomposition of fundamental based on matrix R theory. A new color space has orthogonal opponent-color axes with hue linearity because it's created through a linear naive transformation of fundamental in FCS.

The key points lie in that the “chromatic” projector R_C is further decomposed into R_R and R_B opponent-color components and an orthogonal Luma/Chroma FCS is newly created by a set of (R_A, R_R, R_B) , each composed of $n \times n$ matrix. Now image colors are mapped onto Luma/Chroma FCS. First, a tri-stimulus value XYZ from sRGB camera input is transformed back to the fundamental C^* by pseudo-inverse projection. Next, C^* is decomposed into the spectral triplet (C_A^*, C_R^*, C_B^*) through the (R_A, R_R, R_B) . Finally, the achromatic fundamental $C_A^*(\lambda)$, n -dimensional vector, is converted to the luminance value L_A by integral over λ . As well, the chromatic fundamentals, $C_R^*(\lambda)$ and $C_B^*(\lambda)$ are converted to the chrominance values C_R and C_B . The paper shows how the image colors are mapped onto (L_A, C_R, C_B) Luma/Chroma space and introduces its application to the image segmentation in comparison with conventional CIELAB and IPT color spaces.

Orthonormal Fundamental Color Space

This paper discusses a Luma/Chroma opponent-color space created from a spectral decomposition of fundamental based on matrix R theory. A new color space has orthogonal opponent-color axes with hue linearity because it's created through a naive linear transformation of fundamental in FCS.

A color matching function (CMF) transforms n -dimensional spectral input C into tri-stimulus vector $T = XYZ$. While, according to “matrix R ” theory by Cohen [1], C is decomposed into the fundamental C^* and metameric black B in spectral space as

$$C = [C(\lambda_1), C(\lambda_2), \dots, C(\lambda_n)]^t \quad (1)$$

$$C = C^* + B, \quad C^* = RC, \quad B = (I - R)C \quad (2)$$

I denotes unit matrix and R is the projector onto HVSS (Human Visual Sub-Space) derived from CMF A as

$$R = A(A^t A)^{-1} A^t \quad (3)$$

A is the $n \times 3$ matrix of 1931CIE $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ CMF.

The fundamental C^* is the intrinsic color stimulus that causes a unique XYZ sensation to human vision. The metameric black B is the residue insensitive to human vision and spans $n-3$

dimensional null space. C^* carries the essential spectrum to human vision and the tri-stimulus value of B is zero as follows.

$$T = [X, Y, Z]^t = A^t C = A^t C^*, \quad T_B = A^t B = 0 \quad (4)$$

The projector R is the $n \times n$ symmetric matrix whose i -th column vector E_i is composed of the fundamental corresponding to each single spectrum e_i at wavelength λ_i .

$$R = [E_1, E_2, \dots, E_i, \dots, E_n] \quad (5)$$

$$E_i = R e_i; \quad e_i = [0, 0, \dots, 1, \dots, 0]^t, \quad i\text{-th element} = 1 \quad (6)$$

Since the rank of R is 3, it has only 3 independent vectors and the remaining $n-3$ are redundant. We can recreate R by choosing arbitrary triplet from the column (row) vectors. The selected triplet is called “matrix E ” and $i=r, g, b$ show the spectral primaries at wavelength $\lambda_r, \lambda_g, \lambda_b$ as follows.

$$E = [E_r, E_g, E_b]^t \quad (7)$$

$$R = E(E^t E)^{-1} E^t \quad (8)$$

Indeed, Fig.1 shows the reconstructed matrix R from the middle three entries $[E_1, E_2, E_3]$ at $\lambda=540, 550$, and 560 nm. The FCS is a color space spanned by a triplet of basis vectors called “matrix F ”, which is orthonormalized version of matrix E using Gram Schmidt method as

$$F = [F_1, F_2, F_3] = \text{GramSchmidt}[E_1, E_2, E_3] \quad (9)$$

$$F^t F = I; \quad \langle F_j \bullet F_k \rangle = \begin{cases} 1 & \text{for } j = k \\ 0 & \text{for } j \neq k \end{cases} \quad (10)$$

The symbol $\langle u \bullet v \rangle$ denotes the inner product of u and v . When the $n \times 3$ matrix F is orthonormal,

$$F F^t = R \quad (11)$$

The selection of matrix E is very important to construct orthonormal FCS as suggested by Brill, Finlayson, et al [2]. For example, Burns, Cohen, and Kuznetsov [3] created an ortho-normal FCS called “ R - L - V ” choosing the matrix E with quasi-orthogonal axes of Red, Luminosity, and Violet. The historical orthogonal CMF by MacAdam¹ is also used as a matrix E and classical Guth's opponent CMF was recently orthonormalized by Worthey et al [4]. Kotera [5] reported that these CMFs span orthonormal FCSs close to R - L - V but slightly different.

Since the entry of matrix E may be a linear combination of arbitrary column vectors $\{E_i\}$; $i=1 \sim n$, for instance, the second vector E_2 may be given by the weighted sum of $\{E_i\}$ by illuminant D_{65} that means the fundamental of D_{65} itself.

$$E_2 = \sum_{i=1}^n D_{65}(\lambda_i) E_i = R D_{65}; \quad D_{65} = \text{SPD of illuminant} \quad (12)$$

$$D_{65} = [D_{65}(\lambda_1), D_{65}(\lambda_2), \dots, D_{65}(\lambda_n)]^t$$

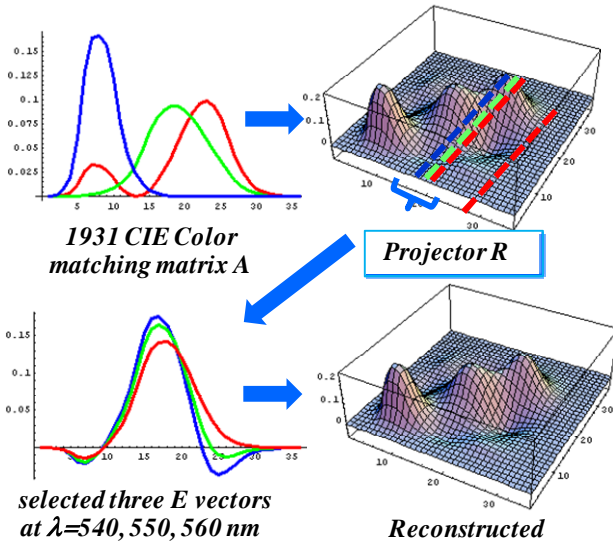


Figure.1 Matrix R and its reconstruction from matrix E

Spectral Decomposition of Fundamentals

Achromatic/Chromatic Fundamentals

Setting the basis F_2 to the achromatic fundamental and (F_1 , F_3) to arbitrary orthogonal chromatic fundamentals in matrix F , the matrix R is decomposed [6] as

$$\begin{aligned} R &= FF^t = R_A + R_C \\ R_A &= F_2 F_2^t, \quad R_C = R - R_A \end{aligned} \quad (13)$$

R_A and R_C are the orthogonal projectors to decompose the basic fundamental metamer C^* into the achromatic and the chromatic fundamentals C_A^* and C_C^* .

$$\begin{aligned} C^* &= C_A^* + C_C^* \\ C_A^* &= R_A C = R_A C^* \\ C_C^* &= R_C C = R_C C^* \end{aligned} \quad (14)$$

When the chromatic projector R_C is orthogonal to the achromatic projector R_A , the inner product between them should be zero as

$$\langle R_A \bullet R_C \rangle = 0 \quad (15)$$

Indeed, the matrix F in “ $R-L-V$ ” FCS is composed of orthonormal basis functions with achromatic luminosity axis and satisfies the condition of Eq. (15). As well, FCSs derived from CMF by MacAdam or Guth also satisfy the same condition.

Orthonormal Opponent FCS

As well known, Luminance/Chrominance color models with opponent-color axes assigned to “Red-Green ($R-G$)” and “Yellow-Blue ($Y-B$)” have been widely used for color imaging, analysis, and picture coding. YIQ used in NTSC Television is a opponent-color system by a simple linear transformation from XYZ . CIELAB is a most popular uniform color space mapped on the $R-G$ and $Y-B$ rectangular opponent-color coordinates. Hence, a foundation of FCS with orthogonal and opponent-color structure is a lot of fun in practical use.

The chromatic projector R_C is further decomposed into two opponent-color components R_R and R_B by choosing an appropriate pair of (F_1 , F_3). Thus, the chromatic fundamental

C_C^* is decomposed into opponent-color fundamentals C_R^* and C_B^* corresponding to $R-G$ and $Y-B$ hue axes as follows.

$$\begin{aligned} R_C &= R_R + R_B \\ R_R &= F_1 F_1^t, \quad R_B = F_3 F_3^t \end{aligned} \quad (16)$$

$$\begin{aligned} C_C^* &= C_R^* + C_B^* \\ C_R^* &= R_R C_C^*, \quad C_B^* = R_B C_C^* \end{aligned} \quad (17)$$

To be perfectly opponent for C_R^* and C_B^* , it is desirable that the projectors R_R and R_B satisfy the following zero-sum conditions.

$$\begin{aligned} \text{Sum}[R_R] &= \sum_{j=1}^n \sum_{k=1}^n R_R(j, k) = 0; \quad R_R(j, k) \text{ is } jk \text{ element} \\ \text{Sum}[R_B] &= \sum_{j=1}^n \sum_{k=1}^n R_B(j, k) = 0; \quad R_B(j, k) \text{ is } jk \text{ element} \end{aligned} \quad (18)$$

Although the matrix F in “ $R-L-V$ ” FCS surely satisfies the orthonormal condition in Eq. (10) and the orthogonality between achromatic and chromatic components in Eq. (15), the zero-sum condition in Eq. (18) didn’t hold good as well as MacAdam’s or Guth’s FCSs.

In the “ $R-L-V$ ” FCS, the fundamental of $\lambda_g = 563$ nm single spectrum is selected as the vector E_2 to reflect the Luminosity L axis, but this doesn’t exactly mean the luminance which is defined as a linear mixture of R , G , B components.

The zero sum condition can be obtained by replacing the vector E_2 with the fundamental of white illuminant such as EE (Equal-Energy) or $D65$. However EE is not popular as an illuminant in practice, while $D65$ is recommended as the most widely used illuminant. Hence “ $R-D65-V$ ” FCS was created by introducing the fundamental of $D65$ into vector E_2 in Eq. (13) and applying the GramSchmidt orthonormalization for matrix E to get matrix F .

Fig.2 summarizes how the matrix R is decomposed into the achromatic R_A and chromatic R_C components and further into the opponent-color projectors R_R and R_B . The left half is the Cohen’s “ $R-L-V$ ” and the right half is the proposed “ $R-D65-V$ ” FCS. Although R_A and R_C are orthogonal each other in both models, the zero-sum conditions in the projectors R_R and R_B don’t hold good for “ $R-L-V$ ” FCS, while they are almost maintained in the proposed “ $R-D65-V$ ” FCS under the negligibly small errors.

Orthogonal Luma/Chroma Color Space

Now a new Luma/Chroma FCS is founded using a set of achromatic and chromatic fundamentals (C_A^* , C_R^* , C_B^*). Since these decomposed fundamentals are denoted as $n \times 1$ vectors or scalar functions of λ , they are converted to the luminance and chrominance values to be located at luminance /chrominance coordinates (L_A , C_R , C_B) as follows. By taking the inner products with each basis vector in matrix F ,

$$\begin{aligned} L_A &= \langle C^* \bullet F_2 \rangle = \sum_{j=1}^n C^*(\lambda_j) F_2(\lambda_j) \\ C_R &= \langle C^* \bullet F_1 \rangle = \sum_{j=1}^n C^*(\lambda_j) F_1(\lambda_j) \\ C_B &= \langle C^* \bullet F_3 \rangle = \sum_{j=1}^n C^*(\lambda_j) F_3(\lambda_j) \end{aligned} \quad (19)$$

That is, the decomposed fundamentals (C_A^* , C_R^* , C_B^*) are simply related to

$$\begin{aligned} C^* &= C_A^* + C_R^* + C_B^* \\ C_A^* &= L_A F_2, \quad C_R^* = C_R F_1, \quad C_B^* = C_B F_3 \end{aligned} \quad (20)$$

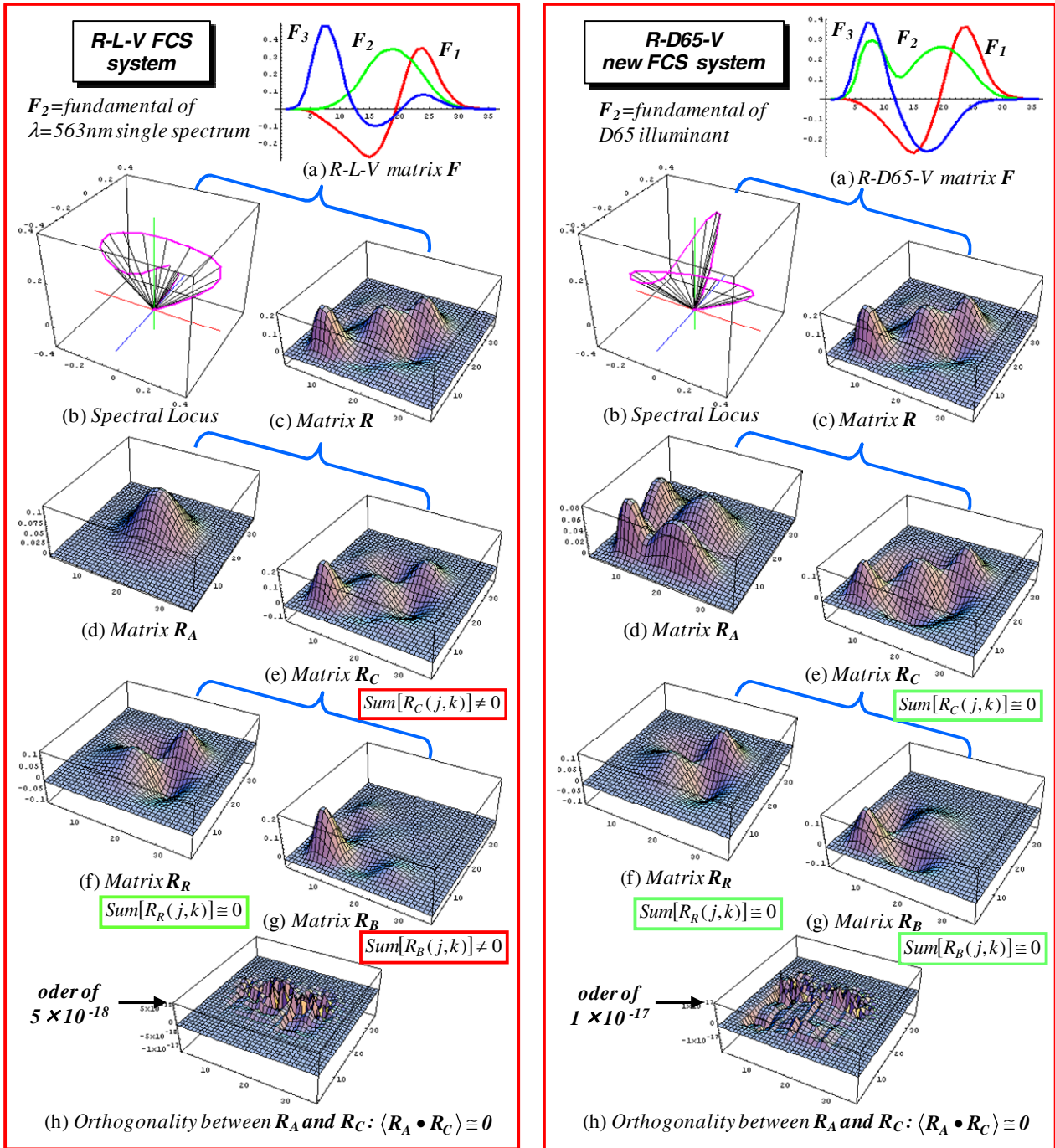


Figure.2 Matrix F and Luminance/Chrominance decomposition of matrix R (left: R-L-V, right: proposed R-D65-V model)

Here L_A reflects the luminance component as the integration of basis vector F_2 weighted by the fundamental C^* in λ_j . As well C_R and C_B correspond to the opponent-color R-G and Y-B chromatic components. Since the opponent-color fundamentals C_R^* and C_B^* are the function of λ_j , the chromaticity of an input fundamental spectrum C^* draws its specified locus in 2-D (C_R^* , C_B^*) plane.

Fig.3 illustrates the spectral decomposition of fundamental C^* for the IT8/7.2 skin color chip #125 and its spectral locus in 2-D chrominance (C_R^* , C_B^*) plane and its chrominance (C_R , C_B) is pointed by the vector.

Now we can map the fundamental C^* onto the new 3-D orthogonal luminance/chrominance color space pointed by the coordinates (L_A , C_R , C_B) after spectral decomposition in FCS.

However (L_A , C_R , C_B) is not a uniform color space like as CIELAB, because it is basically derived from the linear transformations from

original tri-stimulus value XYZ. Then in practical use, it'll be better for (L_A , C_R , C_B) values to be compressed in nonlinear manner. Following the IPT opponent-color model [8] with excellent hue linearity, the power function with exponent of γ was taken into account for the compression as follows.

$$\tilde{L}_A = L_A^\gamma, \quad \tilde{C}_R = C_R^\gamma, \quad \tilde{C}_B = C_B^\gamma \quad (21)$$

Here we call the γ -compressed ($\tilde{L}_A, \tilde{C}_R, \tilde{C}_B$) as Luma/Chroma notated by popularly used words for the signals after γ -correction, that is, "Luma" for luminance and "Chroma" for chrominance.

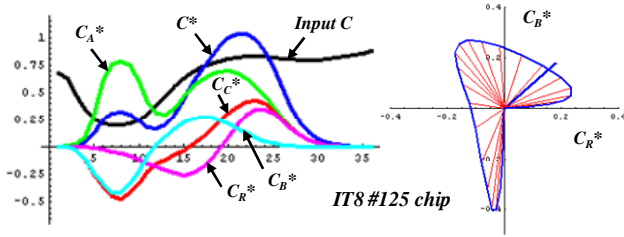


Figure.3 Spectral decomposition of IT8 #125 chip and its chromatic locus

Image Color Mapping Experiments

The characteristics of the proposed *FCS* have been examined by the color mapping experiments on some test color chips and natural color images. Again, it should be taken notice that the proposed model transforms an input color into $(\tilde{L}_A, \tilde{C}_R, \tilde{C}_B)$ space based on the spectral decomposition of *fundamental*, though it needs not any spectral input but just a normal tri-stimulus *XYZ* or tri-color *RGB* camera input.

Transformation of XYZ to Fundamental

The proposed *Luma/Chroma* color model needs not any spectral reflectance input *C* but needs the *fundamental C** of each pixel to map the image colors. Since *C** carries the correct tri-stimulus value $T = XYZ$ as given by Eq. (4), we can get *C** by the *inverse projection* from *T* to *C** [7].

Applying the generalized pseudo-inverse projector P_{INV} ,

$$C^* = P_{INV}T \quad (22)$$

$$P_{INV} = A(A'A)^{-1} \quad (23)$$

Now, the *sRGB* image data $X_j = [R(j), G(j), B(j)]^t$ ($j=1 \sim N$ pixels) are transformed firstly to *XYZ* tri-stimulus values, secondly to the *fundamental C*(j)* ($j=1 \sim N$) by Eq.(22) ~ (23), and finally to the corresponding *Luma/Chroma Z_j* = $[L_A(j), \tilde{C}_R(j), \tilde{C}_B(j)]^t$ through Eqs. (14) ~ (21).

Fig.4 illustrates the *pseudo-inverse projector P_INV* and the *fundamental C** of IT8/7.2 color chip #125 reconstructed from its *XYZ* value taken under D65 illuminant.

Hue Linearity of RGBCMY Color Tone

The defect in hue non-uniformity of CIELAB or CIECAM97 has been much improved by IPT color model [8]. As well, the proposed model is basically expected to hold the hue linearity, because the model is derived from a "naïve" linear transformation of *fundamental* based on *matrix R* theory, a root of human vision.

Fig. 5 compares a hue linearity for the pure primary *RGB* and secondary *CMY* color tones. Six mono-color tones with 256 gradation were generated by computer, where $C=G+B$ (for $R=0$ and $G=B=0 \sim 255$), $M=R+B$ (for $G=0$ and $R=B=0 \sim 255$), and $Y=R+G$ (for $B=0$ and $R=G=0 \sim 255$) are displayed as *sRGB* data. Against the non-linear distorted hue lines in CIELAB, the proposed *Luma/Chroma FCS* resulted in the straight hue linearity as well as IPT. It's notable that the six *RGBCMY* hue lines are mapped in almost perfect opponent-color directions in the new model. Through all the experiments, $\gamma=0.43$ is used as same as IPT.

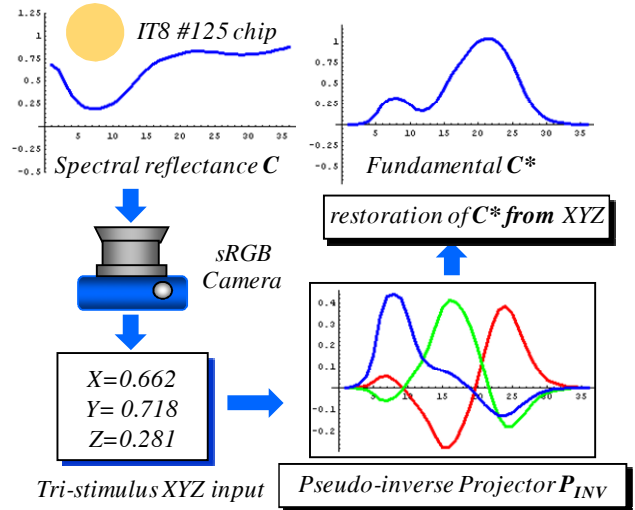


Figure.4 Pseudo-inverse transform from XYZ to fundamental

Mapping of Uniform OSA Color Chips

A set of radial sampled chips in *OSA uniform color scales* by Moroney [9] was tested in comparison with CIELAB and IPT. As shown in Fig.6, *IPT* looks to have the best uniformity, while the proposed *FCS* also gave the better uniformity than CIELAB.

Mapping of sRGB Color Chips

Fig.7 compares the mapping results in computer generated *sRGB* chips. Totally 9621 combinations of chip data are distributed to fill the *sRGB* display gamut. Different from Fig.6, the mapped colors by proposed model look to be widely and uniformly spreading than CIELAB and IPT.

Application to Image Color Clustering

Since the proposed *Luma/Chroma FCS* has orthogonal *opponent-color* axes and well-separated hue linearity, it may be useful for color clustering that is a key technology to image segmentation.

Fig.8 shows a sample of image color mapping and segmentation in the new *FCS*. As shown in Fig.5, the gradations of pure primary and secondary colors are mapped on straight lines in the new *FCS*. The color maps of *sRGB* image "parrot" shown in (b) are classified by the most popular *k-means clustering* method [10] and segmented as shown in the outline contours in (a).

Fig.8 (c) and (d) show another sample for Scid2 image "flower".

In the discrimination of clustered colors, the proposed *Luma/Chroma FCS* resulted in the better performance than conventional CIELAB and roughly close to IPT.

Conclusions

The paper proposed a new *Luma/Chroma FCS* with orthogonal *opponent-color* axes. Since the model is introduced from a *naïve* transformation by spectral decomposition of *fundamental* based on *matrix R* theory, it has the excellent hue linearity and perfect opponent axes. Image colors mapped onto the proposed new coordinate system are expected to reflect that naïve *achromatic/chromatic* spectral separation characteristics of human color vision. Further research on the better *FCS* and applications of its unique features to better color imaging are to be continued.

References

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Author Biography

Hiroaki Kotera joined Matsushita Electric Industrial Co., in 1963. He received Doctorate from Univ. of Tokyo, Japan. After worked in image processing at Matsushita Res. Inst. Tokyo, during 1973-1996, he was a professor at Information and Image Sciences, Chiba University until his retirement in 2006. He received 1995 Johann Gutenberg prize from SID, 2005 Chester Sall award from IEEE, 2006 journal award from ISJ and 2007 Raymond Bowman award from IS&T. He is a Fellow of IS&T.

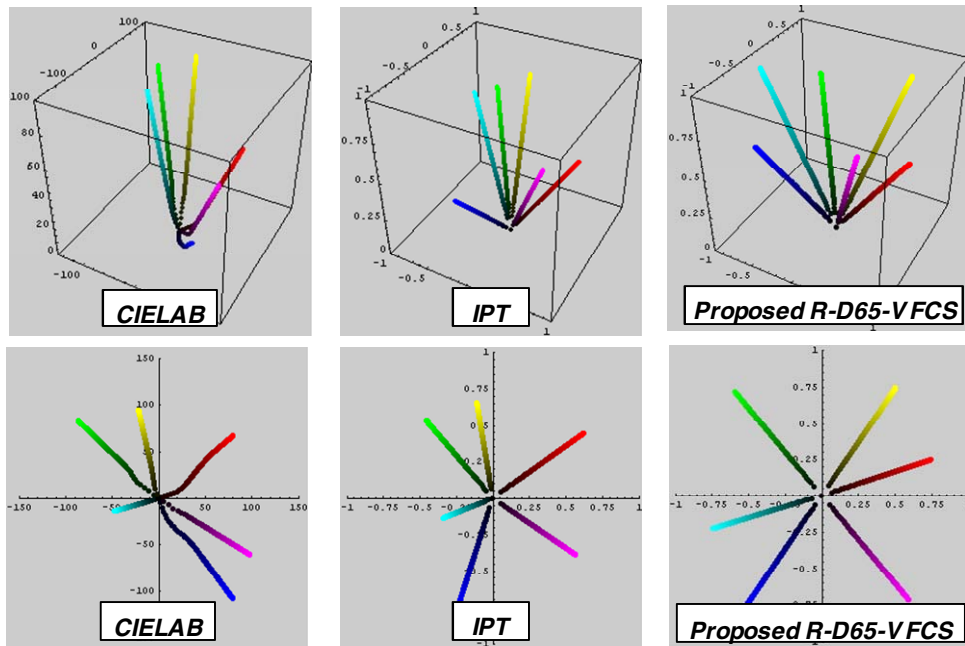


Figure 5. Color map of RGBCMY Color Tone (256 gradations)

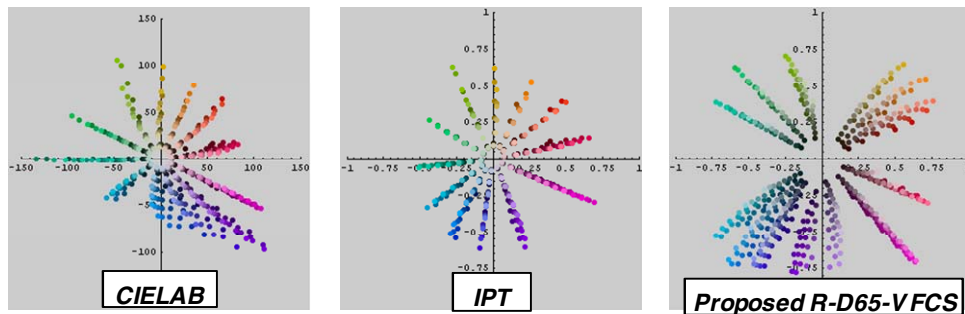


Figure 6 Color map of radial sampled uniform OSA chips

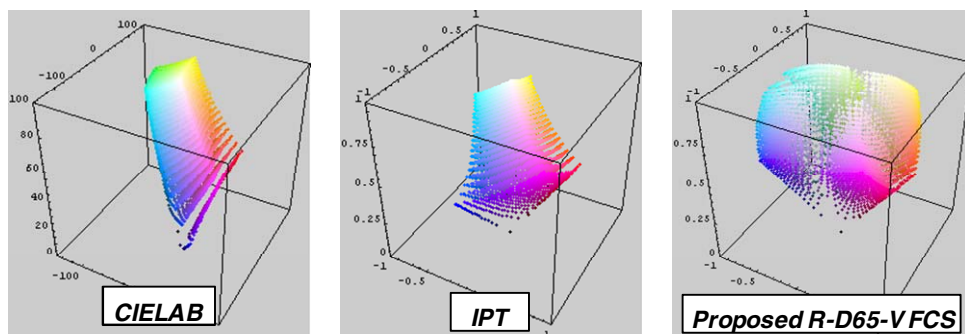
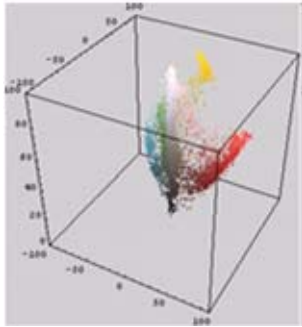


Figure 7. Color map of sRGB 9621 chips

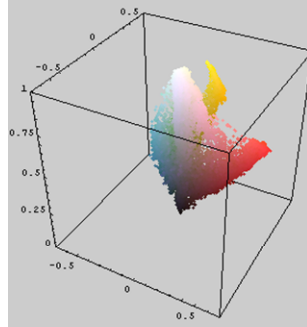


CIELAB **IPT** **Proposed R-D65-VFCS**

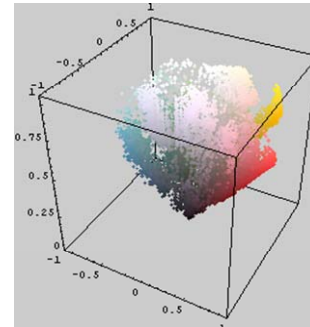
(a) Segmented results for "parrot" by k-means clustering method (K=8)



CIELAB



IPT



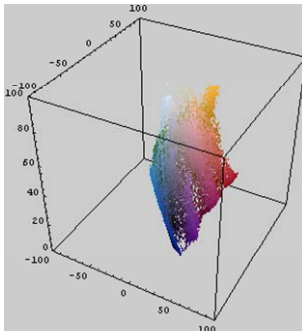
Proposed R-D65-VFCS

(b) Clustered color distribution for "parrot" in 3-D color space

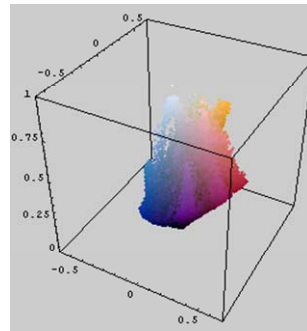


CIELAB **IPT** **Proposed R-D65-VFCS**

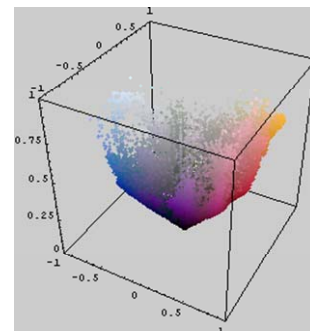
(c) Segmented results for "flower" by k-means clustering method (K=13)



CIELAB



IPT



Proposed R-D65-VFCS

(d) Clustered color distribution for "flower" in 3-D color space

Figure 8 Image segmentation results by k-means clustering in proposed R-D65-V Luma/Chroma FCS in comparison with CIELAB and IPT