Broad Band Filter Selection by Approximating Principal Components of Reflectance Spectra

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Abstract

In this paper, a multispectral camera equipped with a number of broad band filters arranged in a filter wheel is in focus. The different spectral transmittances of the filters allow for the capture of a number of different image separations from which the spectral color stimulus of each image pixel is estimated. A large number of thin film and low cost filters is available on the market. Here, a method selecting a limited number of filters allowing for effective spectral reconstruction is proposed. The strategy is based on the concept of selecting the filters in such a way that a linear combination of the resulting camera sensitivities approximates the principal components of a representative spectral reflectance set as well as possible. The filter selection consists of an iterative method that eliminates filters from the basic set of available filters until the desired number of filters is left. The spectral estimation is based on estimating the weights of the basis vectors from the sensor response on one hand and using Wiener inverse on the other hand. Simulated spectral estimation results based on a multispectral camera equipped with the selected filters are given as well.

Introduction

The spectral separation in a multispectral camera can be obtained using optical broad band or narrow band filters combined with a greyscale sensor. Usually, narrow band filters are quite expensive and if mounted between lens and sensor tend to degrade the image quality due to a certain thickness and other reasons [1, 2]. On the other hand, a large number of inexpensive broad band filters are available on the market. This paper deals with the selection of a set of appropriate broad band filters from a comprehensive set of available filters.

Some research has been carried out on this subject in the past. Ng and Allebach presented a subspace matching filter design method [3], Berns et. al. studied a combination of a conventional RGB-camera and absorption filters [4], Imai et. al. compared narrow band with wide band filtering for spectral reflectance reconstruction [5], Baribeau examined the selection of optimal laser wavelengths for estimation of object reflectances [6], and Hardeberg compared a number of filter selection methods [7]. Schettini et. al. proposed a filter selection method for narrow band filters and a second method based on eigenvectors of sensor responses [8]. Maître et. al. proposed a method based on maximizing the projections of filter transmittance functions onto eigenvectors of a representative spectral dataset [9].

The approach shown in this paper is based on the idea to select a subset of filters in such a way that a linear combination of the resulting sensor sensitivities approximates a given set of basis vectors. The filter selection is carried out as an iterative elimination of filters from a given set of filters until the desired number of filters is left. Filters are eliminated if they exhibit a low contribution to the linear combination. The basis vectors are derived in advance by performing a Principal Component Analysis on a representative spectral data set. If an approximate linear transform between the resulting channel sensitivities and basis vectors can be formulated, the same transform will link sensor responses to weights of the basis vectors. Hence, these weights can be estimated from the sensor response and used for spectral reconstruction.

Basic Concept

Applying a Principal Component Analysis (PCA) onto a spectral reflectance data set $\mathbf{R}$ consisting of a large set of reflectance functions $\mathbf{r}_j$ yields a system $\mathbf{B}$ of orthonormal basis vectors $\mathbf{b}_i$ (row vectors):

$$\underbrace{\begin{pmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_B \end{pmatrix}}_{\mathbf{B}} = \mathrm{PCA}\left(\mathbf{R}\right), \tag{1}$$

where B is both the number of basis vectors and the number of rows in the matrix $\mathbf{B}$. The projection of any given spectral column vector $\mathbf{r}_j$ onto $\mathbf{B}$ yields the column weight vector $\mathbf{w}_j$

$$\mathbf{w}_j = \mathbf{B} \cdot \mathbf{r}_j \tag{2}$$

that describes to the weights of the basis vectors $\mathbf{b}_i$ in $\mathbf{r}_j$. Due to the orthonormality of $\mathbf{B}$ there is an inverse operation $\mathbf{B}^{-1} = \mathbf{B}^T$ that reconstructs exactly the original spectral vector $\mathbf{r}_j$ from the weight vector $\mathbf{w}_j$:

$$\mathbf{r}_j = \mathbf{B}^T \cdot \mathbf{w}_j \tag{3}$$

where $\mathbf{X}^T$ denotes the transposed matrix of $\mathbf{X}$. An important characteristic of $\mathbf{B}$ is that most of the information is concentrated in the first few components $\mathbf{b}_i$ with $i \ll B$, so that most of the spectra $\mathbf{r}_j$ can typically be described at very small error by a linear combination of these first vectors while the components of higher order are omitted. The weights $w_{j,i}$ in $\mathbf{w}_j$ statistically decrease with increasing i. The first six basis vectors calculated from a PCA of the dataset by Vrhel et. al. [10] are shown in fig. 1.

If it would be possible to realize a set of sensor sensitivities $\mathbf{A}$ identical to the basis vectors $\mathbf{B}$, the sensor signals $\mathbf{a}$ would equal the projection of the recorded spectrum onto the orthonormal basis system (eq. 2):

$$\mathbf{a} = \mathbf{A} \cdot \mathbf{r} = \mathbf{B} \cdot \mathbf{r} = \mathbf{w}, \tag{4}$$

where $\mathbf{A}$ contains the spectral sensor sensitivities $\mathbf{a}_i$ as row vectors. In this case, an exact spectral reconstruction would be possible by applying eq. 3. However, the basis vectors derived by PCA have negative parts, so they are not directly physically realizable as sensor sensitivities. What can be done, though, is trying to find a set of sensor sensitivities that can be linearly combined to approximate the basis vectors. The method proposed here is to select a subset of K filters from a big set of F available filters in such a way that these K filters form an approximate linear combination given by the matrix $\mathbf{K}_K$ of B basis vectors and vice versa:

$$\underbrace{\begin{pmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_B \end{pmatrix}}_{\mathbf{B}} \approx \underbrace{\begin{pmatrix} k_{1,1} & \dots & k_{1,K} \\ \vdots & & \vdots \\ k_{B,1} & \dots & k_{B,K} \end{pmatrix}}_{\mathbf{K}_K} \cdot \underbrace{\begin{pmatrix} \mathbf{a}_1 \\ \vdots \\ \mathbf{a}_K \end{pmatrix}}_{\mathbf{A}_K}, \tag{5}$$

where the row vectors $\mathbf{b}_i$ and $\mathbf{a}_j$ denote the basis vectors and sensor sensitivities, respectively, and $k_{i,j}$ are the coefficients describing the linear combination.

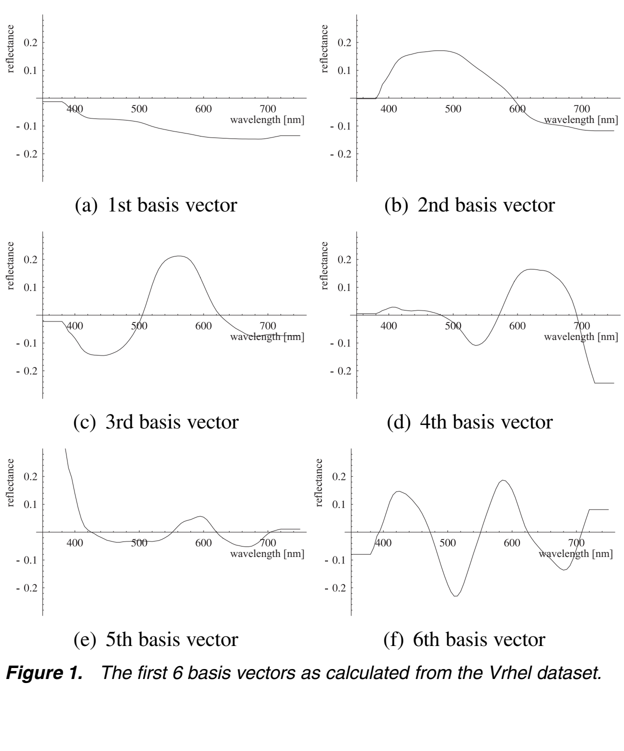
The sensor response vector $\mathbf{a}$ obtained by the sensor $\mathbf{A}_K$ can be transformed by the same matrix $\mathbf{K}_K$ into the estimated weights of the basis vectors $\widehat{\mathbf{w}}$, and, thus, the projection of the original spectrum $\mathbf{r}$ onto the orthonormal basis system $\mathbf{B}$ can be estimated:

$$\widehat{\mathbf{w}} = \mathbf{K}_K \cdot \mathbf{a}, \tag{6}$$

Finally, the spectral vector $\hat{\mathbf{r}}$ can be estimated based on these weights $\widehat{\mathbf{w}}$, i.e. $\mathbf{w}_j = \widehat{\mathbf{w}}$ in eq. 3. Alternatively, the inversion of $\mathbf{A}_K$ can be conducted using known linear inversion methods such as pseudoinverse or Wiener inverse, as well.

Selection Algorithm

Depending on the characteristics of the given filter set, it might be important to carry out a reduction of the number of available filters F in advance. There might be pairs of filter transmittances $\mathbf{f}_p$ and $\mathbf{f}_q$ that are approximately colinear, so that $\mathbf{f}_p \approx k\mathbf{f}_q$, with $k \in \mathbb{R}$. Furthermore, if a pair of filters fulfills $\left|\mathbf{f}_p - k\mathbf{f}_q\right| \le \varepsilon$ the corresponding sensor responses will approximately be $\mathbf{a}_p \approx k\mathbf{a}_q$, i.e. the signals are highly redundant. Thus, one of the two filters can be eliminated from the data set in advance without loss of information. Apart from that, similar sensor sensitivities degrade the system performance by introducing strong noise problems.



(a) 1st basis vector (b) 2nd basis vector (c) 3rd basis vector (d) 4th basis vector (e) 5th basis vector (f) 6th basis vector

***Figure 1.** The first 6 basis vectors as calculated from the Vrhel dataset.*

The elimination of redundant filter pairs is realized in two steps. First, all filter transmittance functions are normalized, so that $|\mathbf{f}_i| = 1$ for any filter i. Then, the $F \times F$ dimensional matrix

$$\mathbf{C} = \begin{pmatrix} \mathbf{f}_1\mathbf{f}_1 & \dots & \mathbf{f}_1\mathbf{f}_F \\ \vdots & \ddots & \vdots \\ \mathbf{f}_F\mathbf{f}_1 & \dots & \mathbf{f}_F\mathbf{f}_F \end{pmatrix} \tag{7}$$

is calculated from the scalar products $\mathbf{f}_i\mathbf{f}_j$ of the respective filter transmittances. The largest numbers in $\mathbf{C}$ indicate highly redundant filter pairs, one of which is eliminated from the data set. Orthogonal filter transmittances lead to zero covariance. Practical broad band filters do not show this. The following considerations are based on the filter set that results from the reduction described here.

The effective sensor sensitivity $\mathbf{a}_i$ of channel i resulting from the greyscale sensor sensitivity $\mathbf{s}$, the respective filter transmittance $\mathbf{f}_i$ and other effects such as objective lens transmittance, etc., $\mathbf{o}$ can be expressed as the component-wise multiplication of these vectors:

$$\mathbf{a}_i = \mathbf{s} \cdot \mathbf{f}_i \cdot \mathbf{o}. \tag{8}$$

In order to find a subset of K filters of the set of F available filters, an iterative algorithm has been implemented. It successively eliminates filters from the set of all F filters until the desired number of filters is left. In the first step, a linear combination $\mathbf{K}_F$ of all F available sensor sensitivities $\mathbf{A}_F$ is calculated. $\mathbf{A}_F$ results from applying all F filters to eq. 8. If the filter set is sufficiently big compared to the set of basis vectors (i.e. $F \gg B$), this linear combination will approximate the given B basis vectors $\mathbf{b}_i$ in the matrix $\mathbf{B}$ at very little error. The matrix $\mathbf{K}_F$ transforming $\mathbf{A}_F$ onto $\mathbf{B}_B$ can be obtained by computing the pseudoinverse $\mathbf{A}_F^+$ of $\mathbf{A}_F$ and multiplying it on both sides of eq. 5, which leads to:

$$\mathbf{K}_F \approx \mathbf{B} \cdot \mathbf{A}_F^+. \tag{9}$$

Now, the filter with the smallest influence on the linear combination is selected from $\mathbf{A}_F$ by searching for the column in $\mathbf{K}_F$ that contains the smallest absolute mean value. This filter is eliminated from $\mathbf{A}_F$ leading to a matrix $\mathbf{A}_{F-1}$ with the number of rows decreased by one. Next, the pseudoinverse $\mathbf{A}_{F-1}^+$ of $\mathbf{A}_{F-1}$ is calculated and used to compute the new linear combination $\mathbf{K}_{F-1}$ of the remaining $F-1$ filters following eq. 9. Again, the filter with the smallest, absolute, mean weight is deleted from $\mathbf{A}_{F-1}$ and the new matrix $\mathbf{K}_{F-2}$ is computed. This step is repeated successively until the desired number of filters K has been achieved or, a maximum error in terms of root mean square error between the original basis vectors and the approximating linear combination of the selected filters is exceeded.

Results

In the present study, the basic set of available filters consists of three commercially available filter sets: the Rosco,[1] Gamcolor,[2] and Lee filters,[3] resulting in a set of 827 filters. The filters were measured using a spectral photometer[4] providing spectral transmittance data between 200 nm and 800 nm in 0.6 nm steps. By cutting off UV and IR parts the spectra were reduced to comprehend only the visual part of the spectrum from 350 nm to 750 nm. They were reinterpolated in 1 nm steps.

[1] Rosco Laboratories, USA
[2] GAM Products, USA
[3] Lee Filters, Great Britain
[4] Dr. Gröbel UV-Elektronik, Germany

Since the filter set created in this way contained a lot of very similar filter pairs, a reduction of the number of filters performed before the execution of the selection algorithm. The reduction was done using the method described in the previous section. Due to the high redundancy of the original filter set, approximately 600 filters could be eliminated without observable loss of information. The method resulted in $F = 227$ remaining filter transmittances. The set of basis vectors $\mathbf{B}$ was calculated using PCA from the spectral remission dataset of Vrhel et. al.

The CCD sensor IXC085AL by Sony is a commonly used image sensor and was chosen for the simulations performed here. Its spectral sensitivity $\mathbf{s}$ (eq. 8) is given in fig. 2. The spectral transmittance function of the objective lens and all other optical components concatenated in $\mathbf{o}$ were assumed to be without influence, i.e. $\mathbf{o} = (1, \ldots, 1)^T$ inside the spectral range considered here. In practice, all filters used in this study tend to transmit close to 100% in the infrared part of the spectrum. Therefore, an additional filter is necessary that blocks any light outside the spectral range studied here, because the CCD is still sensitive to such wavelengths. In this work, an ideal filter was assumed with 100% transmittance inside and 0% transmittance outside the interval [350 nm, 750 nm].

Two main characteristics of eq. 5 were examined in this work. Firstly, what is the number B of basis vectors, i.e. the number of rows in the matrix $\mathbf{B}$, that should be approximated? Secondly, how many sensor channels K are necessary so that a sufficiently exact linear combination of the given B basis vectors is constituted? The achievable quality of the spectral estimation served as a quality criterion. As indicated in the previous section, the spectral estimation was done in three ways, the first of which consisting of an estimation of the weights $\widehat{\mathbf{w}}$ of the basis vectors and superposing the weighted basis vectors obtained by a PCA on the Vrhel dataset. In the second case, the estimated weights $\widehat{\mathbf{w}}$ were used to superpose the approximation of the PCA basis vectors. In the third case, the sensor matrix $\mathbf{A}_K$ was inverted using Wiener inverse. In order to have a different test set than the one used for PCA, the dataset by Pointer [11] served as a representative collection of reflectance spectra. It was used to calculate the mean color distance expressed in CIE ΔE_{ab} between the original colors and the captured and estimated colors as a function of B, K, and the spectral estimation method. The results of the simulations are summarized in figs. 6 - 8. The bottom line marked "id" shows the simulation results for the theoretical case that the sensor sensitivities exactly match the PCA basis vectors, i.e. $\mathbf{A} = \mathbf{B}$ as a function of the number of channels K.

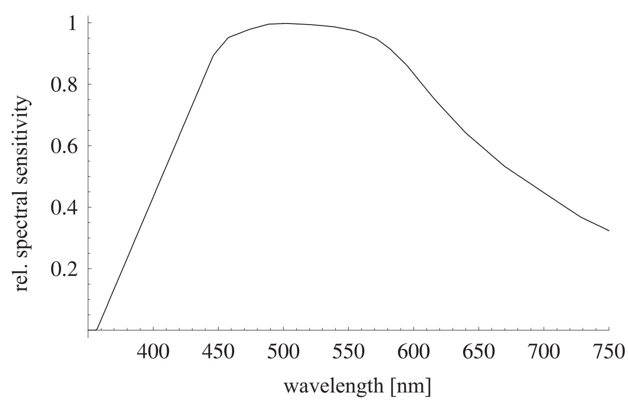


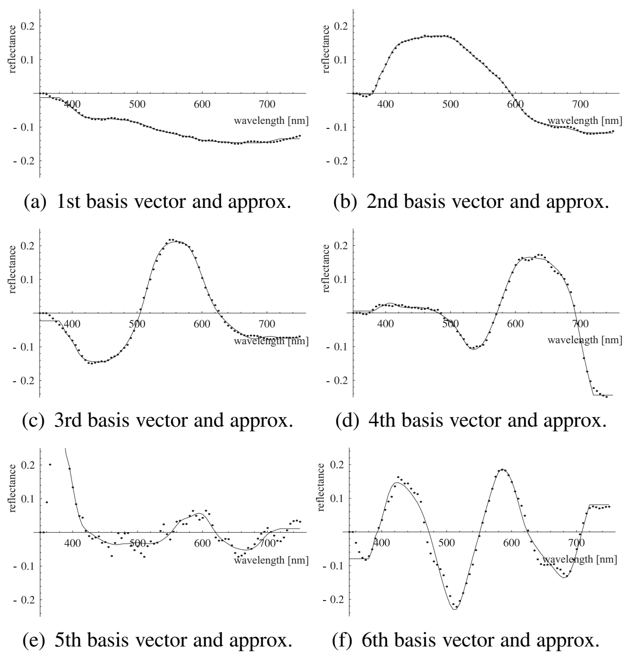
Figure 2. *Sony ICX085AL sensor sensitivity.*

In the first case (fig. 6), the spectra were reconstructed by a weighted linear combination of the basis vectors generated by PCA. The weighting was done based on the estimated weights $\widehat{\mathbf{w}}$ calculated from the sensor response (see eq. 6). The simulation results show that two conditions must be fulfilled in order to assure a mean color distance below 1: the number of basis vectors B should not be below 8 and the number of channels K should not be below 13.

In the second case (fig. 7), the spectra were again reconstructed by a weighted linear combination of basis vectors. Unlike the first case, these were not exactly the basis vectors generated by PCA, but their approximations given by the linear combination $\mathbf{K}_K \cdot \mathbf{A}_K$ of the sensor sensitivities. The results are in general quite similar to those in the first case, though they tend to be slightly worse.

In the third case (fig. 8), the spectra were estimated using Wiener inversion of the sensor sensitivity matrix $\mathbf{A}_K$. In this case no dependency can be observed between the quality of the estimation results and the number of basis vectors B that are approximated. There is merely a dependency on the number of channels B showing that if 9 or more channels are used the estimation results will drop below $\Delta E_{ab} = 1$.

A typical result of the approximation of the basis vectors generated by PCA using the algorithm is given in fig. 3. In the case depicted here, $B = 6$ basis vectors were approximated using $K = 16$ channels. There is quite a good match between the basis vectors and the linear combination of sensor sensitivities. Fig. 4 shows the sensor sensitivities selected by the algorithm.



(a) 1st basis vector and approx. (b) 2nd basis vector and approx.
(c) 3rd basis vector and approx. (d) 4th basis vector and approx.
(e) 5th basis vector and approx. (f) 6th basis vector and approx.

Figure 3. *The first 6 basis vectors as calculated from the Vrhel dataset (solid lines) and their respective approximations (dotted lines) by performing a linear combination of the spectral sensitivities selected by the algorithm with the number of channels $K = 16$ and the number of approximated basis vectors $B = 6$. In all cases a quite good match can be observed.*

The basis vectors used in this research were computed from the Vrhel dataset using equal energy white as light source. In order to examine the estimation results if other light sources than E are used, the capture and estimation processes were simulated for illuminants A, B, C, D50, D65, as well. In all of these cases, the spectral estimation was done using Wiener inverse. It was found that the mean and maximum ΔE_{ab} values did not increase significantly as a function of different light sources. On the contrary, they even decreased in some cases. The results are summarized in fig. 5.

Conclusion

In this paper, a filter selection method is proposed and examined. It is based on the concept of iteratively eliminating filters from a large set of available broad band filters. The iterative deselection of filters results in a set of sensor sensitivities that approximates a linear combination of basis vectors of a representative spectral remission data set. Based on the resulting sensor response, spectral estimation can be carried out by estimating the weights of the basis vectors or using Wiener inversion. In this paper, minimum numbers of channels and basis vectors are given that are required to gain a certain quality of spectral estimation as a function of the spectral estimation method. The best estimation results were obtained by using Wiener inverse.

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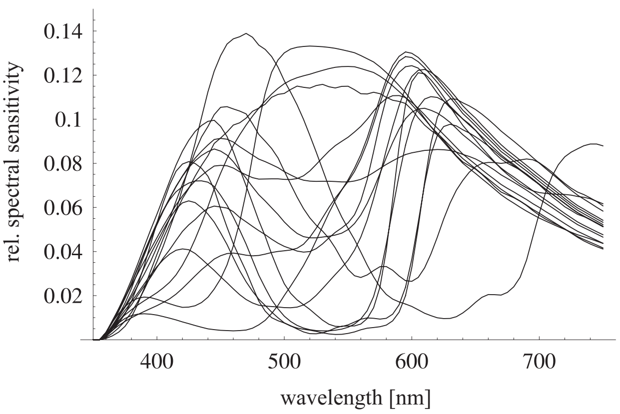

Figure 4. *The 16 sensor sensitivities resulting from the selection of 16 transmittances from the filter set using the proposed algorithm.*

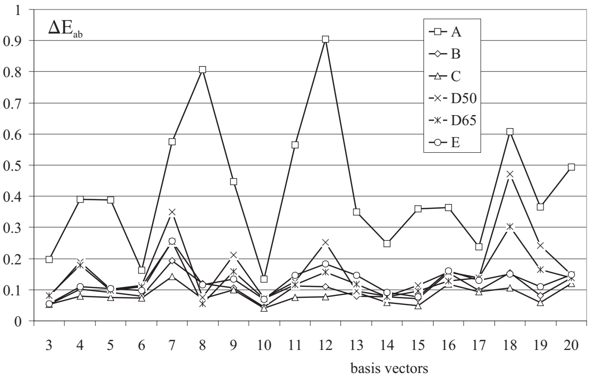

Figure 5. *The influence of the use of different illuminants for the capture process on the quality of the spectral estimation.*

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Author Biography

Stephan Helling received his diploma degree in Electrical Engineering from the RWTH Aachen University in 2001. He is now engaged in research on multispectral imaging systems with focus on multispectral cameras at the same university in the Color and Image Processing Research Group. He also joined Color AIXperts GmbH, Germany. He is member of IS&T and the German Society for Color Science and Application DfwG.

number of channels K

number of basis vectors B	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3	17.67	17.85	14.93	14.87	15.00	15.95	15.79	15.67	15.63	15.62	15.63	15.60	15.61	15.61	15.61	15.61	15.60	15.59
4	39.03	30.23	9.00	9.29	9.35	9.30	9.74	9.69	9.92	9.92	9.84	10.03	10.03	9.98	10.01	10.02	9.98	9.98
5	19.88	13.59	10.06	10.62	9.30	9.24	8.33	9.31	9.46	9.69	9.66	9.68	9.68	9.70	9.68	9.69	9.69	9.71
6	28.19	4.22	6.02	5.18	3.35	3.23	3.21	3.19	2.98	2.99	2.99	2.98	2.97	2.95	3.00	2.98	2.93	2.94
7	10.39	10.16	6.36	4.22	4.43	2.18	2.28	1.99	2.04	2.08	2.04	1.79	1.78	1.78	1.79	1.80	1.80	1.80
8	11.77	9.86	9.67	9.45	7.42	4.35	3.23	2.56	2.35	2.39	2.03	2.03	2.02	1.97	1.93	1.91	1.91	1.92
9	34.16	14.70	10.23	6.72	5.11	4.00	3.84	4.61	4.65	4.34	1.31	1.31	0.86	0.82	0.81	0.85	0.83	0.84
10	31.22	9.85	9.84	4.99	4.01	4.15	2.45	2.45	2.99	1.48	1.35	0.90	0.90	0.90	0.91	0.76	0.74	0.62
11	10.43	9.87	10.97	9.18	5.93	5.68	5.24	3.56	0.91	0.71	0.68	0.50	0.57	0.60	0.58	0.56	0.57	0.56
12	8.92	10.12	9.59	6.26	6.48	7.09	2.99	0.61	0.68	1.34	1.35	0.81	0.85	0.77	0.68	0.55	0.58	0.50
13	31.40	10.16	10.05	10.00	4.85	4.80	2.98	2.78	3.32	1.52	1.45	0.64	0.48	0.32	0.34	0.33	0.35	0.30
14	31.42	10.22	10.11	4.70	4.65	3.55	2.86	2.05	2.00	1.18	1.10	0.30	0.35	0.45	0.43	0.29	0.28	0.18
15	31.37	10.20	10.06	10.22	4.15	4.03	2.33	2.68	1.68	1.03	0.87	0.88	0.95	0.85	0.69	0.21	0.16	0.16
16	10.45	9.69	12.15	13.76	3.08	2.67	2.08	2.04	2.16	2.49	2.18	0.72	0.64	0.68	0.30	0.31	0.20	0.17
17	8.97	10.68	11.05	5.93	5.61	2.06	1.10	0.89	0.60	0.61	0.73	0.42	0.35	0.36	0.37	0.22	0.19	0.18
18	24.17	10.33	8.68	8.44	6.52	6.68	4.21	2.75	2.48	0.93	0.53	0.17	0.26	0.25	0.36	0.12	0.11	0.07
19	97.49	22.80	21.53	20.60	4.94	3.40	3.35	2.49	2.10	1.58	0.95	0.49	0.50	0.40	0.28	0.26	0.26	0.23
20	31.41	10.22	10.09	4.86	4.70	3.66	3.20	2.96	2.49	1.23	1.15	1.16	0.70	0.70	0.52	0.21	0.18	0.15
id	15.47	10.01	9.85	2.78	1.79	1.96	0.88	0.50	0.62	0.55	0.32	0.11	0.07	0.08	0.03	0.02	0.02	0.02

Figure 6. *Mean color distances ΔE_{ab} between original and estimated colors of the dataset of Pointer with illuminant E as a function of the number of basis vectors B and the number of channels K. Dark grey indicates an error $> 5\ \Delta E_{ab}$, light grey indicates an error $> 1\ \Delta E_{ab}$. The spectral estimation was done by estimating the weights of the basis vectors from the channel response and summing up the weighted PCA basis vectors (case 1).*

number of channels K

number of basis vectors B	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3	13.56	13.88	19.89	19.48	19.34	15.64	15.77	15.47	15.40	15.60	15.58	15.62	15.63	15.64	15.61	15.60	15.59	15.57
4	46.90	33.95	11.13	10.64	10.09	9.85	9.88	9.91	10.18	10.18	10.03	10.11	10.10	10.02	10.08	10.09	10.05	10.05
5	22.17	20.01	14.02	14.64	12.04	9.89	8.51	9.57	9.66	9.95	9.91	9.86	9.84	9.76	9.69	9.70	9.68	9.70
6	27.78	11.57	7.61	7.07	3.33	3.21	3.76	3.29	2.96	2.88	2.94	2.99	2.94	2.93	2.99	2.97	2.90	2.90
7	10.66	9.62	9.43	6.07	6.39	2.54	2.66	1.82	1.80	1.75	1.77	1.75	1.73	1.79	1.79	1.78	1.78	1.77
8	11.88	9.90	9.82	10.44	6.74	4.24	3.54	3.94	2.28	2.37	2.12	2.12	2.08	1.99	1.93	1.93	1.93	1.94
9	34.39	15.00	10.15	6.64	5.85	5.29	4.97	6.96	6.91	6.55	1.68	1.71	0.96	0.89	0.84	0.86	0.87	0.85
10	31.16	9.85	9.28	8.61	5.15	5.56	3.53	3.22	3.72	1.80	1.58	1.23	1.22	1.26	1.16	0.89	0.93	0.63
11	10.48	9.66	10.89	10.22	6.24	5.95	5.00	3.35	0.88	0.89	0.75	0.55	0.65	0.63	0.58	0.57	0.57	0.57
12	8.97	9.95	10.21	6.70	6.71	7.29	2.93	0.66	0.68	1.50	1.48	0.81	0.92	0.78	0.68	0.49	0.56	0.48
13	31.66	10.28	9.87	10.10	3.87	3.75	3.11	3.69	4.34	2.04	1.95	0.84	0.83	0.42	0.38	0.36	0.37	0.33
14	31.66	10.30	9.93	5.07	4.96	3.86	3.04	2.29	2.04	1.60	1.05	0.29	0.37	0.44	0.46	0.35	0.35	0.21
15	31.65	10.29	9.94	10.19	3.99	3.85	1.96	2.57	2.17	1.54	1.33	1.33	1.46	1.42	1.25	0.28	0.21	0.23
16	10.43	9.67	11.86	12.98	3.32	3.21	2.04	2.27	2.37	2.72	2.23	0.86	0.62	0.80	0.39	0.39	0.27	0.25
17	8.98	10.58	10.94	5.97	5.51	1.95	1.31	0.96	0.60	0.56	0.62	0.49	0.39	0.47	0.40	0.32	0.28	0.25
18	24.22	10.29	8.64	8.44	6.47	6.59	4.24	2.73	2.26	0.78	0.51	0.26	0.25	0.25	0.40	0.14	0.14	0.08
19	97.20	23.39	22.17	21.22	4.86	3.27	3.24	2.59	2.22	1.67	1.20	0.66	0.66	0.51	0.32	0.29	0.32	0.28
20	31.60	10.26	9.97	4.88	4.75	3.78	3.30	3.04	2.64	1.17	1.09	1.09	0.65	0.64	0.48	0.22	0.22	0.17
id	15.47	10.01	9.85	2.78	1.79	1.96	0.88	0.50	0.62	0.55	0.32	0.11	0.07	0.08	0.03	0.02	0.02	0.02

Figure 7. *Mean color distances ΔE_{ab} between original and estimated colors of the dataset of Pointer with illuminant E as a function of the number of basis vectors B and the number of channels K. Dark grey indicates an error $> 5\ \Delta E_{ab}$, light grey indicates an error $> 1\ \Delta E_{ab}$. The spectral estimation was done by estimating the weights of the basis vectors from the channel response and summing up the weighted PCA basis vectors as approximated by the sensor (case 2).*

number of channels K

number of basis vectors B	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3	10.65	10.72	3.22	2.99	2.85	2.58	0.93	0.60	0.29	0.22	0.21	0.18	0.14	0.12	0.11	0.10	0.09	0.07
4	47.18	24.82	4.67	2.23	1.49	1.08	0.80	0.40	0.40	0.35	0.24	0.26	0.20	0.18	0.18	0.07	0.07	0.07
5	14.01	9.71	2.85	2.60	2.22	1.17	0.94	0.41	0.34	0.31	0.25	0.27	0.22	0.17	0.16	0.06	0.06	0.06
6	17.34	3.33	2.93	2.84	0.96	0.97	1.11	0.70	0.37	0.35	0.22	0.18	0.18	0.15	0.14	0.12	0.04	0.05
7	10.84	10.72	3.29	2.55	1.36	0.92	0.93	0.33	0.30	0.26	0.25	0.16	0.13	0.07	0.06	0.03	0.04	0.02
8	12.57	10.19	7.00	4.96	3.15	1.34	1.19	0.52	0.36	0.24	0.15	0.15	0.14	0.14	0.12	0.11	0.11	0.08
9	26.86	12.11	9.13	3.91	2.26	1.48	1.30	0.73	0.57	0.31	0.20	0.16	0.12	0.12	0.10	0.11	0.07	0.07
10	32.53	4.96	5.27	2.64	1.83	1.40	1.18	0.60	0.54	0.47	0.29	0.11	0.10	0.10	0.09	0.06	0.06	0.04
11	10.84	10.72	10.25	4.96	3.15	3.08	1.98	1.02	0.40	0.20	0.09	0.08	0.08	0.08	0.04	0.03	0.03	0.03
12	10.45	10.26	6.75	3.67	3.38	2.87	0.90	0.37	0.33	0.14	0.11	0.08	0.06	0.03	0.03	0.03	0.02	0.02
13	27.60	10.93	7.00	4.96	1.84	1.47	1.23	0.50	0.48	0.41	0.40	0.31	0.15	0.07	0.05	0.05	0.03	0.02
14	27.60	10.93	7.00	2.35	2.49	1.92	0.94	0.56	0.55	0.44	0.44	0.15	0.12	0.11	0.09	0.04	0.03	0.03
15	27.60	10.93	7.00	4.96	1.84	1.47	1.04	1.09	0.29	0.19	0.15	0.14	0.12	0.11	0.10	0.06	0.05	0.05
16	10.84	10.72	3.22	3.53	1.48	1.48	0.83	0.56	0.56	0.47	0.41	0.30	0.29	0.13	0.11	0.11	0.06	0.05
17	10.45	10.26	9.88	1.81	1.74	1.07	0.49	0.49	0.47	0.47	0.45	0.21	0.15	0.15	0.15	0.08	0.04	0.03
18	27.04	9.46	4.41	4.35	3.33	2.51	1.50	1.03	1.01	0.40	0.13	0.13	0.12	0.11	0.10	0.03	0.02	0.02
19	33.19	8.25	7.95	7.53	4.82	2.55	2.50	0.91	0.86	0.67	0.26	0.12	0.10	0.07	0.08	0.06	0.05	0.05
20	27.60	10.93	7.00	2.35	2.49	1.92	1.77	1.77	1.59	0.91	0.80	0.72	0.45	0.44	0.16	0.10	0.07	0.07
id	16.41	10.28	10.61	2.30	1.74	2.11	0.95	0.38	0.48	0.45	0.25	0.08	0.05	0.04	0.03	0.01	0.01	0.01

Figure 8. *Mean color distances ΔE_{ab} between original and estimated colors of the dataset of Pointer for illuminant E as a function of the number of basis vectors B and the number of channels K. Dark grey indicates an error $> 5\ \Delta E_{ab}$, light grey indicates an error $> 1\ \Delta E_{ab}$. The number of approximated basis vectors does not seem to have a significant influence, though. The spectral estimation was done using Wiener inverse (case 3).*