Color-weak correction by discrimination threshold matching

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Abstract

The correction of the effects of color-weakness is difficult if not impossible. One of the reasons is that are no objective criterions available to evaluate corrections since it is difficult to quantify the color perception of an individual. Also the degree and the character of color-weakness vary widely among individuals and different color stimuli. In this paper, we use Riemann geometry to construct a new method for color-weakness correction. The basic mechanism is the matching between the discrimination thresholds of color-weak and of color-normal observers. The goal is to provide the same color perception of color-normal observers to the color-weak observer: it takes into account the individual differences between observers and since it defines a global deformation of the whole color space it is applicable for correction of complex visual inputs such as natural images.

Introduction

The properties of human color perception vary widely. There is, for example, a significant part of the male population that is color-blind. Human color perception depends on the existence of the three (LMS) of cones in the retina. They have different spectral sensitivities and if one or two of them are missing the observer cannot discriminate between certain pairs of colors. Dichromats are observers with one missing cone type. But color perception varies even among the population that is not color blind. These observers may make color discriminations that are different from the discriminations of persons with normal color vision, an effect known as anomalous trichromacy or color-weakness. Finally, it is also known that the distribution of the cones in the retina of observers with normal color vision varies widely. More details can be found in books on color vision and in [6],[7].

Color-blindness and color-weakness are also of great practical importance since many communication processes make use of color in various degrees. For critical applications colorblind and colorweak users must be able to discriminate color differences.

A typical approach used in universal color design to solve this problem is to amplify the color-contrast between figures or symbols and the background. However in order to apply this approach to natural images, one needs to know how much and in which direction he/she should correct each color. Unfortunately, there is no objective criterion for such a correction because the color perception properties of users is difficult to estimate. Furthermore, the characteristics of color perception vary greatly between individuals. Since it is difficult to characterize the color weakness of an observer exactly, there is no way to control the extent we should compensate his or her color defect. For these reasons, people used to believe that exact correction is simply impossible in principal.

In this paper we introduce a color-weak correction method based on Riemann geometry. The goal of this method is to find a way to correct color-weakness, so that the corrected color reproduction gives color-weak observers the same color perception as the original to color-normals. This correction should take into account individual differences, and it should be applicable to natural images to be reproduced on different media like prints, TV or movies.

Color-blind model and color-weak correction

Recently, simulation models for color-blind particularly dichromatic color perception are devoloped in [2], [3]. We deal here with a popular method to generate color displays for color-blind observers described in [2]. In its simplest form it maps the input space of a monitor (given by the unit cube of normed RGB vectors) to the three-dimensional LMS coordinate space describing the stimulation states of the three cone types.

The stimuli perceived by the dicromats consists of the two planes which are spanned by three invariant hues which are perceived equally by both color-normal and color-blinds.

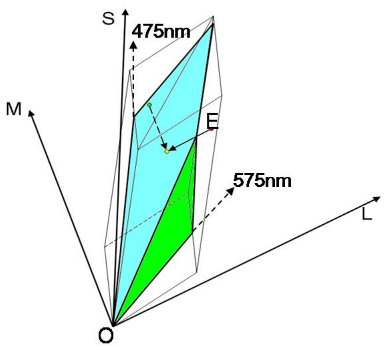

Figure 1. *Brettel's dichromat model*

This model describes color-blind vision as a projection operation along L axis for protanopia and M axis for deuteranopia, which are in correspondence to the confusion lines.

Color-weakness or partial color-blindness is known as an intermediate state between color-normal and color-blind vision. Its origin is more complicated to explain. Possible causes are certain malfunctions of either the retina or the visual path or different mechanisms in the brain. The result is however a large variations in the forms of color-weakness and it is therefore difficult to characterize and to compensate its effects.

A typical approach in universal color design is to simulate or to detect color-blindness by e.g. wearing color-blocked glasses and then to intensify the color-contrast between figures or symbols and background. In this way letters of an alphabet or symbols can be made easier to distinguish for color-handicapped people by presenting them on a background of the complementary color. For such a correction, Brettel's color-blind model provides reference information besides the direction information of the confusion lines .

Unfortunately, even for these simple symbols and an average color-handicapped observer, one needs to know how much the color should be corrected. Much more is required if one wants to construct color correction methods applicable to natural images to be shown as still-pictures, TV sequences and movies, and if one wants to take into account the characteristics of individuals. We will show latter that for the same observer, the extent of color-weakness could vary a great deal, and that the properties are often more complicated than Brettel's model could describe. This holds even true for the observers classified as color-normal for which there is a great variation of their color vision characteristics.

Color spaces as Riemann spaces

The only observable data in color perception are usually color differences. In particular, the most accessible and reliable measurement are small or local color differences. The best-known such measurement are the so-called just noticeable-difference (jnd) thresholds or the discrimination thresholds, measuring the minimal color-difference from the test colors that the observer can detect.

These threshold measurements provide at every color a measure of local distance in color space as follows: denote the test color x as the origin and a color vector close to x as y with respect to the origin, then the discrimination ellipses/ellipsoids are the unit circles/spheres centered at the test color x, which can be expressed by the following equation:

$$y^T G(x) y = 1 \tag{1}$$

Here the positive definite matrix $G(x)$, varying with the location of the test color x, is uniquely determined by the ellipses/ellipsoids and vice versa. With such a matrix $G(x)$ defined at every x, the local distance around x can be expressed as

$$\parallel dx \parallel^2 = dx^T G(x) dx. \tag{2}$$

Such a space with a smoothly defined local distance or the matrix $G(x)$ (known as Riemann metric) is called a Riemann space [**?**].

Another yet even more important quantity in color perception is large color differences to be distinguished from small color differences. They are however more subjective and hence harder to deal with.

In our model the color difference between two points in a color space is expressed as distance between the two points. In a Riemann space, the distance between points x and x' is defined as the length of the shortest curve connecting the two points. This shortest curve is known as a geodesic.

Consider a map f from a color space C_1 to another color space C_2. Denote the Riemann metric of C_1 as $G_1(x)$, and the Riemann metric of C_2 as $G_2(y)$ where $y = f(x)$. The condition for f to be local or small color-difference preserving at x is that f maps the discrimination threshold at x to the threshold at y, or

$$G_1(x) = (D_f)^T G_2(y) D_f, \tag{3}$$

where D_f is the Jacobian matrix of f,[**?**].

In Riemann geometry, such a color-difference preserving mapping at every points is called a local isometry. A map preserving large color-differences are called global isometry, which means that the distance between any pair of points in one space is equal to the distance between the corresponding pair of points in the other space.

Using tools from Riemann geometry it can be shown that two locally isometric spaces are also globally isometric. In other words, if we can match the thresholds at every corresponding colors, such that the small color differences are adjusted to be always the same everywhere, then the large color difference between any corresponding pair of colors is also identical. [4],[8].

Correction by discrimination threshold matching

In order to present the same color stimuli to a color-weak observer as perceived by color-normal observers, the most natural way is to transform the color space of the color-weak observer so that it has the same geometry, and therefore the same color differences, as the color space of color-normal observers.

From the previous section, we know that all we need is an isometry between the color space of the color-weak observer and the color space of color-normal observers. In order to construct such an isometry, it is enough to meet the local isometry condition (3) everwhere.

Therefore we propose a new criterion for color-weak correction to match discrimanation threshold ellipses/ellipsoids between color-weak and color-normal observers.

This isometry or color difference preserving map which transforms the color space C_w of the color-weak observer to C_n of color-normal observers is central to our method. We call this map "*the color-weak*" map and denote it as

$$w : C_n \longrightarrow C_w, \qquad x \longmapsto y = w(x) \tag{4}$$

In fact, the color-weak map for an observer can be estimated thresholds matching as follows. Assume that a color stimulus x perceived by color-normals is mapped by w to $y = w(x)$ perceived by a color-weak observer. If we have discrimination thresholds $G_n(x)$ of the color-normal and the corresponding thresholds $G_w(y)$ for color-weak, then the Jacobian matrix D_w of w can be obtained by the threshold matching condition:

$$G_w(y) = (D_w)^T G_n(x) D_w \tag{5}$$

To correct the color-weakness, we invert the color-weak map w. In particular, before showing an image to the color-weak observer we apply the inverse color-weak map w^{-1} to it. Then this preprocessed image will be perceived by the color-weak observer in the same way the original image by the color-normal.

On the other hand, applying w to the input image and showing it to color-normal observers will provide them the same experience as the color-weak observer.

Measurement and Estimation of thresholds

Before we can apply the Riemann geometric strategy described above we first need to estimate the discrimination thresholds from measurement data. This is done as follows. First we select the center color of an ellipsoid as the origin . Then n points $\mathbf{x}_i = (x_i, y_i, z_i), i = 1 \cdots n$ on the surface of the ellipsoids are sampled. We then substitute the coordinates of these points into the defining equation of the ellipsoid,

$$ax^2 + by^2 + cz^2 + dxy + exz + fyz = 1 \tag{6}$$

The coefficients in the equation are obtained by e.g. least squares fitting.

In particular, define

$$\alpha := (a, b, \cdots, f)^T \quad \beta := (1, 1, \cdots, 1)^T \tag{7}$$

$$A := \begin{pmatrix} x_1^2 & y_1^2 & z_1^2 & x_1 y_1 & x_1 z_1 & y_1 z_1 \\ x_2^2 & y_2^2 & z_2^2 & x_2 y_2 & x_2 z_2 & y_2 z_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^2 & y_n^2 & z_n^2 & x_n y_n & x_n z_n & y_n z_n \end{pmatrix} \tag{8}$$

Then one has

$$A\alpha = \beta \tag{9}$$

This can be solved by e.g. an generalized inverse matrix A^+:

$$\alpha = A^+\beta, \qquad e.g. \qquad A^+ = \left(A^T A\right)^{-1} A^T \tag{10}$$

The Riemann metric is

$$G := \begin{pmatrix} a & d/2 & e/2 \\ d/2 & b & f/2 \\ e/2 & f/2 & c \end{pmatrix} \tag{11}$$

In our experiments we measured the discrimination threshold ellipsoids using a 10 degree visual field of size $14cm \times 14cm$ seen from a distance of 80cm. The discrimination threshold data were measured for 45 college students (38 male, 7 female, 1 color-weak) in CIEXYZ coordinates. We choose 13 points among the 25 centers of MacAdam ellipsoids within the gamut of the monitor as test colors.

We estimate 3D threshold ellipsoids at the above 13 test colors. Around each of them 16 directions are measured. The distribution of the 16 direction is not uniform but denser around the direction of the confusion lines and long axes of threshold ellipsoids. The average thresholds of color-normal and thresholds of the color-weak observer are shown in Fig. 2 and Fig. 3

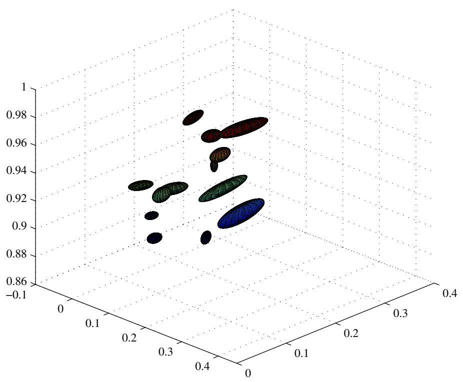

Figure 2. Measurement of threshold ellipsoids for color-normal observers

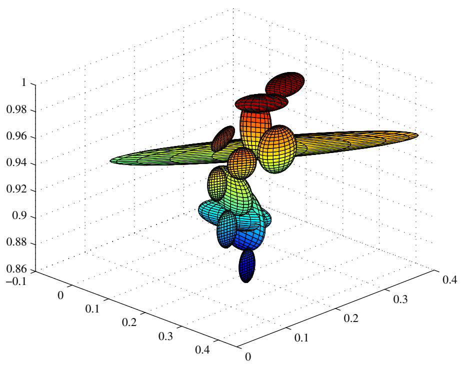

Figure 3. Measurement of threshold ellipsoids for a color-weak observer

Correction based on Brettel's model

We now consider color-weakness and its correction using Brettel's model. The color-weak map and correction are shown in Fig. 4. We assume that the color-weak map will map every stimulus towards the color-blind stimuli plane without reaching it. In other words, every color-normal stimuli Q is mapped along the confusion lines to a point Q'' between Q and color-blind stimulus Q'.

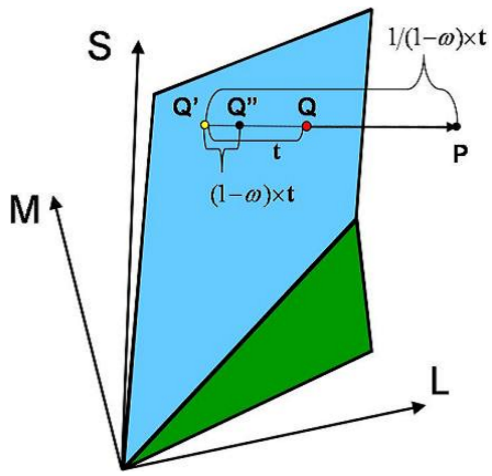

Figure 4. Correction based on Brettel's model

The 1D color-weak mapis defined as follows:.

$$\begin{aligned} Q" &= w(Q) = \omega Q' + (1-\omega)Q \quad (0 \le \omega \le 1) && (12) \\ &= Q' + (1-\omega)(Q - Q') && (13) \end{aligned}$$

We will call ω the color-weak index which indicates degree of color-weakness in percentage of color-blindness. The observer is completely color-blind if $\omega = 1$ and a color-normal if $\omega = 0$.

The color map w simulates color-weak vision when applied to the original image. The correction map is the inverse of the color-weak map w^{-1}, which preprocesses the original image as follows.

$$P = w^{-1}(Q) = Q' + \frac{1}{1-\omega}(Q - Q') \tag{14}$$

Substituting the corrected color P into (12) one can confirm that the color-weak observer actually perceived the same color as the color-normals do.

To estimate the color-weak map w or the color-weak index ω, we use the threshold matching condition as follows: Denote the average length of the discrimination threshold of color-normals as a_n and the length of discrimination of threshold of the color-weak observer as a_w, then apply (5) to obtain

$$1 - \omega = \frac{a_n}{a_w}, \qquad \omega = 1 - \frac{a_n}{a_w}. \tag{15}$$

The discrimination threshold along the confusion lines of protanopia, of the protanopic color-weak observer and an average of color-normal observers are shown in Fig 5 and Fig. 6. Fig. 7 shows the distribution of ω for the color-weak observer.

Extension to 2D and 3D cases

As we have seen in Brettel's model of color-blindness, modeling and correction of color-weakness all occurred in one dimension, on the confusion lines. This is an over-simplified modeling of color-weakness. In fact, our experiments showed that color-weakness usually occurs in more than one direction, and takes more complicated forms.

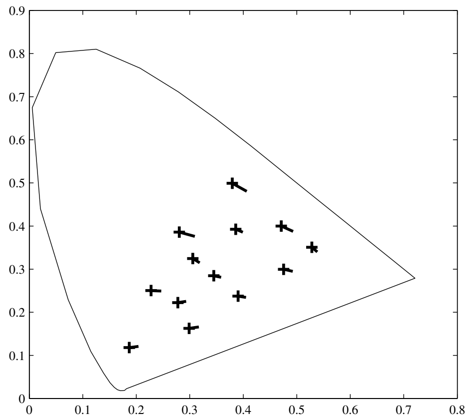

Figure 5. *1D Thresholds of color-normal*

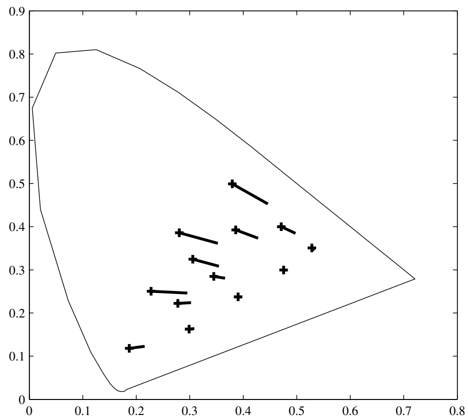

Figure 6. *1D Thresholds of a color-weak protanope*

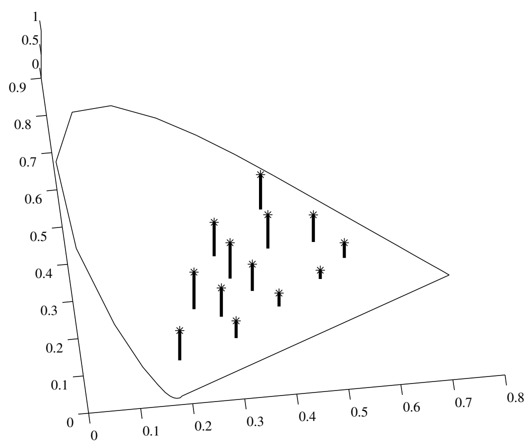

Figure 7. *The distribution of the color-weak indices*

Now we extend the color-weak map and correction alThe distribution of the color-weak indicesgorithm to 2D and 3D cases. In the 1D case, the color weak map can always be characterized by one parameter, the color-weak index ω. However, in the 2D and 3D cases, the color-weak map becomes a 2 by 2 or 3 by 3 matrix W defined locally.

Estimation of the color weak matrix W is based on matching the threshold ellipses using (5). Below, we show a method to compute W for the 2D case. The 3D case can be done similarly.

Denote the Jacobian of w at i-th test color as W_i, then locally

$$\mathbf{y} = W_i \mathbf{x}. \tag{16}$$

The matrix W_i can estimated using the corresponding points (e.g. long and short axes) on the two threshold ellipses. For an example, the sections of the ellipsoids in Fig. 3 with the chromaticity plane are shown in Fig. 8 and Fig. 9.

Assume we have

$$\mathbf{x}_1 = \begin{pmatrix} x_{11} \\ x_{12} \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} x_{21} \\ x_{22} \end{pmatrix} \tag{17}$$

which correspond to

$$\mathbf{y}_1 = \begin{pmatrix} y_{11} \\ y_{12} \end{pmatrix}, \mathbf{y}_2 = \begin{pmatrix} y_{21} \\ y_{22} \end{pmatrix} \tag{18}$$

$$W_i = \begin{pmatrix} a_i & b_i \\ c_i & d_i \end{pmatrix} \tag{19}$$

$$\begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{x}_1^T & \mathbf{0}^T \\ \mathbf{0}^T & \mathbf{x}_1^T \\ \mathbf{x}_2^T & \mathbf{0}^T \\ \mathbf{0}^T & \mathbf{x}_2^T \end{pmatrix} \begin{pmatrix} a_i \\ b_i \\ c_i \\ d_i \end{pmatrix} \tag{20}$$

Then the entries of W can be obtained as

$$\begin{pmatrix} a_i \\ b_i \\ c_i \\ d_i \end{pmatrix} = \begin{pmatrix} \mathbf{x}_l^T & \mathbf{0}^T \\ \mathbf{0}^T & \mathbf{x}_l^T \\ \mathbf{x}_s^T & \mathbf{0}^T \\ \mathbf{0}^T & \mathbf{x}_s^T \end{pmatrix}^+ \begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix} \tag{21}$$

here $()^+$ denotes a generalized inverse matrix.

Once the color-weak map is obtained, one can use it in correction and simulation as before.

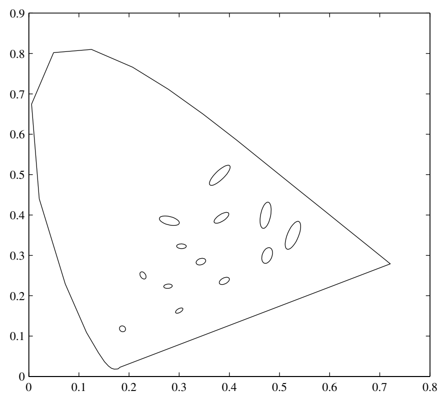

Figure 8. *Threshold ellipses of color-normal observers*

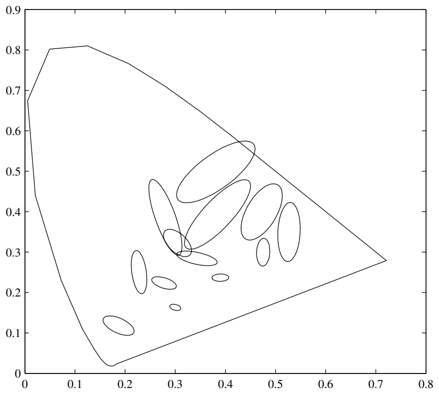

Figure 9. *Threshold ellipses of the color-weak observer*

Simulation and Correction

In Fig. 10, the color-weak map is estimated using Brettel's model of protanopia. The color-weak simulated image (top) and the corrected image (bottom) are produced by applying the color-weak map and its inverse to the original image (middle).

In Fig. 11, the color-weak map is estimated using matching of discrimination threshold ellipsoids. Again, the color-weak simulated image, the original image and the corrected image are shown in the same order. Obviously the strong tritanopia (blue-orange) of the observer (see Fig. 9) is also corrected by the color-weak map in higher dimension.

Evaluation

We evaluated the corrected images and color-weak simulation images using the semantic differentiation (SD) test described in [5]. This is a standard culture-independent procedure to quantitatively evaluate subjective impressions. First a selection of concept-pairs of adjectives related to the test images were chosen by an individual group. Then the test images are scored by another group and the color-weak observer using a 5 points or 7 points scale for each concept-pair of the adjectives. The SD curves obtained by connecting the scores on every scale of concept-pairing adjectives are used to compare the impressions.

In particular, two sets of SD tests are compared with each other: (1) between the original image evaluated by color-normals and the corrected image by the color-weak (Fig. 12), (2) between the "color-weak simulation" of the original image evaluated by the color-normals and the original image evaluated by the color-weak (Fig. 13). These two comparisons show that the color-normals obtained very similar impressions from the original image as the color-weak from the corrected image, and that the color-weak's impression on the original image is very close to that of the color-normals from the "color-weak simulation" of the original image.

Conclusions

We proposed a new criterion for color-weak correction and simulation to provide the same color perception of color-normal observers to a color-weak observer. A color-weak map is defined as a color-difference preserving map between the color spaces of color-normal and color-weak observers, which can be obtained by matching of discrimination thresholds between these two kind of observers. This method is applied using measurement of threshold data to Brettel's dichromat model and extended to higher dimension. The performence is evaluation by

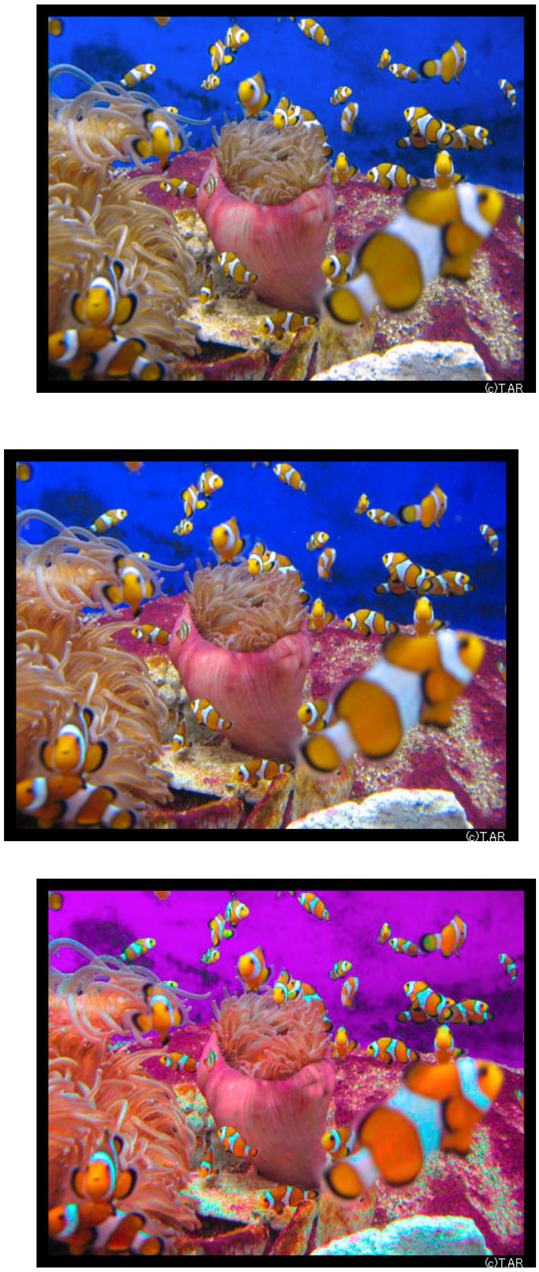

Figure 10. *Color-weak simulated, the original and corrected images using Brettel's model*

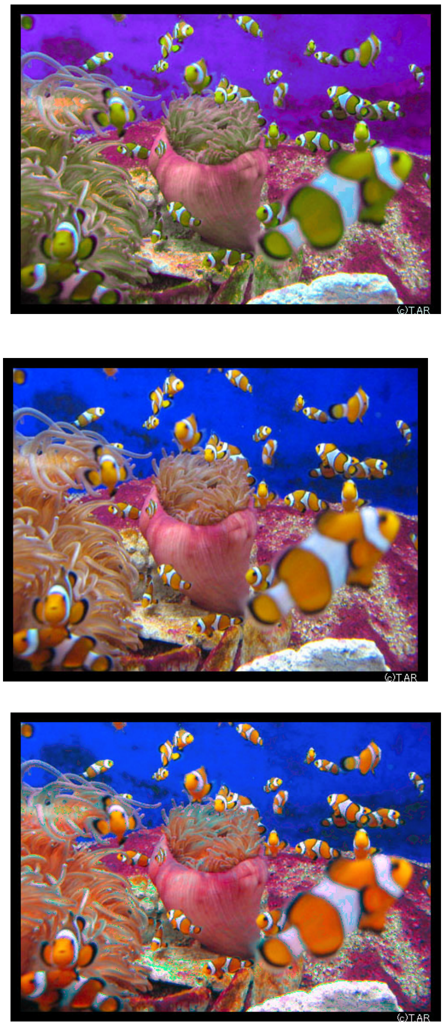

Figure 11. Color-weak simulated, the original and corrected images by ellipsoids matching

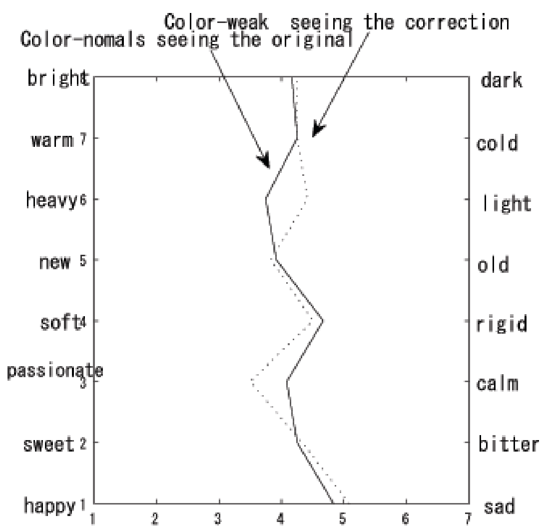

Figure 12. SD evaluation for color-weak simulation

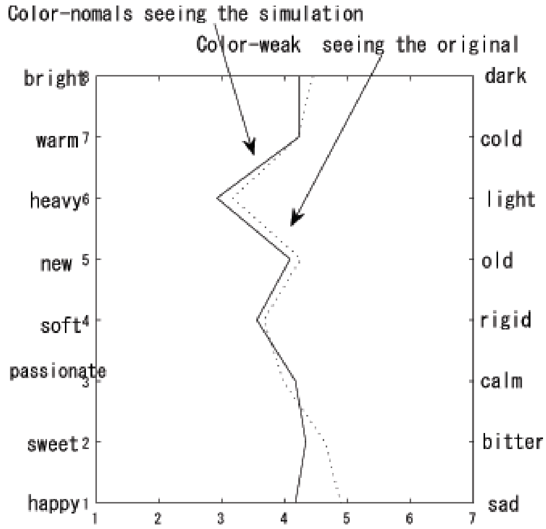

Figure 13. SD evaluation for color-weak correction

SD method. An even more accurate approach is to use geodesic coordinates [9]. This method can be extended to simulate on one individual aother one's color vision, as long as their discrimination threshold data are available.

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