# Combined Matrix Based Determination of Control Values for a 6-Primary Display Considering Different Observers 

Thomas Boosmann; Color and Image Processing Research Group, Aachen University of Technology; Aachen, Germany


#### Abstract

In contrast to three primary displays, multi-primary displays provide an essential enlargement of the color gamut and a higher degree of freedom to reproduce colors. The degree of freedom can be used to optimize color differences between color stimuli of the reproduction and the original for different observers.

The simplest and least time consuming approach to control the primaries from input stimuli is the application of a matrix. Yet, one matrix is performing well for a certain number of stimuli only. Another matrix might achieve improved results for other stimuli but is performing worse for the first group. The basic idea of the algorithm presented here is to combine different matrices to fill the gap between oppositional requirements and to obtain adequate results in face of expense and quality. Altogether, four different matrices have been chosen in this study. None of the four matrices is appropriate on its own to achieve small color differences as desired for all colors.

The matrices are constructed from the optimization of color differences for different observers ( 24 in total) and a large number of typical color stimuli. To display a specific color stimulus, one of the four matrices is selected with respect to minimum color errors for all observers. By combining different matrices, a fast control of a display at high color quality is realized.


## Introduction

In recent years, several groups started working on displays with enlarged color gamuts. Some approaches are still based on three primary colors but these primaries exhibit a higher saturation than commonly used. The largest gamuts which are achievable by three primaries are obtained by laser primaries [1]. Nevertheless, the size of these gamuts is restricted to a larger extent than desired. The only way to exceed this limit is to enhance the number of primaries on the one hand and to reduce the spectral width of the channels on the other. At present, several groups are working on such displays and have realized experimental ones already $[2,3,4,5]$. They are equipped with up to six primary colors. Typically, the spectral width of the display channels is approximatively 50 nm wide. So, these displays offer an essential enlargement of the color gamut. In addition, more than three primaries offer a degree of freedom to control primaries to minimize color differences for different observers. So, errors by observer metamerism can be reduced as far as possible. Yet, not all approaches make use of this possibility and concentrate on the expansion of the gamut only. However, publications demonstrate that metamerism may neither be underestimated nor neglected $[6,7,8]$. The control of more than three primaries is a basic problem because on the one hand, colorimetric control is excessively determined and on the other, a number of up to six primaries is not sufficient to realize a match to spectral color stimuli well. This paper contributes to the evaluation of this essential problem.

## Existing approaches

A number of proposals has been published on the problem of controlling multi-primary displays. These approaches imply quite a number of different methods. Some of them are concentrated on the CIE 1931 standard observer $\left(2^{\circ}\right)$ only $[9,10,11]$, though, multi-primary displays offer the possibility to optimize the control for more than one observer or even to minimize metamerism for a set of different observers.

In another proposal the color stimulus is matched exactly to two observers, the CIE 1931 standard observer and the CIE 1964 supplementary standard observer $\left(10^{\circ}\right)$ by using six equations [12]. Even more advanced approaches are minimizing the color differences for a larger number of observers by methods of linear programming or on the basis of stochastic iterations [ $13,14,15]$. A gradient based iteration of minimizing color differences for a set of observers is applied in [16]. An alternative way is presented in [7] to minimize metamerism for several observers. This publication proposes to optimize differences of energy in the spectral domain and to apply a spectral-approximation method to roughly match the spectral stimulus. A completely different approach is presented in [5]. Here, the application is based on HDTV technology and a gamut mapping method is used to expand the gamut of the television signal to full multi-primary gamut. This approach is limited to one single observer because of the constitution of input data.

The performance varies by the different approaches and methods. Though, the different approaches cannot be compared directly because of different test data and error criteria. In [14], a method of linear programming is presented resulting in a mean color difference of about $\overline{\Delta E_{a b}} \approx 0.5$ if several observers, illuminant $E$, and the data set of Vrhel with representative color stimuli are tested. Maximum color differences of approximately $\Delta E_{94}=1.6$ are achievable by stochastic iteration as proposed in [13]. The study is applied to 24 observers, illuminant $D_{65}$, the data set of Vrhel, and the spectral characteristics of experimental channels, but without consideration of the black offset which is always present in practical set ups. If idealized channels are applied, the maximum color difference is reduced to even $\Delta E_{94}=1.2$ for all 24 observers. Another stochastic iteration using illuminant $D_{65}$ as well as the data set of Vrhel and 24 observers obtains a reduction of the maximum color difference to a threshold beneath $\Delta E_{94}=1.5$ [15]. Yet, the algorithm does not really guarantee the finding of the control vector of the absolute minimum or the local minimum of color differences. The gradient based iteration published in [16] applies the data set of Vrhel as well as both kinds of channel characteristics, experimental and ideal ones. In addition, the investigation is expanded to a series of illuminants. In a maximum, a color difference of $\Delta E_{94}=1.16$ is achievable for $D_{65}$ and 24 observers if experimental characteristics are considered. For illuminant $E$, the maximum color difference rises to $\Delta E_{94}=1.71$. Applying ideal Gaussian character-
istics, the maximum color difference is reduced to $\Delta E_{94}=0.84$ for $D_{65}$ and $\Delta E_{94}=1.58$ for illuminant $E$. In addition in [16], it turns out that illuminant $E$ causes the worst results for common illuminants. Moreover, the average of color differences is much beneath of the maximum. In [7] the quality of the spectral approximation is rated by visual comparison of several human observers only. Hence, no values of color differences are available.

## Ambition

The determination of control values for a multi-primary display is always a trade-off between expense and quality. The approaches published and cited so far demonstrate this point of view. Fast linear determinations using matrices are available on the one hand and time consuming iterations on the other. In general, the quality of control values is in close connection to the computing expense. The fastest algorithms are problematic with respect to metamerism. The ones achieving the least color differences so far are inappropriate for common use by current available and affordable computers.

## $\uparrow$ tristimulus values



Figure 1. Spectral color matching functions of 24 human observers including the CIE 1931 standard- and the CIE 1964 supplementary standard observer [18, 20], the standard deviator [19] and curves measured by Stiles and Burch [20].

The aim of this study is to fill the gap between these two oppositional requirements and to obtain adequate results in face of expense and quality. To get comparable results to the prior mentioned methods, the data set of Vrhel [17] and several observers are used to describe color quality in this study likewise. The data set of Vrhel is consisting of 354 representative color stimuli and the set of observers is represented by 24 color matching functions published in [18, 19, 20] (fig. 1). In addition, the spectral channel characteristics of an experimental setup applied in [16] (fig. 2) are used for this study to maintain comparability with earlier studies. Part of this investigation is the reproduction for different illuminants which are characterized by uniform spectral radiating power on the one hand and spiky spectral power distributions on the other. Hence, various light sources are applied to verify the performance including measured and standardized ones [18, 21], e.g. $D_{50}, D_{55}, D_{65}, D_{75}, A, B, C, E$, and $F_{1}$ to $F_{12}$. The maximum color difference CIE $\Delta E_{94_{\max }}$ of all 24 observers is used as error criterion to provide small color differences for every human observer.


Figure 2. Spectral power distribution of the laboratory model measured in the center of the screen; each channel is approximatively 50 nm wide, the black offset is considered in the calculation but not plotted here.

## Algorithm

## Preliminary remarks

First of all the quality of color reproduction is defined by maximum color reproduction errors of the 354 color stimuli of the data set of Vrhel and a number of 24 different observers as mentioned in the previous section. So, a number of 24.354 color differences of reproduced and input stimuli are calculated using the definition of CIE $\Delta E_{94}$.

The simplest and least time consuming approach to determine control values for reproducing color is the application of a matrix which maps the input values to the control values of the primaries of the display. If there are $N$ primaries to control, the determination of the control values requires $N$ equations provided by the matrix $A$. To determine this matrix, far more equations are necessary. There are many different methods to determine matrices. Four of them are discussed in more detail below.

If the maximum errors as defined above are calculated for each of these matrices the errors are very large and unacceptable. Yet, a more detailed study of the test stimuli reproducing the largest color differences shows that matrices derived from different criteria result in maximum color differences for some color stimuli only. It even happens that the reproduction errors are unacceptably large for one of the matrices but remarkably smaller for another one. Moreover, a large problem is to avoid negative control values which would have to be clipped to zero. In many cases the maximum color errors result from such necessary modifications. Therefore, it is proposed in this paper to calculate control values for each input stimulus and resulting colors for a number of different matrices in parallel together with the calculation of the maximum reproduction errors and select the best resulting one finally. By this way, the time consumption is increased a little but the weak point of each matrix can be compensated by another one. However, the time consumption remains remarkably lower than in the case of using stochastic iterations or gradient based algorithms as published in [15] or [16]. An alternative to accelerate the calculation would be to calculate the results for one matrix after the other and stop the process if the result is beneath a threshold. The advantage of the second mentioned approach will become more obvious after discussing the results at the end of this proposal. Hence, the algorithm to determine control values is described in more detail in the following subsection first.

## General determination of control values

The possibility to display a specific color for one single observer using a multi-primary display is not definite. By applying $N$ display channels a degree of freedom of $f=N-3$ is offered. Hence, a color can be matched for $m=\left\lfloor\frac{N}{3}\right\rfloor$ observers. ${ }^{1}$ A different approach is to minimize color differences for a set of observers.

Using $N=6$ channels, the simplest way to determine the control values

$$
\vec{c}=\left(c_{1}, \ldots, c_{6}\right)^{T}
$$

is to combine the color primaries of the $N$ channels for two different observers within the $6 \times 6$ matrix

$$
A=\left(\overrightarrow{X_{C h_{1}}}, \ldots, \overrightarrow{X_{C h_{6}}}\right),
$$

where

$$
\overrightarrow{X_{C h_{n}}}=\left(X_{1}^{n}, Y_{1}^{n}, Z_{1}^{n}, X_{2}^{n}, Y_{2}^{n}, Z_{2}^{n}\right)^{T}
$$

denote the color of the primary of display channel number $n$ for observer one and two. The control values are achieved by

$$
\vec{c}=A^{-1} \cdot \vec{X}
$$

Here,

$$
\vec{X}=\left(X_{1}^{\varphi}, Y_{1}^{\varphi}, Z_{1}^{\varphi}, X_{2}^{\varphi}, Y_{2}^{\varphi}, Z_{2}^{\varphi}\right)^{T}
$$

denotes the color values of observer one and two of the spectral input color stimulus $\varphi_{\lambda}^{\text {input }}$. The matrix $A^{-1}$ signifies the inverse of $A$.

If an arbitrary number of primaries is assumed, matrix $A$ can be constructed by different ways. Even a $B x N$ matrix with $B>N$ is possible. In such a case the inverse $\left(A^{-1}\right)$ has to be substituted by the pseudo inverse $A^{+}$. In the following, the description of methods will just be concentrated on the problem of determining adequate matrices $A$

## Considering a black offset

The determination of control values does not permit to consider a spectral black offset $\varphi_{\lambda}^{K}$ of the display system in direct manner. Yet, the consideration of a black offset is possible by applying just a small modification. Instead of the original spectral color stimulus $\varphi_{\lambda}^{\text {input }}$, a substituted color stimulus

$$
\tilde{\varphi}_{\lambda}^{\text {input }}=\varphi_{\lambda}^{\text {input }}-\varphi_{\lambda}^{K}
$$

has to be used and the matrix $A$ has to be substituted by the matrix $\tilde{A}$. This matrix is obtained in almost the same manner as $A$ but with the difference that the color values

$$
\overrightarrow{X_{C h_{n}}}=\left(X_{1}^{n}, Y_{1}^{n}, Z_{1}^{n}, X_{2}^{n}, Y_{2}^{n}, Z_{2}^{n}\right)^{T}
$$

are free of black offset as well. Of course, if $\tilde{\varphi}_{\lambda}^{\text {input }}$ becomes negative, the respective color can no more be displayed.

## Methods of obtaining matrices

The matrices considered here are obtained in different ways. One of them is based on a matrix matched to two different observers as applied in [12]. In [12], a combination of CIE 1931 standard and CIE 1964 supplementary standard observer is proposed. In contrast a different pair of color matching functions is applied in this investigation because of better performance (method 2). In addition, three more matrices are applied. Yet, not all of these ones are based on a combination of two different color matching functions as within the first mentioned matrix. In the strict sense none of the remaining three matrices results directly from two color matching functions, though, two of them are still related to color matching functions. More precisely, the four different matrices are obtained by:

1. basis vectors
2. two different observers
3. regression
4. the CIE 1931 standard and an average observer

The determination of the different matrices and the determination of the control values out of the stimuli will be specified in the following in more detail.

## Basis vectors

The first method considered here to obtain the control values for display channels is the only one within this investigation which is totally independent of particular color matching functions. The initial point of this method is a number of $B$ basis vectors of modified representative natural spectra. These basis vectors are derived from the application of principal component analysis (PCA) to the set of test stimuli. These test stimuli are modified by a pre-distortion on the one hand and a window function on the other. The pre-distortion of the set of test stimuli is following the sensitivity of human observers. A more detailed description of the modifications can be found in [22]. The obtained vectors are used to expand stimuli and display primaries into a series of basis vectors. Within this study a number of $B=12$ basis vectors has been applied.

The basic idea of this method is to reproduce a number of superposed and weighted basis vectors instead of the spectral input stimulus itself. To achieve this, the input values which have to be mapped to control values are the $B$ weights resulting from the expansion of the spectral input color stimulus $\varphi_{\lambda}^{\text {input }}$ into basis vectors:

$$
\vec{X}=\left(g_{1}^{\varphi}, \ldots, g_{B}^{\varphi}\right)^{T}
$$

Consequentially the mapping matrix $A$ has to consist of the weights of the spectral channel characteristics

$$
\overrightarrow{W_{C h_{n}}}=\left(g_{1}^{n}, \ldots, g_{B}^{n}\right)^{T}
$$

in such a way that $A$ is given by:

$$
A=\left(\overrightarrow{W_{C h_{1}}}, \ldots, \overrightarrow{W_{C h_{6}}}\right) .
$$

Of course, the number of values in vector $\vec{X}$ has to be equal to the number of rows within matrix $A$. In general, the number of basis vectors is not equal to the number of channels. Then, the application of a pseudo inverse is required.

[^0]
## Combination of two different observers

The combination of two different observers has been used in the prior section to describe the procedure of achieving the control values in principle. For that reason this will not be carried out here again. Additionally the best combination of two different observers will be presented in place.

For reasons of wide spread, the CIE 1931 standard observer has been chosen as one of the two observers. The second observer is one of the remaining 23 ones. For this purpose, an investigation has been carried out numerically by testing all observers to find the most suitable one resulting in least color differences. Because of shortage of space, the detailed results are not presented here. But recapitulating, best results have been achieved by the combination of the CIE 1931 standard observer and the standard deviator described by the second deviation function applied to the CIE 1964 supplementary standard observer [19].

## Regression

An alternative to applying two different observers is an integration of all 24 observers within one matrix. Equivalent to the procedure of applying two observers the color values for all 24 observers have to be calculated here. On the one hand the input vector is resulting form color values of the input stimuli

$$
\vec{X}=\left(X_{1}^{\varphi}, Y_{1}^{\varphi}, Z_{1}^{\varphi}, \ldots, X_{24}^{\varphi}, Y_{24}^{\varphi}, Z_{24}^{\varphi}\right)^{T}
$$

and matrix $A$ consists of the color values of the channel primaries on the other hand. Matrix $A$ is therefore a $72 x N$ matrix. The system of equations for 24 observers is overdetermined if only $N=6$ primaries are present. Hence, an inverting of the matrix $A$ is possible by the application of a pseudo inverse only. The pseudo inverse is minimizing the average difference. So, all observers are considered to a greater or lesser extent.

## CIE 1931 standard and average observer

## $\uparrow$ tristimulus values



Figure 3. Spectral color matching functions of the average observer. This observer neither represents the average observer of the whole set of color matching functions nor the one of the remaining 23 observers. Yet, it leads to the same results compared to the second mentioned one.

The fourth method considered is another combination of two different sets of color matching functions. Because of wide spread, the first observer is the CIE 1931 standard observer likewise. However, in contrast to the prior mentioned combination
of two observers, in this case the other set of the color matching functions does not belong to a real observer but is originated from calculations. They are achieved by an average determination of color matching functions of the remaining 23 observers. The method of determing this functions is pointed out for $\bar{x}(\lambda)$ only but is equivalent for the other two ones ( $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$ ) as well. The color matching function $x^{*}(\lambda)$ of the average observer (fig. 3) is obtained by

$$
x^{*}(\lambda)=\bar{x}_{1}(\lambda)+\Delta x^{*}(\lambda)
$$

where $\bar{x}_{1}(\lambda)$ is the one of the color matching functions of the CIE 1931 standard observer and $\Delta x^{*}(\lambda)$ is the average of the differences of all 23 remaining color matching functions to the one of the CIE 1931 standard observer (fig. 4):

$$
\Delta x^{*}(\lambda)=\frac{1}{M-1} \sum_{m=2}^{M}\left(\bar{x}_{1}(\lambda)-\bar{x}_{m}(\lambda)\right)
$$

where $M$ denotes the number of observers $(M=24)$. The calculated color matching function $x^{*}(\lambda)$ presented here neither represents the average observer of the whole set of color matching functions nor the one of the remaining 23 observers. Yet, it leads to the same results compared to the second mentioned one.

## $\uparrow$ relative sensitivity



Figure 4. Spectral differences of the color matching functions of the average observer to the CIE 1931 standard observer. These differences represent the average of differences of the remaining 23 observers to the CIE 1931 standard observer.

## Results

The determination of control values is conducted using six primaries of an experimental multi-primary display [4, 16]. The spectral power distribution has been measured at the laboratory model in the center of the screen. For the experimental system the black offset of the display is taken into account. The simulation is carried out within a spectral range from 380 to 720 nm using 1 nm increments to consider even illuminants characterized by narrow-band peaks. The performance of these matrix combinations is analyzed using 354 representative color stimuli of the data set of Vrhel [17]. In total 52 different illuminants are considered to value the input stimuli and to define the white reference of the display. The color difference $\Delta E_{94}$ is calculated for a set of 24 observers. The maximum of these color differences $\Delta E_{94_{\max }}$ defines the color quality. This measure is considered to cover a large variety of human observer with normal color vision.

As mentioned previously, in many cases the maximum color errors of a specific control methods result from necessary modifications related to negative control values resulting from the

Table 1: Summary of color differences $\Delta E_{94}$ between calculated reproductions and originals for 354 spectral stimuli using 24 observers and a specified illuminant (maximum, average, and median color differences complemented by the maximum color differences for each single matrix method)

|  | combination of method 1-4 |  |  | method 1$\Delta E_{94_{\max }}^{\text {meth. } 1}$ | method 2$\Delta E_{94_{\max }}^{\text {meth. } 2}$ | method 3$\Delta E_{94_{\text {max }}}^{\text {meth. } 3}$ | method 4$\Delta E_{94_{\max }}^{\text {meth. } 4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\Delta E_{94}{ }_{\text {max }}$ | $\overline{\Delta E_{94}}$ | $\Delta E_{94}^{\text {median }}$ |  |  |  |  |
| $D_{40}$ | 2.54 | 0.50 | 0.41 | 5.22 | 16.7 | 17.3 | 18.9 |
| $D_{50}$ | 2.46 | 0.53 | 0.44 | 4.48 | 20.1 | 20.7 | 22.2 |
| $D_{55}$ | 2.63 | 0.54 | 0.45 | 4.23 | 21.3 | 21.9 | 23.4 |
| $D_{65}$ | 3.02 | 0.58 | 0.48 | 4.00 | 23.2 | 23.6 | 25.1 |
| $D_{75}$ | 3.32 | 0.62 | 0.51 | 4.17 | 24.6 | 24.9 | 26.4 |
| E | 2.44 | 0.74 | 0.62 | 4.53 | 19.8 | 18.7 | 20.9 |
| A | 1.59 | 0.39 | 0.34 | 6.85 | 10.6 | 10.7 | 12.5 |
| $B$ | 2.59 | 0.50 | 0.42 | 4.20 | 20.8 | 21.1 | 22.9 |
| C | 3.61 | 0.55 | 0.48 | 3.65 | 25.9 | 26.2 | 27.9 |
| $F_{1}$ | 1.98 | 0.46 | 0.38 | 4.03 | 25.7 | 21.4 | 27.3 |
| $F_{2}$ | 2.72 | 0.43 | 0.35 | 5.51 | 20.7 | 15.5 | 21.6 |
| $F_{3}$ | 2.20 | 0.47 | 0.37 | 7.23 | 17.7 | 11.8 | 17.4 |
| $F_{4}$ | 2.70 | 0.59 | 0.47 | 9.33 | 15.1 | 8.29 | 14.2 |
| $F_{5}$ | 1.89 | 0.46 | 0.38 | 3.95 | 24.1 | 20.3 | 25.6 |
| $F_{6}$ | 2.51 | 0.43 | 0.35 | 5.81 | 18.9 | 14.0 | 19.5 |
| $F_{7}$ | 1.79 | 0.44 | 0.37 | 3.62 | 25.3 | 21.8 | 27.3 |
| $F_{8}$ | 2.13 | 0.46 | 0.34 | 4.36 | 24.0 | 19.2 | 26.8 |
| $F_{9}$ | 1.95 | 0.39 | 0.31 | 4.49 | 21.8 | 16.4 | 24.0 |

mathematical algorithm. Those values are unavoidably and have to be clipped to zero for practical applications. Surprisingly, the number of necessary modifications does not vary significantly between the different methods. Just as an example, the modifications for illuminant $E$ are $18,12,17$, and 17 starting from method 1 to method 4 for all 354 stimuli of the data set. The number of the appearance of negative values is small in general and the suitability of the four matrices is not only determined by these numbers of stimuli. Besides the site where negative values appear, one matrix results in quite uniform and medium color errors and another one leads to lower errors for most stimuli but to much higher errors for a limited number of stimuli.

Parts of the results are presented in table 1. The results have been derived for 354 reflectances and 52 illuminants ( 18 of them are shown in table 1) assumed to serve as light sources to capture reflectance spectra. The results for the application of the four different methods (matrices) are given in $\Delta E_{94_{\max }}$ for each illuminant. In the first three columns the respective best results selected from the four methods are presented. The maximum color differences are given in $\Delta E_{94_{\max }}$ in the first column and it is demonstrated impressively that the combination by selection leads to much lower errors than each method on its own. In the second column the average errors are given and in the third the median.

The majority of the best performing control values is obtained by method 3. In total 281 times out of 354 , the method of regression performs best for illuminant $E$. Yet, the rest of the spectra results in higher errors. Method 4 provides the best values 62 times respectively and method 1 and 2 only 4 and 7 times. Though, method 1 results in relatively low maximum errors in a first view compared to the others, the other methods are responsible for small average color differences $\overline{\Delta E_{94}}$. Nevertheless, method 1 and 2 should not be omitted since otherwise the maximum errors would increase in the 4 and 7 cases noticeably.

As mentioned, the temporal expense for the calculation can be reduced by defining a threshold and a well chosen order of applying the matrices. For example, for illuminant $E$, the calcu-
lation should start with method 3 followed by method 4 and 2 and be completed by method 1 . Of course, the full use of four matrices results in better quality for many samples.

The results of table 1 are based on experimental primaries shown in fig. 2. These spectra are very spiky but the spikes are located at positions where many fluorescent lamps exhibit large spikes as well. This special feature of the spectral primaries are the reason why the smooth spectral characteristic such as $D_{50}$ or $D_{65}$ result in larger errors than the $F$ light sources. If the spectral primaries are developed as smooth characteristics the results would be vice versa.

## Comparison to other methods

Compared to the best results achieved by the gradient based method [16] the maximum errors are up to 2 or 3 times larger depending on the illuminant considered. For illuminant $E$, the two methods achieve a maximum color difference of $1.71 \Delta E_{94}$ or 0.35 in average and 2.44 or 0.74 respectively. If illuminant $D_{65}$ is considered the difference is larger. Although the average color differences are closer to each other ( 0.29 and $0.58 \Delta E_{94}$ ) the maximum color differences diverge stronger. The gradient based procedure achieves 1.16 and the matrix combination 3.02 $\Delta E_{94}$ in a maximum. For the gradient based method illuminant $D_{65}$ performs better than illuminant $E$ whereas for the result of table 1 illuminant $E$ performs better than $D_{65}$. Yet, it should be noticed that the iterative methods such as proposed in [15] or the gradient based approach proposed in [16] are time consuming and not applicable to real time processing.

The color differences by linear programming as published in [14] are comparable to the results achieved by methods 2 to 4 and are even larger than the results of method 1 , not to mention the combination of matrices by selection. However, it should be noted that these results have been derived from different spectral characteristics of the display channels. The results of [12] are comparable to method 2 which is understandable because method 2 is based on two observers as well.

## Conclusions and Outlook

In conclusion, the combination of matrices succeeds in filling the gap between single matrix methods to determine the control values of a multi-primary display and stochastic iterations. This is reached in a double sense. On the one hand the expense is only a little bit larger than using a single matrix but obviously smaller than for an iterative solution. On the other hand the size of color differences lies between these two approaches likewise. In particular, the maximum color differences are located closer to the results of the iteration than to the ones achieved by one matrix although the time consumption is far beneath from an iterative solution.

Furthermore, the results presented in this paper demonstrate that six primaries are a good compromise between expense and quality of color reproduction. As well, it shows that six primaries offer a good compromise between the limited color quality of conventional three channel displays and a complete spectral reproduction.

In addition, it is certainly a big step to reproduce colors at acceptable quality not only for the CIE 1931 standard observer but for a large variety of different observers.

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## Biography

Thomas Boosmann received his diploma degree in Electrical Engineering from the Aachen University of Technology in 2000. He is now engaged in research on multispectral imaging systems with focus on multichannel displays. He is member of IS\&T and the German society for color science and application DfwG.


[^0]:    ${ }^{1}\lfloor k\rfloor$ means the next integer smaller than or equal to $k$.

