Modeling Light Scattering in Paper for Halftone Print

Patrick Jenny*, Markus Vöge**, Safer Mourad** and Tobias Stamm**

* Swiss Federal Institute of Technology (ETH),
Institute of Fluid Dynamics, Zürich, Switzerland

** Swiss Federal Laboratory for Materials Testing and Research (EMPA),
Laboratory for Media Technology, Dübendorf, Switzerland

Abstract

This paper is concerned with the study and description of light scattering in turbid material such as paper. We present a new modeling approach, which is based on the transport theory. A statistical description for the scattering and absorption behavior inside the material is used to obtain spatial distributions of wavelength dependent photon number density and fluxes. Opposed to previous methods, the new approach can account for complex multi-point statistics of the material properties in an efficient and general way. This is achieved by solving a model equation for photon number density and propagation direction distribution. The present method is able to account for the spacial distribution of specific, wavelength dependent scattering and absorption characteristics. Thereby, it offers a general framework which allows to predict for example the appearance of colors in halftone prints.

Introduction

The study of light scattering in paper is of major concern and importance for the description of phenomena related to halftone printing. However, often it is not possible to predict or explain experimental data with existing models. There are two fundamentally different approaches [2, 3]. The first one, the analytical theory [2, 4, 9], is rigorous, but computationally very expensive. More adequate for practical problems [2] is the second approach, the transport theory [1], which was developed on a heuristic basis dealing with transport of energy through turbid media directly [10].

In this paper we present a new modeling approach, which is based on the transport theory. A statistical description for the scattering and absorption behavior inside the paper is used to obtain spatial distributions of wavelength dependent photon number density and fluxes. Opposed to previous methods, our approach allows to account for the typically complex paper structure and multi-point statistics in a general, but still efficient way. This is achieved by solving a modeled evolution equation for photon number density and joint probability density function (PDF) of propagation direction. In order to solve the high dimensional PDF equation efficiently, a particle method is employed. In this framework, each computational particle represents a number of photons indicated by a weight. Additional particle properties are position in physical space, propagation direction and wavelength. To account for absorption, the particle weight decreases with time. We first describe the transport equation for the photon density and direction distribution. Then a particle solution algorithm is devised. With numerical results we demonstrate that the method shows the correct tendencies in terms of absorption coefficient and correlation length scales in the paper. Finally,

we discuss how the new model can be extended to account for more specific scattering behavior in a very general way. Since the model honors wavelength dependency and spatial distribution of the coefficients, it can be used to predict the effect of optical dot gain on the colors in halftone prints. Moreover, it is interesting that in the 1D case our model is consistent with the existing random walk [7, 8] and Kubelka and Munk [5, 6] models.

PDF Transport Equation

First, a brief outline of the transport theory is given. The most important quantity considered is the radiance, $I(\boldsymbol{x}, \boldsymbol{s})$, which is the average energy flux per solid angle at location $\boldsymbol{x}$ in direction $\boldsymbol{s}$. The change of $I(\boldsymbol{x}, \boldsymbol{s})$ experienced along the path from $\boldsymbol{x}$ in direction $\boldsymbol{s}$ is expressed by the differential equation

$$\frac{dI(\boldsymbol{x}, \boldsymbol{s})}{ds} = -\gamma_t I(\boldsymbol{x}, \boldsymbol{s}) + \frac{\gamma_t}{4\pi} \int_{4\pi} p(\boldsymbol{s}, \boldsymbol{s}') I(\boldsymbol{x}, \boldsymbol{s}') d\omega', \qquad (1)$$

where γ_t is the extinction coefficient (which is composed of the absorption and scattering coefficients γ_a and γ_s, respectively), $p(\boldsymbol{s}, \boldsymbol{s}')$ is the phase function describing the part of photon flux scattered from the direction $\boldsymbol{s}'$ into the direction $\boldsymbol{s}$, and $d\omega'$ is the elementary solid angle about the direction $\boldsymbol{s}$. Knowing the photon number density, ρ, and the PDF, $f_{\hat{\boldsymbol{s}}}(\boldsymbol{s}; \boldsymbol{x}, t)$, of photon propagation direction, $\boldsymbol{s}$, one can extract all statistics of interest ($\boldsymbol{s}$ is the sample space variable of $\hat{\boldsymbol{s}}$). The objective of the following modeling approach is to compute ρ and $f_{\hat{\boldsymbol{s}}}(\boldsymbol{s}; \boldsymbol{x}, t)$ by solving the PDF evolution equation

$$\frac{\partial \rho f_{\hat{\boldsymbol{s}}}}{\partial t} + \frac{\partial}{\partial x_i} \left\{ \left\langle \frac{d\hat{x}_i}{dt} \middle| \boldsymbol{s}; \boldsymbol{x}, t \right\rangle \rho f_{\hat{\boldsymbol{s}}} \right\}$$
$$+ \frac{\partial}{\partial s_i} \left\{ \left\langle \frac{d\hat{s}_i}{dt} \middle| \boldsymbol{s}; \boldsymbol{x}, t \right\rangle \rho f_{\hat{\boldsymbol{s}}} \right\} = -\frac{\rho f_{\hat{\boldsymbol{s}}}}{\tau_a}, \qquad (2)$$

where c is the speed of light and following the Einstein summation convention we sum over the index i. The first term describes the change of photon number density in the $\boldsymbol{x}$-$\boldsymbol{s}$-space with time, the second term accounts for transport in physical space, the third term for evolution in direction sample space, and the term on the right-hand side for absorption. The conditional expectations, $\langle d\hat{s}_i/dt | \boldsymbol{s}; \boldsymbol{x}, t \rangle$ and $\langle d\hat{s}_i/dt | \boldsymbol{s}; \boldsymbol{x}, t \rangle$, require modeling. Note that in general one is only interested in a steady state solution of Eq. (2).

Stochastic Model for Photon Scattering

We propose a modeling framework, which is based on solving Eq. (2). A particular difficulty is the high dimensional $\boldsymbol{x}$-$\boldsymbol{s}$-space, in which $\rho(\boldsymbol{x}) f_{\hat{\boldsymbol{s}}}(\boldsymbol{s}; \boldsymbol{x}, t)$ evolves. Therefore, and due to an easier approach to modeling photon propagation, a Lagrangian particle method is employed. In fact, such Monte Carlo particle

methods are widely used in computational physics to solve high-dimensional problems, since the computational cost increases only linearly with the number of dimensions.

Particle Method

In our framework, we consider a cloud of computational particles in the $\boldsymbol{x}$-$\boldsymbol{s}$-space such that $\rho(\boldsymbol{x}) \, f_{\hat{\boldsymbol{s}}}(\boldsymbol{s};\boldsymbol{x},t)$ is represented by the particle number density. The computational particles have a weight w^*, a position $\hat{\boldsymbol{x}}^*$ in physical space, a propagation direction $\hat{\boldsymbol{s}}^*$ (position in $\boldsymbol{s}$-space) and possibly further properties, e.g. a wavelength. Position, $\hat{\boldsymbol{x}}^{*^{n+1}}$, and propagation direction, $\hat{\boldsymbol{s}}^{*^{n+1}}$, at the new time $t^{n+1} = t^n + dt$ are modeled in terms of $\hat{\boldsymbol{x}}^{*^n}$ and $\hat{\boldsymbol{s}}^{*^n}$ at the previous time t^n. Moreover, the weight w^* is modified depending on the absorption time scale τ_a as

$$w^{*^{n+1}} = w^{*^n} e^{-dt/\tau_a}. \tag{3}$$

Scattering behavior as well as absorption time scale depend on the material properties and are therefore position dependent. Next, we describe how $\hat{\boldsymbol{x}}^{*^{n+1}}$ and $\hat{\boldsymbol{s}}^{*^{n+1}}$ are modeled as functions of $\hat{\boldsymbol{x}}^{*^n}$ and $\hat{\boldsymbol{s}}^{*^n}$, while we assume that the medium is isotropic. We consider a local, orthogonal coordinate system with its origin at $\hat{\boldsymbol{x}}^{*^n}$ and the unit vectors $\boldsymbol{e}^1 = \hat{\boldsymbol{s}}^{*^n}$, $\boldsymbol{e}^2 \perp \boldsymbol{e}^1$ and $\boldsymbol{e}^3 = \boldsymbol{e}^1 \times \boldsymbol{e}^2$. In this coordinate system, the new particle location and propagation direction, $\hat{\boldsymbol{x}}'^{*^{n+1}}$ and $\hat{\boldsymbol{s}}'^{*^{n+1}}$, are determined by random lookup from pre-computed evolution tables. These tables depend on wavelength, material properties (and therefore on the position $\hat{\boldsymbol{x}}^{*^n}$) and time increment dt. Each entry represents an equally possible new state $(\hat{\boldsymbol{x}}^{*^{n+1}}, \hat{\boldsymbol{s}}^{*^{n+1}})$ and the new particle properties in the reference coordinate system are obtained by the transformations

$$\hat{\boldsymbol{x}}^{*^{n+1}} = \hat{\boldsymbol{x}}^{*^n} + \boldsymbol{T} \cdot \hat{\boldsymbol{x}}'^{*^{n+1}} \tag{4}$$

and

$$\hat{\boldsymbol{s}}^{*^{n+1}} = \boldsymbol{T} \cdot \hat{\boldsymbol{s}}'^{*^{n+1}} \tag{5}$$

using the transformation matrix

$$\boldsymbol{T} = \begin{bmatrix} e_1^1 & e_1^2 & e_1^3 \\ e_2^1 & e_2^2 & e_2^3 \\ e_3^1 & e_3^2 & e_3^3 \end{bmatrix}. \tag{6}$$

At the beginning of each time step, the boundary conditions are set by populating the border regions of the domain Ω with new particles having the properties $\hat{\boldsymbol{x}}^{*^n}$ and $\hat{\boldsymbol{s}}^{*^n}$. Then, for each particle the evolution table corresponding to the position $\hat{\boldsymbol{x}}^{*^n}$ and a uniformly distributed random integer $k \in \{1,...,m\}$ are determined, where m is the number of table entries $(\hat{\boldsymbol{x}}'^{*^{n+1}}, \hat{\boldsymbol{s}}'^{*^{n+1}})$. The local orthogonal coordinate system is obtained by setting the first unit vector $\boldsymbol{e}^1$ equal to $\hat{\boldsymbol{s}}^{*^n}$, choosing an arbitrary second unit vector $\boldsymbol{e}^2 \perp \boldsymbol{e}^1$ and computing the third unit vector as $\boldsymbol{e}^3 = \boldsymbol{e}^1 \times \boldsymbol{e}^2$. Finally, the new particle position and propagation direction are obtained by the transformations (4) and (5) and the change of the weight w^* due to absorption is computed according to Eq. (3). At the end of the time step, all particles outside the domain Ω are deleted and the statistics of interest, $\boldsymbol{\phi}$, is extracted from the particle field. In order to obtain smooth results, exponentially weighted moving time averaging, i.e.

$$\boldsymbol{\Phi}^{n+1} = \mu \boldsymbol{\Phi}^n + (1-\mu)\boldsymbol{\phi}, \tag{7}$$

is applied, where $\boldsymbol{\phi}$ and $\boldsymbol{\Phi}$ represent the instantaneous and the averaged statistics. Note that the smoothness of $\boldsymbol{\Phi}$ depends on the

memory factor μ. Moreover, n_{max} has to be large enough to guarantee that the solution becomes statistically stationary. There exist various techniques to extract statistics of interest from a particle field. Here, we simply sample over small grid cells Ω^i. For example, the radiant energy flux rate is estimated as

$$\psi^i \approx \frac{c\hat{e}_p}{|\Omega^i|} \sum_{\forall \text{ particles} \in \Omega^i} \{w^*\} \tag{8}$$

and the average energy flow per unit area as

$$\boldsymbol{F}^i \approx \frac{c\hat{e}_p}{|\Omega^i|} \sum_{\forall \text{ particles} \in \Omega^i} \{w^* \hat{\boldsymbol{s}}^*\}. \tag{9}$$

Alternatively one can estimate such quantities at grid nodes using kernel functions, but this is not relevant for the modeling approach and is not further discussed here. Next, the computation of the evolution tables is explained.

Evolution Tables

The entries in the evolution tables represent possible combinations of photon position $\hat{\boldsymbol{x}}'^{*^{n+1}} = \hat{\boldsymbol{x}}'^*(t = dt)$ and propagation direction $\hat{\boldsymbol{s}}'^{*^{n+1}} = \hat{\boldsymbol{s}}'^*(t = dt)$ at time dt conditioned on $\hat{\boldsymbol{x}}'^{*^n} = \hat{\boldsymbol{x}}'^*(t = 0) = (0,0,0)^T$ and $\hat{\boldsymbol{s}}'^{*^n} = \hat{\boldsymbol{s}}'^*(t = 0) = (1,0,0)^T$. First, the time t is set to zero and a particle with initial position $\hat{\boldsymbol{x}}'^* = (0,0,0)^T$ and direction $\hat{\boldsymbol{s}}'^* = (1,0,0)^T$ is created. These properties are then modified by a time-stepping scheme, in which the time is incremented until $t = dt$. At the beginning of each iteration, the time until the next scattering event occurs is determined by a random variable τ with specified PDF f_τ. Then, the new position $\hat{\boldsymbol{x}}'^* = \hat{\boldsymbol{x}}'^* + \min(\tau, dt-t)\hat{\boldsymbol{s}}'^* c$ is computed and if $t + \tau > dt$ the iteration loop terminates. Otherwise, the deflection angle due to scattering is determined by a random variable γ with specified PDF f_γ. The new propagation direction in the orthogonal coordinate system with the unit vectors $\boldsymbol{e}^1 = \hat{\boldsymbol{s}}'^*$, $\boldsymbol{e}^2 \perp \boldsymbol{e}^1$ and $\boldsymbol{e}^3 = \boldsymbol{e}^1 \times \boldsymbol{e}^2$ is $\hat{\boldsymbol{s}}''^* = (\cos\gamma, \sin\gamma\sin\beta, \sin\gamma\cos\beta)^T$, where $\beta \in [0, 2\pi]$ is a uniformly distributed random variable. At the end of the iteration, the new direction in the reference system is computed by the transformation $\hat{\boldsymbol{s}}'^* = \hat{s}_1''^* \boldsymbol{e}_1 + \hat{s}_2''^* \boldsymbol{e}_2 + \hat{s}_3''^* \boldsymbol{e}_3$. Finally, once the iteration loop has terminated, the entry $(\hat{\boldsymbol{x}}'^*, \hat{\boldsymbol{s}}'^*)$ is added to the table. Two illustrative examples of such evolution tables with size $m = 10'000$ are shown in Figs. 1a and 2a, where the positions of all entries can be seen. The corresponding cumulated density functions (CDFs) of the single scattering angle γ are depicted in the figures below, i.e. in Figs. 1b and 2b. In both cases $f_\tau = -e^{-dt/\tau}/\tau$ with $\tau = 0.01L/c$ and $dt = 0.01L/c$ were used, where L is a reference length scale, e.g. the size of the domain. Next, the limiting case of 1D scattering is discussed.

Kubelka and Munk and 1D Random Walk

We show that the evolution tables can be constructed such that the particle method presented above becomes identical with the 1D random walk model devised by Simon [8, 7]. In that model, computational particles located at the nodes of a grid with mesh width h are considered. During each iteration n, each particle switches its propagation direction $\hat{s}^* \in \{-1, 1\}$ with the scattering probability q and changes its position according to $\hat{x}^{*^{n+1}} = \hat{x}^{*^n} + h\hat{s}^{*^n}$. It is straight forward to see that our particle method becomes equivalent to this 1D random walk approach, if the evolution table
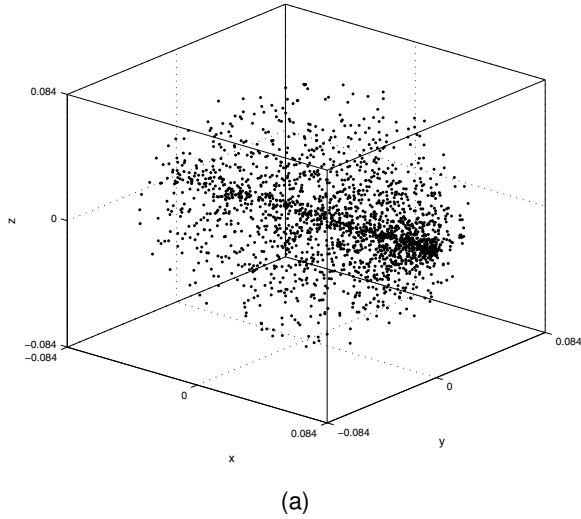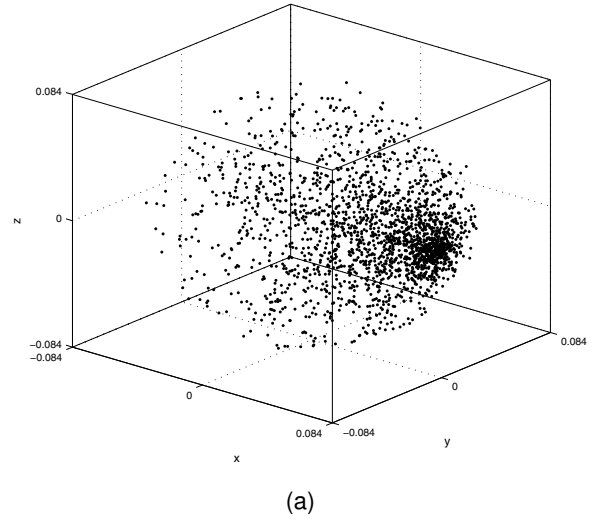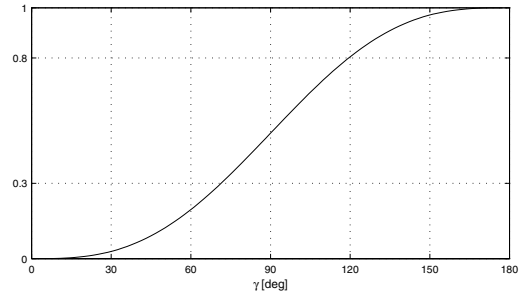
(a)
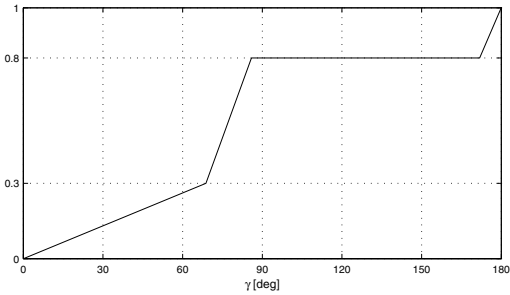
(b)

Figure 1. (a): scatter plot showing all table entry positions for an arbitrarily given γ distribution; (b): the corresponding CDF of γ.

(a)

(b)

Figure 2. (a): scatter plot showing all table entry positions for isotropic scattering; (b): the CDF for isotropic scattering.

entry	$(\hat{\boldsymbol{x}}'^{*},\ \hat{\boldsymbol{s}}'^{*})$
1	$(\boldsymbol{e}^1 c\,\Delta t, -\boldsymbol{e}^1)$
$\vdots$	$\vdots$
m'	$(\boldsymbol{e}^1 c\,\Delta t, -\boldsymbol{e}^1)$
m'+1	$(\boldsymbol{e}^1 c\,\Delta t,\ \boldsymbol{e}^1)$
$\vdots$	$\vdots$
m	$(\boldsymbol{e}^1 c\,\Delta t,\ \boldsymbol{e}^1)$

is used, which is based on the time step size Δt. Scattering probability and 1D mesh size are $q = m'/m$ and $h = c\Delta t$, respectively. A further requirement for exact equivalence is that the initial particle locations are discrete positions $\boldsymbol{x} = ic\Delta t \boldsymbol{e}^1$, where i can be any integer.

It can be shown that the particle method with this evolution table also corresponds to the original model by Kubelka and Munk [5]. For the one-dimensional model one obtains the following relation with the Kubelka-Munk coefficients

$$S = m\Delta t/(m'c), \tag{10}$$
$$K = 1/(\tau_a c). \tag{11}$$

Numerical Experiments

The following numerical experiments shall demonstrate how the photon distribution is influenced by the correlation structure of the material. Using the particle method allows to simulate the scattered light in arbitrary configurations. In Fig. 3 we illustrate contour plots of the logarithm of the particle distribution reflected from two different scattering and absorbing media. The normalized thickness of the media was set in both cases to $1L$ and the scattering coefficient to $\tau = 0.333L$. In order to illustrate the results of different characteristics, the absorption coefficient was set in part (a) to $\tau_a = 0.167L$ and in part (b) to $\tau_a = 1.5L$. In both simulations, the incident light beam was directed perpendicularly to the scattering substrate. Changing the angle of incidence to 45 degrees deflects the backscattered light from the incident axis as expected, which is illustrated in part (a) of Fig. 4. Part (b) of Fig. 4 shows the particle distribution of the oblique light beam transmitted through the same substrate. Note that the most part of the calculated particles are transmitted through the substrate without being scattered (peak in the upper half of Fig. 4 part (b)). It can be seen that the model shows the correct tendency with a statistical noise which depends on to the number of simulated photon particles, completed time steps and employed memory factor.
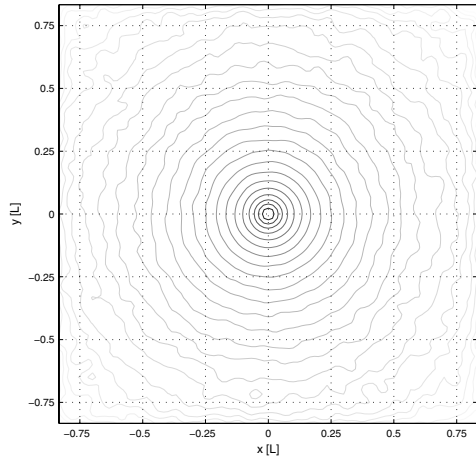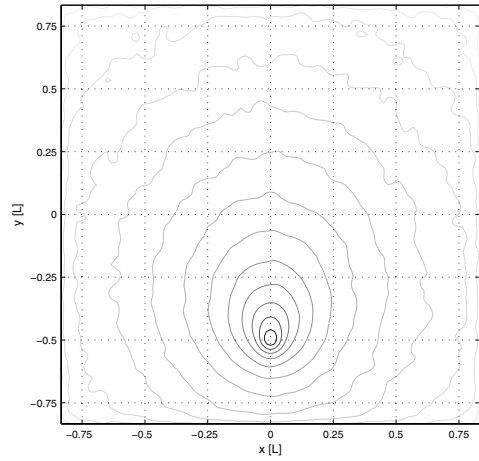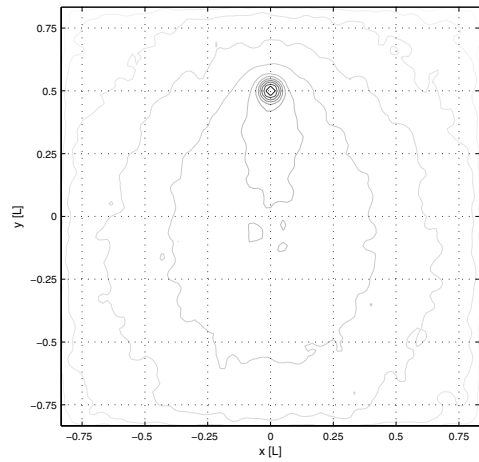
(a)
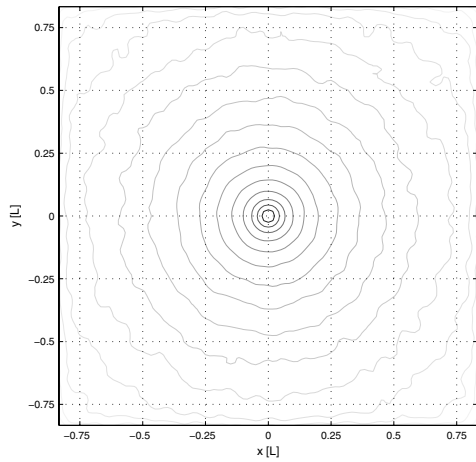
(b)

Figure 3. Logarithmic contour plot of the reflection point spread function of a perpendicular incident light beam. The modeled material has a thickness of $1L$, an isotropic scattering with $\tau = 0.333L$ and an absorption coefficient of $\tau_a = 0.167L$ in plot (a) and $\tau_a = 1.5L$ in plot (b). The depicted noise level is about 1% of the peak value.

(a)

(b)

Figure 4. Plot (a) shows the logarithmic contour plot of the reflection point spread function of a light beam with an incidence angle of 45 degrees entering the substrate in the lower part of the plot. The modeled material has a thickness of $1L$, an isotropic scattering with $\tau = 0.333L$ and no absorption. Plot (b) shows the transmission point spread function of the same configuration.

Conclusion

We presented a general and efficient stochastic modeling framework that allows to simulate light propagation in scattering and absorbing materials. By extracting the relevant statistics, it is possible to compute any scattered light characteristics like the point-spread function in a reflection or transmission configuration. The approach starts form any arbitrary single scattering angle distribution specific for a given material and its scattering and absorption frequencies. In a first step, the joint distribution function of photon position and direction at time $t + dt$ conditional on the photon position and direction at time t is determined. This distribution function is computed by a Monte Carlo simulation, which has to be performed only once for a given material and time step dt. The tabulated joint distribution function is then employed to efficiently compute the particle propagation through the given material. This framework is very general and efficient for isotropic media, however, its extension for anisotropic structures will be a challenge in the future. The approach may be easily extended to account for surface effects and non-isotropic materials by using multiple tables. Moreover, wavelengths dependent absorption coefficients and scattering tables will allow to simulate the color effects in halftone prints.

References

[1] S. Chandrasekhar, Radiative Transfer, Oxford University Press, New York, 1960.

[2] A. Ishimaru, Wave Propagation and Scattering in Random Media, Academic, New York, 1978.

[3] A. Ishimaru, Diffusion of light in turbid material, Applied Optics, 28(12):2210–2215 (1983).

[4] J. A. Kong, Electromagnetic Wave Theory, Wiley, New York, 1986.

[5] P. Kubelka and F. Munk, Ein Beitrag zur Optik der Farbanstriche, Zeitschrift für technische Physik, 12, 593–601 (1931).

[6] Safer Mourad, Patrick Emmel, Klaus Simon, and Roger D. Hersch, Prediction of monochrome reflectance spectra with an extended Kubelka-Munk model, In CIC, pg. 298–304. IS&T - The Society for Imaging Science and Technology (2002).

[7] K. Simon, A stochastic interpretation of Kubelka-Munk, In CGIV 2002: First European Conference on Color in Graphics, Image and Vision, volume 1, pg. 468–472, University of Poitiers, France (2002).

[8] K. Simon and B. Trachsler, A random walk approach for light scattering in material, Discrete Mathematics and Theoretical Computer Science, pg. 289–300 (2003).

[9] L. Tsang, J. A. Kong, and R. T. Shin, Theory of Microwave Remote Sensing, Wiley, New York, 1985.

[10] Li Yang, Björn Kruse, and Reiner Lenz, Light scattering and ink penetration effects on tone reproduction, In PICS, pg. 225–230. IS&T - The Society for Imaging Science and Technology (2000).