A Problem with the Use of XYZ Colour Space for Photorealistic Rendering Computations

Christiane Ulbricht, Alexander Wilkie

Abstract

CIE XYZ colour space is sometimes recommended in literature for photorealistic rendering computations because it allows one to simultaneously handle all perceivable colours at much lower computational cost than by using spectral rendering.

In this paper we show that XYZ is actually a poor choice for such computations, since it is not closed under the component-wise multiplication which is a typical operation in a rendering system. We also discuss why this rather obvious fact has not received much attention yet, and give pointers to alternative solutions.

Introduction

Even though the theoretical desirability of using full spectral representations for light and reflectance values in photorealistic rendering computations has been known for a very long time [4], the overwhelming majority of rendering applications performs those computations which model the interaction of light with matter by using colour values.

Colour values are numerical correlates of human perception, which in turn is an exceedingly complex phenomenon. If one considers this, it is actually somewhat surprising that colour values can be used so well to model light-matter interactions; the main reason why they work is because an RGB triplet basically amounts to a very crude spectral sampling of the visual range.

RGB Rendering

Even though a large number of other colour models – such as CIE XYZ or L*a*b* – exist, the standard choice for the colour space to perform rendering computations in is usually an RGB space; the question of which specific RGB space is suited best for this purpose has been investigated by Ward and Eydelberg-Vileshin [5].

The major disadvantage of RGB colour spaces is of course that none of them contains all visible colours; the obvious problem that arises from this fact is that not all colours that can be found in reality can be represented by positive RGB triplets. This in turn can lead to problems when modelling scenes which contain large numbers of very saturated colours and lights.

CIE XYZ Rendering

A seemingly logical alternative that has been infrequently used in practice is to perform those computations directly in XYZ space [3][2][7], thereby eliminating any gamut restrictions from the rendering process. Figures 1 and 2 show a three-dimensional representation of XYZ space, which forms a closed subspace of the first octant ($X,Y,Z \geq 0$).

The key argument in favour of using XYZ space for rendering calculations is that it assigns positive values to all colours that can be perceived by humans; this property is crucial to ensure a meaningful component-wise multiplication between individual colour values. Nevertheless, Ward et al. [5] came to the conclusion that a carefully chosen RGB space yielded better results than using XYZ space. Unfortunately, the reasons for this behaviour were not analysed further.

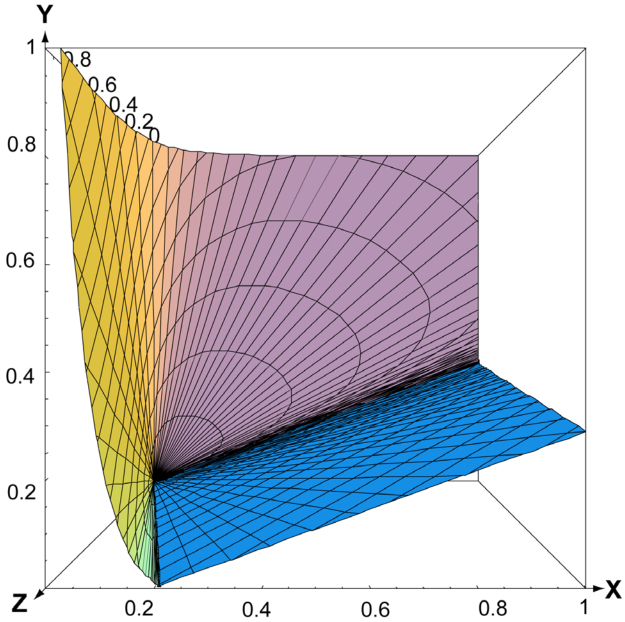

Figure 1. *A Mathematica™-generated plot of the domain of valid XYZ values in the first octant. The parameter lines follow different contours for the curved mantle section and the magenta plane, since these were done using different plots which were joined together afterwards. A better visualisation of this limit surface is shown in figure 2.*

It is known and understood that multiplication between light and reflectance values in colour space is just an approximation of reality; the error incurred by this approximation has been studied by Borges [1]. He compared the result of a multiplication in XYZ space to a reference solution obtained by multiplying two spectra and then converting the result to XYZ space.

Since the potential error bounds were within an acceptable range, the author recommended to use XYZ space instead of spectral rendering because of the lower computational cost and lower storage requirements. However, he did not investigate the error bounds of an RGB space with carefully chosen primaries, which still might be smaller than those of XYZ, especially in the light of Ward's work [5].

A Potential Problem with the Use of XYZ Colour Space

In practice, renderers usually still resort to using an RGB space for their computations, not at least because colourimetric accuracy is usually not a prime concern for such applications at the moment. Therefore actual rendering systems – as opposed to scientific proof of concept implementations – have rarely used XYZ as their internal colour space.

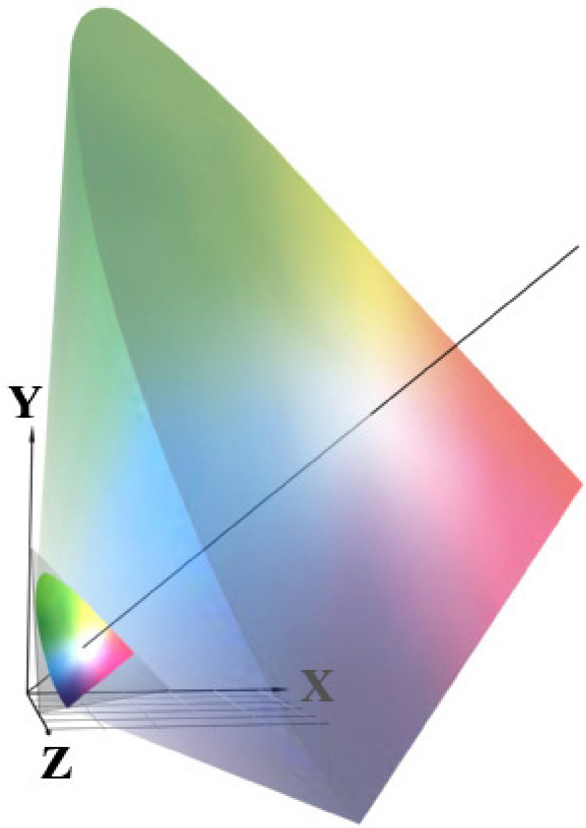

Figure 2. *A more intuitive representation of the subspace of valid XYZ values in the first octant; this figure shows the same shape as figure 1. The geometrical data for the limit surface was exported from Mathematica to 3D Studio Max™and rendered using false-colour shading, transparency and cutting planes which reveal the chromaticity-diagram shaped cross-section of the subspace.*

This might account for the fact that one fundamental problem of using XYZ co-ordinates for computations that model the interaction of light with matter has apparently not received any attention in the computer graphics community so far.

In colour space the interaction of light and matter is approximated through the component-wise multiplication of the tristimulus values of the light and the surface reflectance. In any given RGB space, this operation is not problematical since such an RGB space by definition occupies the entire first octant of the three dimensional co-ordinate system established by the R, G, and B basis vectors; therefore the RGB space is closed with respect to the component-wise multiplication of RGB triplets (i.e. a multiplication of two valid RGB values will always yield a valid, entirely positive RGB triplet as result).

In contrast to this, not all positive XYZ triplets are valid colour values; the gamut of perceivable colours is only a subset of the first octant. This means that a component-wise multiplication of valid XYZ values cannot be guaranteed to yield a meaningful colour value as result, even though all its components will of course be positive.

In particular, multiplications of similar, highly saturated colours almost always generate colours which are outside the range of valid XYZ triplets.

Figures 3 and 4 show the space that is created by "squaring" (i.e. multiplying by itself component-wise) the XYZ triplet of each point that lies on the border of the subspace of valid XYZ values. All results of component-wise multiplication in XYZ space lie within this "squared" space, which turns out to be considerably larger than the gamut of valid XYZ values.

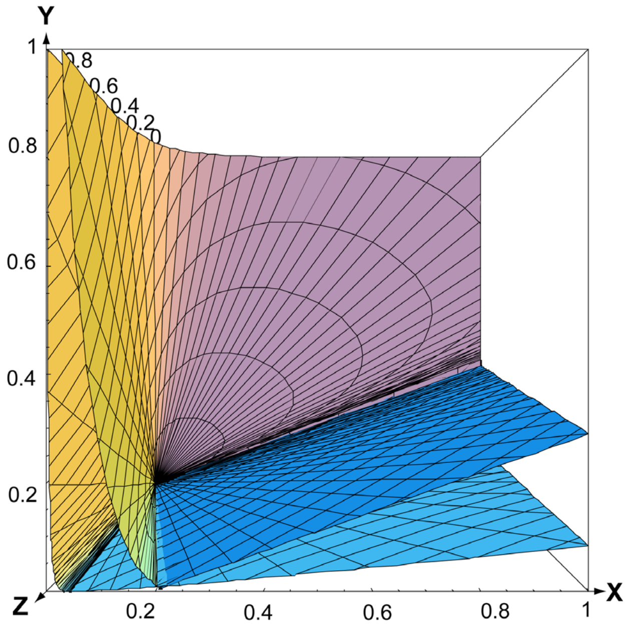

Figure 3. *A Mathematica™-generated plot of the domain of valid XYZ values with the larger surface of possible result values from component-wise multiplication superimposed over it. A better visualisation of the relationship between these two surfaces is shown in figure 4.*

Severity of the Problem in Practice

There is one mitigating circumstance which leads to this problem not being as grave as it might seem at first glance for real rendering systems, and which might also account for the fact that it has apparently not been discussed in computer graphics literature so far.

Meaningful interactions of light and matter usually only occur between values which describe a light – which can be arbitrarily saturated – and surface reflectance values, which form a comparatively small subset of all possible XYZ colours [6]; see figure 5 for a qualitative sketch.

Since the interactions which potentially produce invalid results are those where both operands are highly saturated colours (i.e. those which are already near the boundary of the solid of valid XYZ colours) this means that realistic light and surface interactions are not likely to produce the kind of problem we are trying to illustrate here.

However, the theoretical weakness of XYZ space as a colour space for rendering calculations remains, especially since some kinds of light and matter interactions (such as calculations involving transparency, diffraction effects or dispersion) can lead to situations where invalid colours are easily produced.

Conclusion

Based on the observations made in the preceding sections it can only be concluded that XYZ space should never be used as the internal colour space for rendering computations. While it does offer the benefit of being able to represent all possible colours, the fact that colour multiplications are no longer an operation which can be guaranteed to yield meaningful results pretty much removes any incentive to use it.

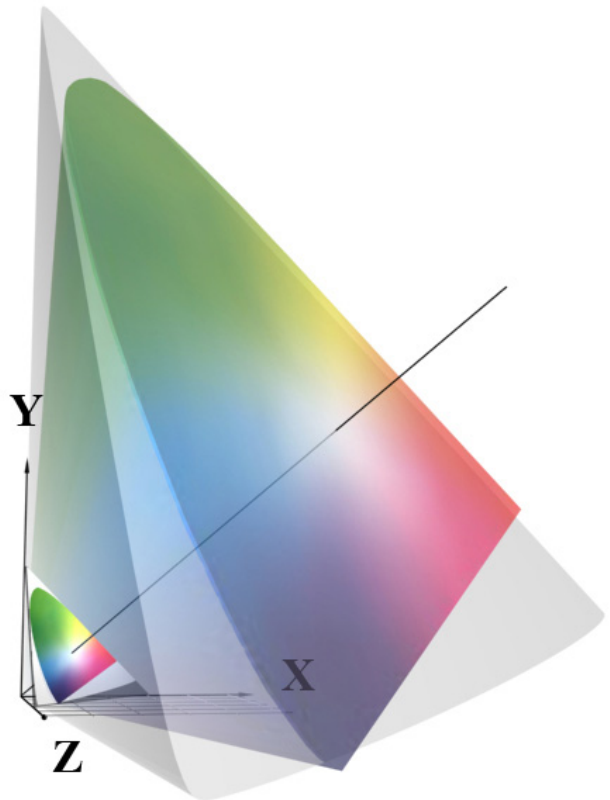

Figure 4. A visualisation of both the gamut of valid XYZ values (in false-colour) and the gamut of triplets that can be generated by component-wise multiplication of these valid XYZ values (grey). This is a 3D Studio Max™rendering of the same data-set that is shown in figure 3, and clearly shows that the grey subspace is outside the range of valid XYZ triplets almost everywhere (the notable exception is a small area of colours in the "purple corner" of the colour sail).

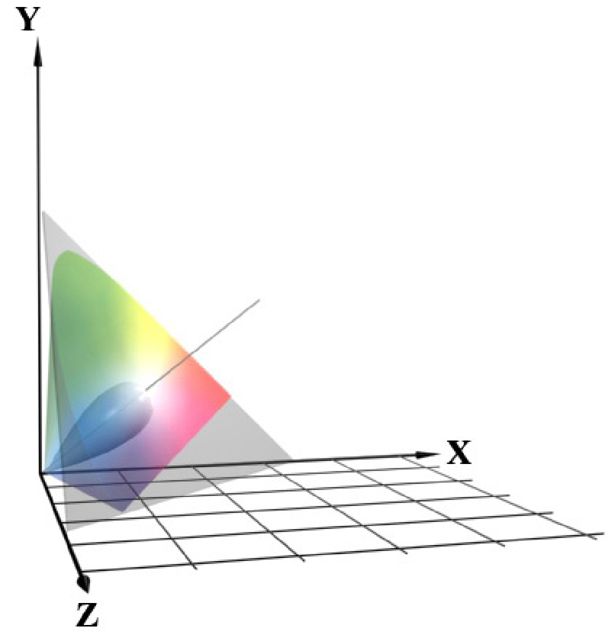

Figure 5. Surface reflectance values form a comparatively small subset of all possible XYZ colours. While the shape of the subset is not correctly represented in this plot it still shows its key feature as far as the topic of this paper is concerned: even its most saturated colours are located at comparatively large distances from the gamut boundary, which means that combinations of surface colours and light values are very unlikely to produce invalid results.

As outlined by Ward et al. [5] one should always use a suitably chosen RGB space instead, or consider full spectral rendering if colourimetric accuracy is of prime concern.

Acknowledgements

We would like to thank Florian Kainz for pointing out the basic problem to us by asking questions about or rendering system (which had – amongst several other options – also supported XYZ rendering up to then) that we had not asked ourselves before. In addition, we thank Andrea Weidlich for providing the 3D Studio Max™ images.

This work was in part supported by the Austrian Science Fund (FWF) under contract number P17558.

References

[1] Carlos F. Borges. Trichromatic approximation for computer graphics illumination models. In *SIGGRAPH '91: Proceedings of the 18th annual conference on Computer graphics and interactive techniques*, pages 101–104, New York, NY, USA, 1991. ACM Press.

[2] Andrew S. Glassner. *Principles of Digital Image Synthesis*, volume 1. Morgan Kaufmann Publishers, Inc., 1995.

[3] Franc Grum and C. James Bartleson. *Optical Radiation Measurements*, volume 2. Academic Press, Inc., 1980.

[4] R. A. Hall and D. P. Greenberg. A Testbed for Realistic Image Synthesis. *IEEE Computer Graphics and Applications*, 3:10–20, 1983.

[5] Greg Ward and Elena Eydelberg-Vileshin. Picture Perfect RGB Rendering Using Spectral Prefiltering and Sharp Color Primaries. In *Thirteenth Eurographics Workshop on Rendering*, 2002.

[6] Günther Wyszecki. *Farbsysteme*. Musterschmidt Verlag, 1962.

[7] Günther Wyszecki and W. S. Styles. *Color Science*. John Wiley & Sons, Inc., 2000.

Author Biography

Christiane Ulbricht studied computer science at the Vienna University of Technology and got her master's degree in 2002. Since then, she is employed as research assistant at the Institute of Computer Graphics and Algorithms and is working on her Ph.D. on the verification of physically based rendering systems. Apart from that her research interests lie in the field of color science and photorealistic rendering.

Alexander Wilkie is an assistant professor at the Institute of Computer Graphics and Algorithms of the Vienna University of Technology. He finished his masters degree in computer science there in 1996, and received his PhD in 2001, also from the Vienna University of Technology. His research interests include predictive rendering, colour science and global illumination.