Computing the Number of Distinguishable Colors under Several Illuminants and Light Sources

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Abstract

The color solid includes all colors perceived by the human visual system associated to physical color stimuli. The optimal or MacAdam colors define its frontier. However, the MacAdam limits, and therefore the shape and volume of the color solid, depend on the illuminant or real light source, even in a uniform color space. In general, the greater the volume of the color solid, the greater the number of distinguishable colors, that is, better colorimetric rendering or quality index. In this work we show two methods to estimate how many distinguishable colors are inside the color solid, particularly using constant lightness planes. The first method fills each constant lightness MacAdam loci in the CIECAM02 chromaticity diagram by squares with unity area. The second method uses the Krauskopf & Gegenfurtner's discrimination model, which permits to fill the constant lightness MacAdam loci with discrimination ellipses increasing in area with increasing distance from the achromatic point. In this way, accumulating the computation of the distinguishable colors for each constant lightness plane, we can estimate the total number of distinguishable colors, so we can establish an absolute ranking of colorimetric quality or color rendering, unlike the CIE color rendering algorithm. Applying both methods for the illuminants A, C, D65, E, F2, F7 and F11 and the real light sources HP1-3, the first position is for the illuminant E, followed by the illuminants C, D65 and F7, and the last positions of this comparative are for the real light sources HP2, HP3 and HP1.

Introduction

Since the human color perception is essentially tri-variant all distinguishable colors by the human visual system are distributed in a 3D structure named color solid. The colors delimiting the borders of the color solid are named optimal colors and they were exhaustively studied by D.L. MacAdam[1-2] in 1935, so the frontiers of the color solid are also named MacAdam limits. The shape of the Rösch-MacAdam color solid does not vary only depending on the color space, but also depending on the illuminant or light source. In the scientific literature there are colorimetric graphs of the color solid under different illuminants[3-5]. In a recent work[6] we have demonstrated that, even in the more uniform color spaces (CIECAM02, DIN99d, etc), the shape and volume of the color solid change with the illuminant or real light source (Figure 1). The greater color solid volume, the greater the number of distinguishable colors, that is, the colorimetric rendering or quality index is better. Therefore, estimating the number of distinguishable colors can be an alternative for the current algorithm of the color rendering of light sources published by CIE[7-11], which is based on the color difference between a pair of corresponding colors (a color chip illuminated under the test lamp and the reference illuminant).

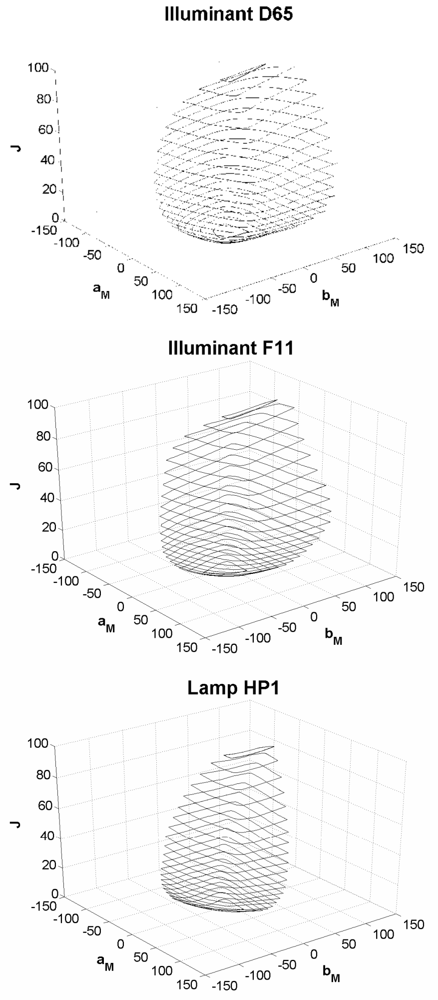

Figure 1. *Rösch-MacAdam color solid in the CIECAM02 color space for some illuminants (D65 and F11) and one real light source (HP1). The step between adjacent lightness planes is taken from the lightness step ΔL^* = 5, from 1 to 100, in order to avoid aliasing.*

In this work we show two methods that are an alternative to those just recently discussed by Kuehni[12] for the computation of the total number of discernible colors. In both methods we firstly calculate the number of distinguishable colors in constant lightness planes of the color solid, and accumulating the partial counts for each lightness plane we can compute the total number of the distinguishable colors. The first method, more easy, intuitive and computationally shorter, consists in filling the constant lightness MacAdam loci with squares with unity area in a uniform chromaticity diagram, for instance, in the CIECAM02 color space. In this case, the color metric would be based on spheres, so the use of squares (cubes) is a good initial approximation, although it would be more adequate to fill the color solid with a spheres packing algorithm. On the other hand, the second method takes into account the Krauskopf & Gegenfurtner's discrimination model[13], based on psychophysical data, and consists in filling the constant lightness MacAdam loci with discrimination ellipses increasing in area with increasing distance from the achromatic point.

Therefore, selecting some illuminants and light sources we can compute the total number of associated distinguishable colors. These results establish an absolute ranking of color rendering, unlike the current CIE color rendering algorithm. In this work we make a comparison between a set of illuminants (A, C, D65, E, F2, F7, F11) and light sources (HP1-3) published by CIE, and their color rendering is evaluated using the proposed methods and also the standard procedure, which is not based on the computation of the discernible colors.

Methods

Before describing the algorithms which constitute the aim of this work, it is necessary to discuss some details about the calculation of the MacAdam limits under several illuminants and light sources.

MacAdam limits under different illuminants

The MacAdam limits are calculated taking into account an improved algorithm[6] developed from the original MacAdam's algorithm. With this new algorithm we can obtain the optimal colors for any lightness value and any illuminant or light source and calculate the complete associated color solid. The illuminants and real light sources used in this work were taken from CIE[14], and they can be separated into three categories: continuous spectra type (illuminants A, C, E and D65), discontinuous spectra type (fluorescent illuminants F2, F7 and F11), and real light sources with very peaked spectra (HP1: standard high pressure sodium lamp; HP2: color enhanced high pressure sodium lamp; and, HP3: high pressure metal halide lamp). Figure 1 shows the color solid in the CIECAM02 color space obtained for these illuminants and light sources. It can be clearly seen that these color solids have different shapes and volumes. From an intuitive point of view, taking into account only the shape and volume of these color solids, we would predict a greater number of distinguishable colors for D65 than for the illuminant F11 and light source HP1, so probably the illuminant D65 would have the highest color rendering value of this restricted comparison.

The number of distinguishable colors inside the color solid can be computed assuming some conditions. A priori, this is achieved by estimating the number of the discrimination ellipsoids filling the color solid. Usually the problem is simplified by fixating the luminance factor Y or the lightness L*, so the computation of ellipsoids is replaced by the simpler computation of the discrimination ellipses plus the interpolation of the just-noticeable lightness differences between a fixed value and the next one[12]. Experimental data about discrimination ellipses abound in the literature[13,15,16]. We have chosen the Krauskopf and Gegenfurtner data[13,16] because they permit a homogeneous sampling of the color solid. This procedure could seem an unnecessary complication of a simple problem, because, once the MacAdam loci are computed, we could assume that their areas (see again Figure 1) in a given color space are a measurement of the size of the color gamut of the human visual system, or even of a digital color device (capture, display or printing). However, in this way the result obtained would be dependent on the color space used, whereas this would not happen with the new procedure we propose. Therefore, using constant lightness planes, and only considering the lightness step or threshold $\Delta L^* = 1$, from 1 to 100, we could pack with ellipses the MacAdam limits taking into account the proposed discrimination model, and after that, sum all partial counts (distinguishable colors) obtained in each constant lightness planes.

However, before describing below the adaptation details of the Krauskopf & Gegenfurtner model, we have considered interesting to use a simpler discrimination model based on squares instead of ellipses. The reason to do this is evident: if we use a true uniform color space, the discrimination ellipsoids are transformed into spheres, so a good initial approximation could be use cubes instead of spheres. Since the CIECAM02 color space is currently the most uniform color space, and we have the color solids for several illuminants and light sources encoded in this color space, we can estimate the number of distinguishable colors in this color space summing how many squares are inside the constant lightness MacAdam loci. It is not the perfect solution, because it would be better to use circles instead of squares, and the CIECAM02 color space also has some uniformity defects, but this partial solution is interesting and powerful due to its simplicity. Comparing a priori both packing methods, with ellipses or squares, we could say that the first one would give the total number of distinguishable colors by defect, while the second one would give a result by excess. Therefore, we have two algorithms to delimit the true total number of discernible colors inside the color solid, as much for the human visual system under several illuminants and light sources as for other color imaging devices.

Squares packing method of the MacAdam loci

This method consists of filling the constant lightness MacAdam loci, encoded by the CIECAM02 color space and plotted in a (a_M, b_M) chromaticity diagram, with squares with unity area. To do this, the initial XYZ data of the optimal colors associated to the illuminant/light source are transformed into perceptual variables (a_M, b_M, J) of the CIECAM02 color appearance model[17]. Then, the packing algorithm draws the first square around the achromatic point and next successive non-overlapping squares are drawn from this first square. This geometric procedure is simple, but the difficulty is in the computational cost due to the very high number of the counted color-squares (around some thousands) for each constant lightness MacAdam loci (from 1 to 100 lightness values with $\Delta L^* = 1$). Figure 2 shows an example of a MacAdam locus packed with squares with this method.

Ellipses packing method of the MacAdam loci

As we advanced above, this method is based on the adaptation of the experimental data of Krauskopf and Gegenfurtner in order to design a packing method with ellipses for the constant lightness MacAdam loci. The optimal colors encoded initially by the CIE-XYZ color space are transformed into a modified MacLeod-Boynton color space[18] taking into account the change of illuminant/light source. This is necessary to apply the discrimination model derived from the experimental data of Krauskopf and Gegenfurtner. In this new color space the chromaticity coordinates for the equal-energy, perceptual or adapted white stimulus are (0,0), and the new chromaticity coordinates (l', s') are $l' = l - l_E$ and $s' = s - s_E$ where $l_E = 0.66537$ and $s_E = 0.01608$, after calculating the cone excitations LMS of the white stimulus E ($X_E = Y_E = Z_E = 100$) using the Smith-Pokorny fundamental matrix. These new chromaticity coordinates are re-scaled in such a way that the threshold around the equal energy white along the cardinal directions is one, that is, l' and s' are divided by, respectively 0.0011 and 0.0012, according to the Krauskopf and Gegenfurtner data.

However, since the MacAdam limits can be associated to illuminants or light sources different to the equal-energy illuminant E, before the color transform between the XYZ and LMS data it is necessary to apply a chromatic adaptation transform from the XYZ data encoded by the test illuminant to the corresponding XYZ data according to the illuminant E. To do this we have used the chromatic adaptation transform of the CIECAM02 color appearance model[17].

Returning to the modified MacLeod-Boynton color space, colors in the same vertical line in the chromatic diagram have constant *L* and *M* values, while colors in the same horizontal line have constant values for *S* and (*L* + *M*). Accordingly, a vertical line contains colors that would give constant response in a red-green mechanism, $T = L - \alpha M$, no matter the value of α Analogously, a horizontal line contains colors yielding constant response in a yellow-blue mechanism of the type $D = S - \beta(L+M)$, no matter the value of β In particular, those colors in the $D = 0$ and $T = 0$ lines elicit responses only from the T or the D mechanism, respectively, and are therefore the cardinal directions of T and D.

With these preliminaries, the discrimination ellipses in this color space are computed as follows. The discrimination ellipse around the equal-energy white ($T = 0$, $D = 0$) defines the unity threshold in each cardinal direction. Thus, with this metric the discrimination ellipse around ($T = 0$, $D = 0$) is a circle of unity radius. Let us consider a pedestal in the T cardinal direction. Thresholds along this direction are proportional to the T response to the pedestal, whereas thresholds along the orthogonal D direction are constant. Analogously, if the pedestal is on cardinal direction D, thresholds along the D direction are proportional to the D response to the pedestal, whereas they are constant along the orthogonal T direction. In consequence, discrimination ellipses around stimuli in one of the cardinal directions are oriented along that direction. The rate at which the major axis of each ellipse changes along each cardinal direction was taken from the experimental data of Krauskopf and Gegenfurtner. When the pedestal is not on one of the cardinal directions, the laws governing thresholds are not so simple. Discrimination ellipses around a pedestal in the first or third quadrant of the modified MacLeod-Boynton space seem to be oriented along the cardinal directions. The sizes of the major and minor axis of the ellipses are proportional to the T or D response elicited by the pedestal. This result can be explained if we admit the existence of two independent discrimination mechanisms, whose cardinal directions are the T and D directions of MacLeod-Boynton's chromaticity diagram, and that interact vectorially. However, discrimination ellipses around pedestals in the second or fourth quadrant seem to be oriented along the direction defined by the pedestal. This result seems to imply the existence of a continuum of mechanisms tuned in along equally spaced directions in the color space. The directions along which are tuned these hypothetical mechanisms could be deduced approximately from the experimental data, but the rate of increment of threshold along each of these directions cannot. Although it could reasonably be admitted that thresholds again would increase with increasing distance to the white stimulus, the actual law of variation would still to be determined. Because our aim is to compare the number of ellipses within the MacAdam limits in the human visual system under several illuminants and light sources, and not to reach the best estimation of this number, the model of two cardinal directions is enough.

The next problem to solve is which method to use to pack the discrimination ellipses. We have followed two different procedures. With what we call the tangent criterion, we determine the position of the centers of the ellipses to verify two conditions: 1) each ellipse is tangent to the other four surrounding it and 2) the center of two adjacent ellipses have either the same T or the same D value. This criterion does not yield optimal packing, because the gaps between ellipses increase with the distance to the achromatic point. The second strategy, that we call dense packing, consists in placing the centers of the ellipses on the centers of the tiles of an hexagonal mosaic covering optimally the space to which we have applied a non-linear transform [x*f(x), y*f(y)]. The functions f(x) and f(y) have been found empirically, and verify that the overlap between ellipses is small. In this way we come nearer to optimum ellipse packing. These ellipse packing algorithm were applied for the first time in making a comparison between the color gamuts of the human visual system and a conventional digital camera[19] assuming the same color metrics for both input devices, and it was found that the two packing criteria produce basically the same results.

Results and Discussion

The results obtained with the two packing algorithms are summarized in Figures 2-5. Firstly, Figure 2 shows an example of a MacAdam locus packed with squares. As we said above, the number of discernible colors obtained with this method would be very large, so the visualization of the MacAdam loci filled with squares with unity area is not easy. For this reason, we have shown the results for a high lightness plane associated to one illuminant because here the number of distinguishable colors is not so high and the drawing of the squares can be seen. Afterwards, we will show a graph and table where the total number of discernible colors for different illuminants and light sources will be shown.

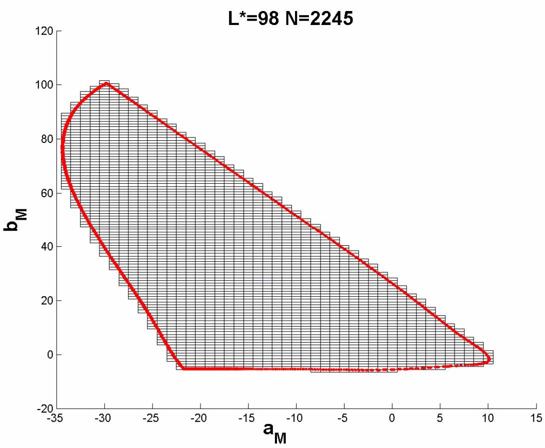

Figure 2: *Packing with squares of unity area of the MacAdam locus for the lightness plane L* = 98 under illuminant D65 in the CIECAM02 chromaticity diagram. N is the number of discernible colors computed inside this MacAdam locus. Take into account that the axis are not regular and the squares seem rectangles.*

As it was also advanced above, and as it can be seen in Figures 4, working with the ellipse packing method the number of distinguishable colors would be a lot smaller than with the squares packing method. However, it is important to remark a visualization effect produced in both methods, above all in the second one. As it can be seen in Figures 3 and 4, and in minor detail in the above figure, there are ellipses (squares) almost outside the MacAdam boundary. This occurs because the MacAdam loci are complex closed curves so any gap, filled partially or completely with a square/ellipse, is considered a new discernible color.

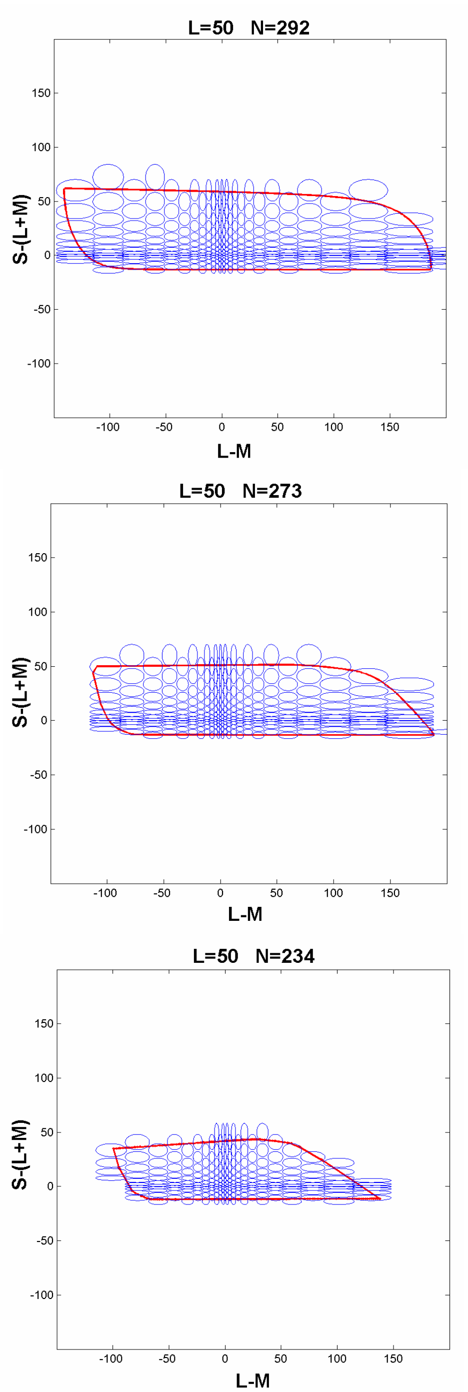

Figure 3: *Dense packing with ellipses of the same constant lightness (L* = 50) MacAdam locus in the modified MacLeod-Boynton's chromaticity diagram for different illuminants and light sources: D65 (top), F11 (center) and HP1 (bottom).*

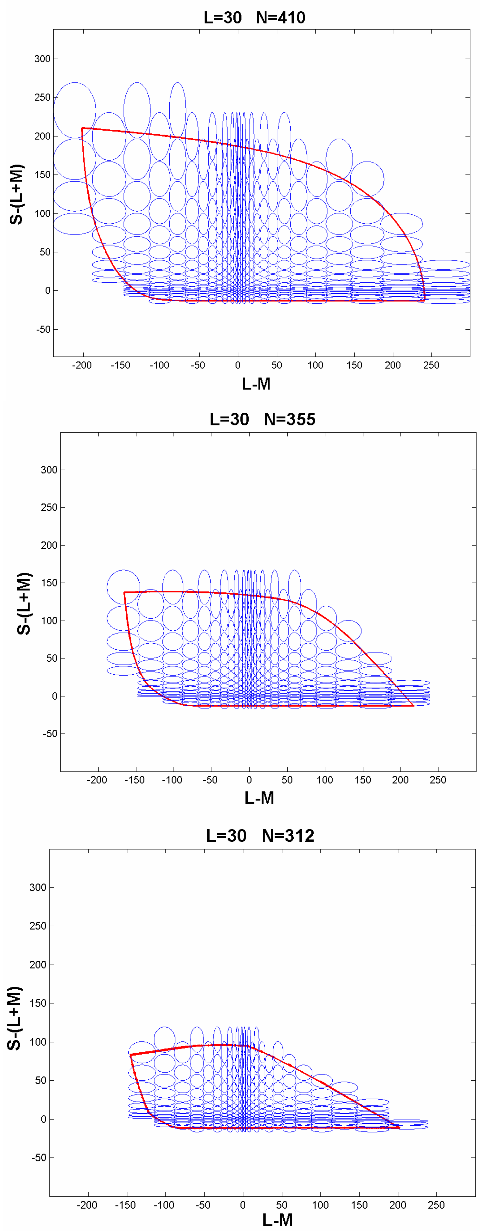

Figure 4: *Dense packing with ellipses of the same constant lightness (L* = 30) MacAdam locus in the modified MacLeod-Boynton's chromaticity diagram for different illuminants and light sources: D65 (top), F11 (center) and HP1 (bottom).*

The number of distinguishable colors, N, for each constant lightness plane and illuminant/light source can be collected as in Figure 5. If the lightness step is $\Delta L^* = 1$, then we can obtain the total number of distinguishable colors calculating the area beneath each curve plotted in Figure 4, i.e., accumulating the partial counts of discernible colors from similar figures as Figures 2-4.

It is interesting to make again a comparison between the packing methods, above all taking into account the color space used for encoding the color solid. In CIECAM02 color space (Figure 1) the MacAdam loci near to absolute black point shrink progressively until they become a single point or black vertex. This behavior is equivalent in the other side (vertex of the absolute white). So, the top graph of Figure 5 clearly shows a maximum in the middle of the curve, near the middle lightness range. However, in MacLeod-Boynton's color space (compare the scaling of Figures 3 and 4), as it also happens in the standard CIE-XYZ or CIE-u'v'Y color spaces, the MacAdam loci associated to a very low lightness are large. So, unlike the top graph, the bottom graph of Figure 5 clearly shows a maximum in the beginning of the curve, in the lightness value L* = 1, when we suppose that the value *N* for L* = 0 is zero. Therefore, as we advanced in the introduction, both methods and used color spaces are an approximations, by defect (ellipses, MacLeod-Boynton) and excess (squares, CIECAM02), of the accurate computation of the total number of discernible colors.

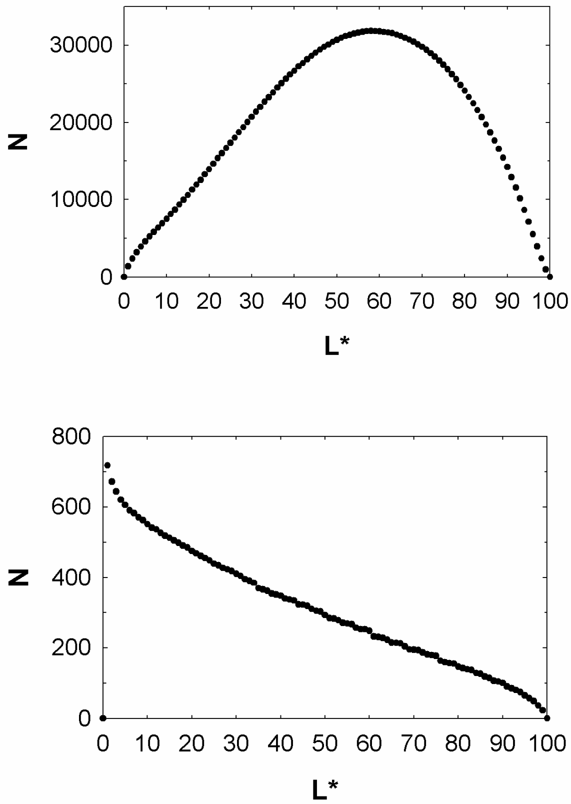

Figure 5: *Partial counts of discernible colors vs. lightness value for the illuminant D65 taking into account both packing methods: squares (top) and ellipses (bottom). The total number of the distinguishable colors is the area beneath each curve.*

Next, it is interesting to make a comparison of the obtained with different illuminants, light sources and the used packing methods with respect to the standard CIE color rendering algorithm[7]. Taking into account these parameters and the selected illuminants and light sources, we have included in Table 1 a colorimetric quality arrangement according to the simplest method used (squares method). It is important again to remember that, unlike both proposed methods, the standard CIE color rendering index (R_a) is a relative colorimetric quality index because each test illuminant or light sources can have a different reference illuminant. In contrast, both proposed methods try to establish an absolute colorimetric quality index without the need to establish a reference illuminant and to calculate color differences between a corresponding color pair, but only calculating the volume of the associated color solid.

Table 1: Total number of the distinguishable colors of several illuminants and light sources according to both packing methods of the constant lightness MacAdam loci.

Light source	Ellipses method	Squares method	R_a (CIE)	Ranking
A	25,851	1,752,861	99.58	5
C	33,500	2,046,392	97.39	2
D65	30,736	2,013,114	99.58	3
E	30,274	2,050,033	95.11	1
F2	26,323	1,665,000	62.83	7
F7	30,732	1,968,210	90.23	4
F11	26,311	1,735,126	82.91	6
HP1	22,609	1,050,525	8.29	10
HP2	25,465	1,663,648	82.59	8
HP3	25,492	1,661,768	82.50	9

It can be seen in this table, and taking into account the ranking order established according to the used simplest method of classifying the colorimetric rendering of illuminants and light sources, that the first position is for the illuminant E, followed by the illuminants C, D65 and F7, and the last positions of this absolute comparative are for the real light sources HP2, HP3 and HP1. Some ranking positions coincide with the standard criterion established by CIE, above all for the low portion of the used ranking value. But, in the other hand, this new analysis permits to distinguish the colorimetric quality of the standard illuminants with CIE color rendering indexes near 100, as for example the illuminants A, C, D65 and E. In particular, we think the logic first position of this comparative should be for the equal-energy illuminant E, and this has been shown in this work, although maybe it could be crucial the natural evolution of the adaptation of the human visual system to the daylight, with more blue spectral component that the equal-energy illuminant E.

Finally, it is interesting to compare the results obtained with the squares method with those recently described by Kuehni[12]. In particular, our result coincides approximately with some studies described in this reference book.

Conclusions

After analyzing our results we can say that we have found an alternative procedure for calculating the color rendering index of the illuminants and light sources, based on the computation of the total number of distinguishable colors inside the associated color solid. It is an easy method, particularly when the squares packing method is used in a uniform color space (CIECAM02), although we will try to apply the best method for packing the MacAdam loci with spheres[20]. Besides, it establishes a colorimetric classification of the illuminants and light sources without the necessity to take a reference illuminant, so the color rendering index we propose is absolute. On other hand, this work can be extended to compare the gamut volumes for color imaging devices[19,21]. Finally, this work also shows that the total number of discernible colors is around some millions, and that this number depends on the light source, so the lighting applications, i.e. in sports, museums[22-24], cinema, etc, can be also potentially interesting.

Acknowledgements

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