Calculation of the Optimal Colors of Linear Input Devices

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Abstract

A new method for calculating the optimal colors of any linear input device is proposed. The algorithm searches systematically the two types of optimal colors in the R+G+B planes of the device. In this way, the loci of optimal colors associated to the input device can be plotted in its own chromaticity diagram, but also in CIE (human) color spaces. Thus, we can make a comparison in CIE color spaces of the color gamut of the linear input device relative to that of the human visual system (MacAdam limits). This algorithm has been applied to some linear input devices, fulfilling or not the Luther condition, associated or not to real digital cameras, even to animal visual systems. We have found that the color gamut of a linear input device is the same as that of the human visual system, regardless of the fulfillment or not of the Luther condition. This is a corollary derived from the definition of optimal color, because it has not got any possible physically metamers except itself, independently of the capture color space. This also implies that the shape and volume of color solid of the positive spectral reflectances depends on the color space used, and, if the input device does not fulfill the Luther condition, the metamerism of the device will work in a different way redistributing the color stimuli. But the number of the distinguishable colors can be different depending on the chromatic discrimination properties of each input device. So, this work may open new ways to compare the color gamuts, actually the number of distinguishable colors, between the human visual system and non-human input devices, above all those associated to animal visual systems, because their color metric may differ from that of the human visual system.

Introduction

Human color perception is essentially tri-variant in nature. Colors are defined by three parameters: lightness, hue and colorfulness (chroma, purity, saturation, etc). This means that colors define a 3D-structure named color solid, in whose upper and lower vertex are the absolute or perceptual white and black, respectively. The colors shaping the intermediate frontiers, obviously with the maximum colorfulness, are called optimal colors and they were exhaustively studied by MacAdam[1,2] in 1935. Due to this, the color solid borders are also known as MacAdam limits. Rösch[3] in 1929, but above all MacAdam, analyzed the theory of optimal colors proving that their spectral reflectance or transmittance can be only zero or one. There are two types of optimal colors (Figure 1): type 1, with "mountain"-like spectral profiles, and, type 2, with "valley"-like spectral profiles. As we know, although these colors are not present in nature, they are very important for Color Science because they constitute the frontier of the human color solid. Therefore the Rösch-MacAdam color solid is the color space derived from the color-matching functions[4]. Due to this, the MacAdam limits are used to analyze the colorimetric quality of colorants in any industrial application[5,6] (textiles, paints, printing, etc). Since there is at the moment an improved method

for calculating the MacAdam limits under several illuminants for any lightness value and hue angle[7], we have applied this new algorithm to calculate the optimal colors of any input device, independently it is associated to a color imaging device or an animal visual system[8-10].
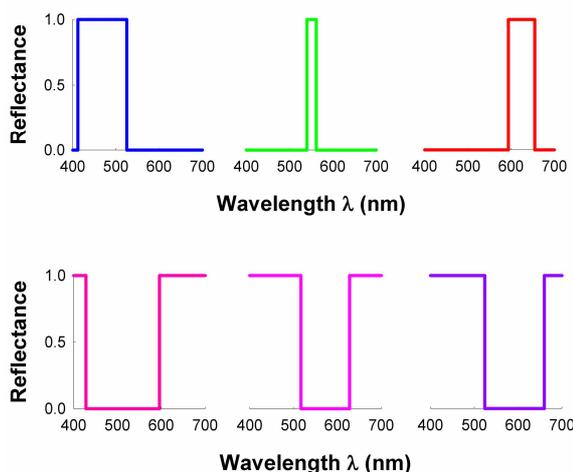
Figure 1: *Six examples of optimal colors (top: type 1; bottom: type 2) with luminance factor Y = 20% under illuminant E and the CIE-1931 XYZ standard observer. The transition wavelengths λ_1 and λ_2 are, from left to right side, as follows: 412.1 – 525.2 nm, 540.0 – 562.0 nm, 594.0 – 654.7, 428.0 – 596.0 nm, 517.1 – 628.0 nm and 524.0 – 660.1 nm.*

As it is well established, for input devices (cameras and scanners, but also for any animal trichromatic vision system) the color encoding is also essentially tri-variant in nature because colors are also defined by RGB or LMS tristimulus values. This means that all the colors encoded by an input device also define a 3D-structure like a color solid, in whose upper and lower vertex are the adopted white and black, respectively. This color solid is the gamut of the input device because it encloses all the possible color-stimuli, including their metamers. The colors shaping the intermediate frontiers, obviously with the maximum colorfulness, are also called optimal colors, but are now associated to the input device.

Nevertheless, a priori these optimal colors associated to an input device, are not necessarily the same, even spectrally, as the optimal colors associated to the human visual system (colorimetric standard observer) due to device metamerism: color-stimuli encoded equal in CIE color space can be encoded different in RGB color space, and vice versa. If the input device is a true colorimeter or if it fulfils the Luther condition[11,12], that is, if their color-matching functions or scaled spectral sensitivities are exact linear combinations of the color-matching functions of the CIE standard observer, both sets of optimal colors would be the same. But, if the input device is not a colorimeter, both sets might be different. However, if we take into account the proper definition of optimal color, the metamerism of the device will not be important because the

optimal colors have not got any spectrally positive metamers except themselves. Therefore, we will test in this work as corollary of the above definition of optimal color if the color gamut of a linear input device encoded and plotted in some CIE chromaticity diagram is the same as the MacAdam limits (human color gamut), independently of the fact that the device may or not fulfill the Luther condition.

In parallel to the tasks indicated above, this work also aims to open a discussion about how the color gamut associated to any input device can be compared with that associated to the human visual system, i.e., if the input devices do not have gamut limitations. Until now, there are two methods in order to do this. One method consists in sampling the space of all possible surface reflectance functions taking into account device metamerism[13], but without taking into account the optimal colors, which are really the borders of the object color solid. The second method[14] takes a different approach from calculating the RGB encoding of the human optimal colors and then transforming these RGB data into XYZ data using a color profile. We think now that the correct approach to solve this discussion is to calculate the own optimal colors of the non-human input device and encode them in any (human) CIE color space.

To summarize, we are going to calculate the optimal colors associated to several input devices, fulfilling or not the Luther condition, associated or not to real digital cameras, even to animal visual systems, in order to compare their color gamuts with the MacAdam limits (human optimal colors), both in the own color spaces of the analyzed input devices and in the (human) CIE color space. Therefore, the aim of this work is to test whether the input devices have gamut limitations, and what limitation means this taking into account the borders of the color solid of each input device, human or non-human.

Materials and methods

Five input devices (Figure 2, left side) were selected for this analysis:

- Two real digital cameras: a 8 bit CMOS color camera (Pixelink PL-662) and a 12 bit 1-CCD color camera (QImaging Retiga).
- Two theoretical color sensors: a set of peaked spectral sensitivities and the MacAdam sensor set[15].
- The spectral sensitivities of the goldfish[16], as representative of an animal visual system.

The first sub-set corresponds to real digital cameras, whose spectral sensitivity set is measured using the monochromator method[17,18]. Nevertheless, any spectral sensitivity set obtained by computational methods[19-21] could be used in this work. The next sub-set is composed by two ideal color sensor sets, the second sensor being the only input device of this analysis that fulfills the Luther condition because it is the exact linear combination of the CIE 1931-XYZ color-matching functions with minimal spectral overlap. Finally, the goldfish has been chosen as representative of the trichromatic animal kingdom[8-10,16]. All these spectral sensitivities have an equienergetic white balance, i.e., the area under the spectral curve for each color channel is the same. We do this because the CIE-1931 XYZ color-matching functions fulfill this condition[22].

The spectral sensitivities or color-matching functions of these input devices, including those associated to the colorimetric standard observer, are denoted by **T** in matrix format (*N* rows for the spectral range and 3 columns for the color channels). If we want to test if the input device fulfills the Luther condition we need firstly to calculate the basic color profile or transform between any input device and the colorimetric standard observer (CIE-1931 XYZ). If we denote by $\mathbf{T_{RGB}}$ and $\mathbf{T_{XYZ}}$ the color-matching functions of an input device and the colorimetric standard observer, respectively, the basic color transform **M** is the connection matrix relating both color spaces with maximum ignorance[23]:

$$\mathbf{M} = \mathbf{T_{XYZ}}^t \cdot \mathbf{T_{RGB}} \cdot \left(\mathbf{T_{RGB}}^t \cdot \mathbf{T_{RGB}}\right)^{-1} \qquad (1)$$

where $\mathbf{T}^t$ is transpose matrix of matrix **T**.
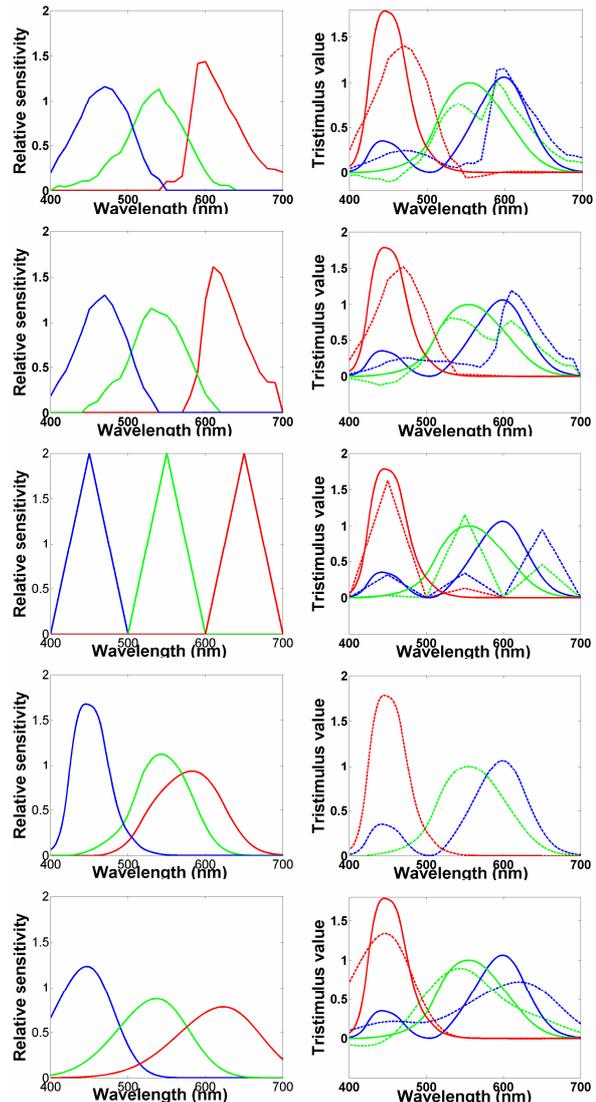
Figure 2: Spectral sensitivities (left side) and the graphical test of the Luther condition (right side) of the analyzed input devices. From top to bottom: CMOS digital camera, CCD digital camera, ideal peaked sensor set, MacAdam sensor set, and goldfish. Solid lines: original CIE color-matching functions; dashed lines: estimated CIE color-matching functions.

The test of the Luther condition ends by calculating the CIE color matching functions estimated by $\mathbf{T_{RGB}} \cdot \mathbf{M}^t$ and comparing them with the original ones. As it can be seen in Figure 2 (right side), only the MacAdam sensor set fulfills the Luther condition in this comparative. So this preliminary test indicates that, although there will always be a matrix **M**

between two linear color spaces, this color transform will not warranty the fulfillment of the Luther condition, unless both linear color spaces are associated to the same visual system (for instance, XYZ and LMS data in the human visual system by the Smith-Pokorny[24] or Hunt-Pointer-Estévez[25] fundamental matrixes, MacAdam sensor set[15], etc). In our case, only the MacAdam sensor set belongs to the same (human) visual system. Therefore, this preliminary test warns us that there will be device metamerism[26,27] in all selected input devices, except one, so color-stimuli encoded as equal in CIE color space can be encoded as different in RGB/LMS color space, and vice versa. Then, the optimal color set of the MacAdam sensor set will be the same than that of the CIE standard observer, so the color gamuts are identical. But, in contrast, it is possible that each optimal color set associated to the rest analyzed input devices may differ from that of the CIE standard observer.

Next, we change the algorithm for calculating optimal colours[7] of the CIE-1931 XYZ observer under equienergetic illuminant, replacing the searching condition for any luminance factor Y value by any R+G+B value, where RGB data are the tristimulus values scaled to 100. With these preliminaries, for each fixed R+G+B value, the routine systematically locates the wavelengths λ_1 and λ_2 where the sudden change of reflectance or transmittance happens (from 0 to 1 or opposite). That is, the spectra of optimal colors in Figure 1 differ in centre and width but not height (always 0 or 1). We can do this because the original MacAdam's algorithm for searching optimal colors is directly related with the calculation of the center of gravity in additive color mixing. So, assuming linear color encoding, i.e., additive color mixing for the analyzed input devices, we have adapted the original MacAdam's algorithm to search the optimal colors along the R+G+B diagonal from black (0,0,0) to equienergetic white (100,100,100) in each RGB/LMS tristimulus color space. To sample all the R+G+B diagonal of the tristimulus color space we use 3 unit step from R+G+B = 1 to R+G+B = 298. If we denote by A the fixed value of the sum R+G+B, the tolerance value ΔA is 0.01, and if we take the spectral range $N = 3001$ (from 400 to 700 nm with 0.1 nm as wavelength step), the routines for calculating the optimal colors in any RGB input devices are as follows:

TYPE 1

for $i = 1\,to\,N = 3001\,do$

for $j = 1\,to\,N = 3001\,do$

if $i < j$ *and*

$$\sum_{k=i}^{j} \bar{\mathbf{r}}(\lambda_k)\Delta\lambda + \sum_{k=i}^{j} \bar{\mathbf{g}}(\lambda_k)\Delta\lambda + \sum_{k=i}^{j} \bar{\mathbf{b}}(\lambda_k)\Delta\lambda \in [A - \Delta A, A + \Delta A]$$

then save $(i \equiv \lambda_1\,,\, j \equiv \lambda_2)$

end for

end for

(2)

TYPE 2

for $i = 1\,to\,N = 3001\,do$

for $j = 1\,to\,N = 3001\,do$

if $i < j$

and $\left[\sum_{k=1}^{i} \bar{\mathbf{r}}(\lambda_k)\Delta\lambda + \sum_{k=j}^{N} \bar{\mathbf{r}}(\lambda_k)\Delta\lambda \right] +$

$\left[\sum_{k=1}^{i} \bar{\mathbf{g}}(\lambda_k)\Delta\lambda + \sum_{k=j}^{N} \bar{\mathbf{g}}(\lambda_k)\Delta\lambda \right] +$

$\left[\sum_{k=1}^{i} \bar{\mathbf{b}}(\lambda_k)\Delta\lambda + \sum_{k=j}^{N} \bar{\mathbf{b}}(\lambda_k)\Delta\lambda \right] \in [A - \Delta A, A + \Delta A]$

then save $(i \equiv \lambda_1\,,\, j \equiv \lambda_2)$

(3)

end for

end for

With each pair of limiting wavelengths, $\lambda_{1(=i)}$ and $\lambda_{2(=j)}$, for each input device and the illuminant E, it is very easy to generate the optimal color stimuli $c_{optimal-RGB}(\lambda)$ as $\rho_{optimal-RGB}(\lambda)$ with N spectral samples. Obviously, from here it is immediate to compute the tristimulus values XYZ from the CIE color-matching functions, CIELAB data, etc. So, we can immediately compare whether the optimal colors obtained for each input device are coincident with those belonging to the colorimetric standard observer encoding and plotting all RGB color solids in any CIE color space.

Results and discussion

Figure 3 shows the color solid of the analyzed input devices in their own tristimulus color space and chromaticity diagram. To avoid aliasing, only a sample of R+G+B values are plotted: 10, 40, 70, 100, 130, 160, 190, 220, 250 and 280 values. As it can be seen, the shapes and volumes of the plotted color solids are different. We can be tempted to assume that the greater the volume of the color solid, the greater the number of distinguishable colors, and conclude, therefore, the color gamut of the ideal peaked sensor set is the best of this comparative. Besides, the color gamut associated to the MacAdam sensor set, which is the same than that of the colorimetric standard observer, would be the worst of this comparative. Other interesting observation of this graphical comparison is the peculiar shape of the optimal color loci in the chromaticity diagram of the MacAdam sensor set (equivalent to the human visual system), particularly in the way of encoding the blue-green color stimuli. However, the correct way to test whether the compared color gamuts are so different is to encode and plot them in the same color space, for instance, in any CIE (human) color space.
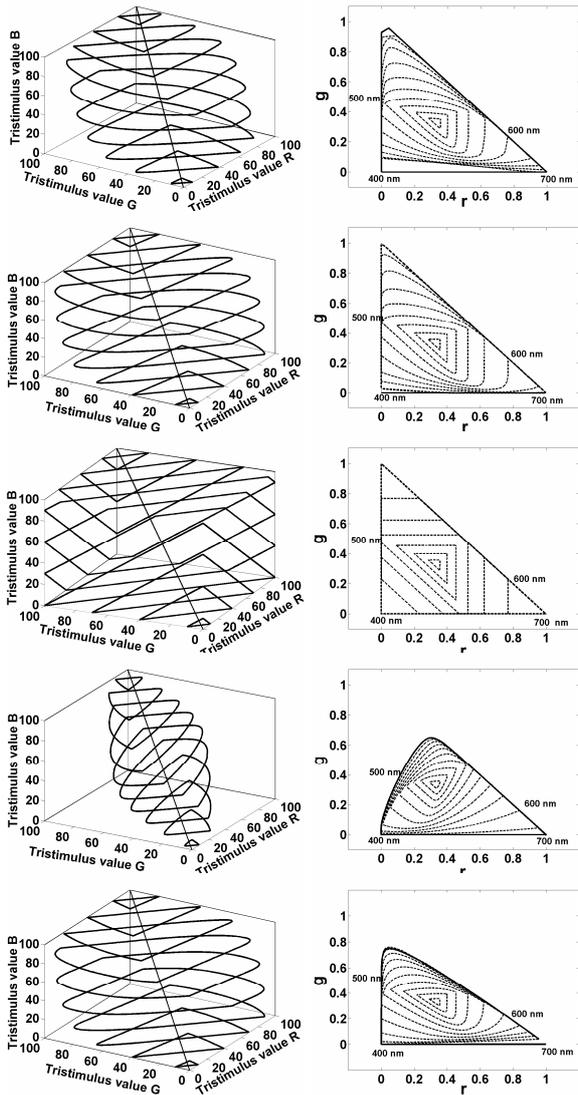
Figure 3: *A point of view (left side) of the Rösch-MacAdam color solid associated to the analyzed input devices in its RGB tristimulus color space. Projections (right side) of the color solid into the own chromaticity diagram (solid line: spectral locus). From top to bottom: CMOS digital camera, CCD digital camera, ideal peaked sensor set, MacAdam sensor set, and goldfish.*

To compare these RGB optimal colors relative to the human MacAdam limits[7], we compute their XYZ and CIELAB data from the transition wavelengths λ_1 and λ_2 considering the equienergetic illuminant. Then, all these data are grouped and ordered by increasing lightness L*. Figure 4 shows the comparison of the loci of RGB optimal colors of one input device relative to the XYZ optimal colors in the chromaticity diagram CIE-(a*,b*). As it can be seen, selecting the same lightness L* planes, both optimal loci are equal except in their color sampling. Therefore, this proves that any linear input device, fulfilling or not the Luther condition, has got the same color gamut as the CIE standard observer. So, this proof or corollary is derived from the fact that any optimal color has not any possible physically metamers except itself.

Therefore, from the question whether the linear input devices, human or non-human, have not gamut limitations, we can say that the key factor is not the shape or volume of the color solid encoded in one or other color space, but how many

distinguishable colors can be inside the color solid[28,29]. A priori, taking into account the results of Figure 3, we could say that the number of distinguishable colors of the MacAdam sensor set is smaller than the corresponding ones of the rest of input devices. However, this preliminary deduction is wrong taking into account Figure 4, as long as we assume the same color metrics or chromatic discrimination model for all input devices. For cameras and scanners it is logical to assign the same color metrics as the human visual system. But, in contrast, there is some evidence in scientific literature[30,31] indicating that the chromatic discrimination properties of animal visual systems could be different to those of the human visual system. Therefore, this work may open new ways to compare the color gamuts, actually the number of distinguishable colors, between the human visual system and non-human input devices, specially those associated to animal visual systems.
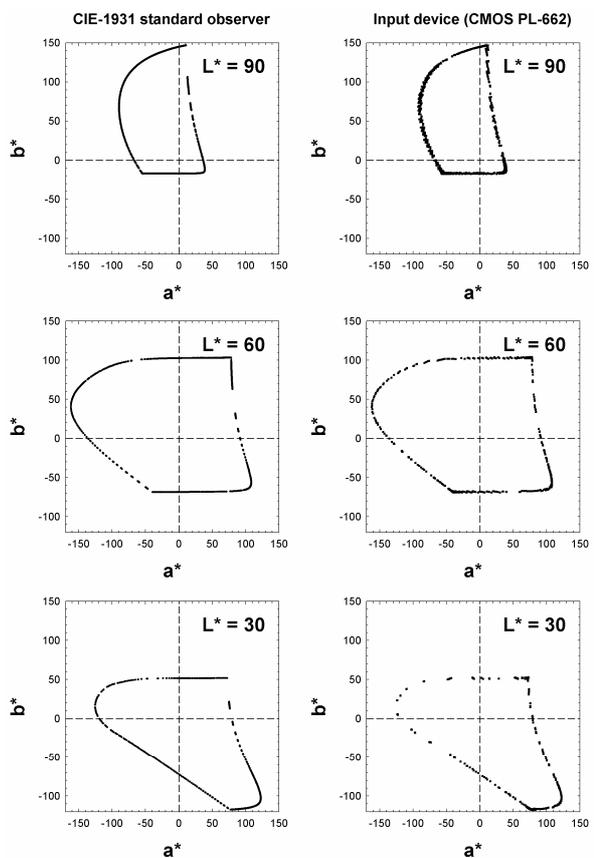
Figure 4: *Comparison of the loci of the optimal colors under the illuminant E in the CIE-(a*,b*) chromaticity diagram in several lightness planes of the CIE standard observer (left side) and the CMOS digital camera (right side).*

Conclusions

We have found that the color gamut of a linear input device, fulfilling or not the Luther condition, is the same as that of the human visual system. This is a corollary of the definition of optimal colors, because they have not got any spectrally positive metamers except themselves, independently of the capture color space. This implies that the color solid of positive spectral reflectances changes in shape and volume according to the capture color space. The non-fulfillment of the Luther

condition only causes the colors inside the color solid to be differently distributed. However, the number of distinguishable colors can be different depending of the chromatic discrimination capabilities in each device. All this, nevertheless, can change drastically if the input device is not linear.

This method for obtaining the color gamut of an input device differs from calculating the RGB encoding of the human optimal colors and then transforming these RGB data into XYZ data using a color profile[14]. On other hand, this alternative method is also slightly different from an accurate determination of the gamut of an input device by sampling the space of all possible surface reflectance functions taking into account the device metamerism[13], but without taking into account the optimal colors, which are really the borders of the object color solid. Therefore, we think that this new approach solves this problem for linear input devices because we have shown that the color gamut is equal for all of types of linear input devices. Obviously, this approach can also be used to compare different forms of trichromatic color vision, by using the spectral sensitivities of non-human or animal eyes as the color-matching functions of the natural "input device". However, all this could be complicated if the natural or artificial input device is not linear (limited dynamic response range, etc).

Acknowledgements

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