Fundamental Considerations related to Chromatic Adaptation

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Abstract

More than one hundred years ago, the von Kries model was developed for a description of the chromatic adaptation phenomena of the human visual system. Various chromatic adaptation models based on the von Kries model have been developed. However, the condition in the wavelength range for which the model works has never been established. Theorems derived for subtractive color mixture problems are applied here to this important issue.

Introduction

Chromatic adaptation is among the most important phenomena of the human visual system. In 1902, von Kries postulated a model for the chromatic adaptation phenomena of the human visual system. However, there remain some important issues related to the von Kries model which have not been clarified up until now. For example, an important issue is what is the conditions in the wavelength range for good von Kries model fitting? The importance of the issue was also stressed[1] by Dr. M. H. Brill who has published historically important articles related to chromatic adaptation.

In this paper, our theorems derived for subtractive color mixture problems will be applied to important issues related to the von Kries model.[2]

In the next chapter, theorems are derived based on our theorems related to subtractive color mixture. Succeeding, the theorems are applied to fundamental considerations related to the von Kries model. Numerical illustrations are also included in relation to the fundamental considerations. Finally, conclusions are provided.

Theoretical model

Notation

$$\langle s(\lambda)\rangle_\tau = \int I(\lambda)s(\lambda)\tau(\lambda)d\lambda = \int I^*(\lambda)\tau(\lambda)d\lambda, \qquad (1.a)$$

$$\langle \rho(\lambda)\rangle_\tau = \int I(\lambda)\rho(\lambda)\tau(\lambda)d\lambda, \qquad (1.b)$$

$$\langle s(\lambda)\rho(\lambda)\rangle_\tau = \int I^*(\lambda)\rho(\lambda)\tau(\tau)d\lambda,$$
$$= \int I(\lambda)s(\lambda)\rho(\lambda)\tau(\lambda)d\lambda, \qquad (1.c)$$

$I(\lambda)$: Illuminant spectral distribution 1,
$I^*(\lambda)$ $(= I(\lambda)s(\lambda))$: Illuminant spectral distribution 2,
$s(\lambda)$ $(0 \le s(\lambda))$: Illuminant operator,
$\rho(\lambda)$ $(0 \le \rho(\lambda) \le 1)$: spectral transmittance,

where

$$\tau(\lambda) := \bar{x}(\lambda) \ or \ \bar{y}(\lambda) \ or \ \bar{z}(\lambda).$$

The notational Eq.(1) indicates that for the original calculation scheme of tristimulus values $\int I^*(\lambda)\rho(\lambda)\tau(\lambda)d\lambda$ $(= \int I(\lambda)s(\lambda)\rho(\lambda)\tau(\lambda)d\lambda)$, $s(\lambda)$ and $\rho(\lambda)$ corresponds to $\rho_1(\lambda)$ and $\rho_2(\lambda)$ in the discussions related to subtractive color mixture (References 3 through 8)), respectively.

The notation $\rho(\lambda)$ is employed for all color types, and the following discussions are consistent not depending on color types.

Tristimulus values

J denotes tristimulus values of X or Y or Z corresponding to $\bar{x}(\lambda)$ or $\bar{y}(\lambda)$ or $\bar{z}(\lambda)$, respectively. Define tristimulus values as follows:

$$J_{I^*} = \langle s(\lambda)\rangle_\tau, \qquad (2.a)$$

$$J_{obj} = \langle \rho(\lambda)\rangle_\tau. \qquad (2.b)$$

Eq.(2.a) corresponds to the tristimulus values of the illuminant $I^*(\lambda)$ derived from $I(\lambda)$ operated by $s(\lambda)$. Eq.(2.b) corresponds to the tristimulus values of an object of $\rho(\lambda)$ under illuminant of $I(\lambda)$. J_I denotes tristimulus values of the illuminant $I(\lambda)$. Equations (2.a) and (2.b) are the defining constraint of the whole problem, and J_{I^*} and J_{obj} are inputs that define the constraints.

Normalization

The following normalizations are performed so Y stimulus value corresponds to 100.0.

$$J_{I^*,norm} = 100.0\langle s(\lambda)\rangle_\tau / \langle s(\lambda) = 1\rangle_{\bar{y}}, \qquad (3.a)$$

$$J_{obj,norm} = 100.0\langle \rho(\lambda)\rangle_\tau / \langle \rho(\lambda) = 1\rangle_{\bar{y}}. \qquad (3.b)$$

For simplicity of the conventional definition of J_{norm} devolves to $J = \langle \rho(\lambda) \rangle_\tau$.

The following theorems are provided.

[Theorem 1]

Posit a random ensemble of pairs $\{s(\lambda), \rho(\lambda)\}$ such that $\langle s(\lambda) \rangle_\tau = J_{I^*}$ and $\langle \rho(\lambda) \rangle_\tau = J_{obj} = J_I / 2$. Then the centroid of $\langle s(\lambda)\rho(\lambda) \rangle_\tau$ is as follows:

$$E\left[\langle s(\lambda)\rho(\lambda) \rangle_\tau\right] = J_{I^*} J_{obj} / J_I \, , \tag{4}$$

where

E : expectation which calculates the centroid.

Proof

Proven in the same way with References 7)8).

Theorem 2 describes the boundary conditions for the centroid formula.

[Theorem 2]

For $J_{obj} = 0$, the centroid of $\langle s(\lambda)\rho(\lambda) \rangle_\tau$ is 0.0, and for $J_{obj} = J_I$, the centroid of $\langle s(\lambda)\rho(\lambda) \rangle_\tau$ is J_{I^*}, and for both cases, the centroid is described in the form of $J_{I^*} J_{obj} / J_I$.

Proof

Proven in the same way with References 7)8).

Combining Theorem 1 ($J_{obj} = J_I / 2$ (middle)) with the boundary conditions of $J_{obj} = 0$ (minimum) and $J_{obj} = J_I$ (maximum) in Theorem 2, an approximation formula of the centroid has been constructed.

Analysis of Von Kries model using the theorems

Theorems 1 and 2 are applied to analysis of the von Kries model. The following equation in Theorems 1 and 2 is transformed as follows:

$$E\left[\langle s(\lambda)\rho(\lambda) \rangle_\tau\right] = E\left[\sum I(\lambda) s(\lambda)\rho(\lambda)\tau(\lambda)\right] \cong J_{I^*} J_{obj} / J_I \, . \tag{5}$$

Eq.(5) is transformed as follows:

$$E\left[\sum I^*(\lambda)\rho(\lambda)\tau(\lambda)\right] \cong J_{obj} J_{I^*} / J_I \, . \tag{6}$$

The right side of X_I, X_{I^*} in Eq.(6) are moved to the left side of Eq.(6), and Eq.(7) is derived.

$$J_I \cdot E\left[\sum I^*(\lambda)\rho(\lambda)\tau(\lambda)\right] / J_{I^*} \cong J_{obj} \, . \tag{7}$$

In Eq.(7), $E\left[\sum I^*(\lambda)\rho(\lambda)\tau(\lambda)\right]$ calculates the centroid of $\sum I^*(\lambda)\rho(\lambda)\tau(\lambda)$ which is represented by using the notation of $\sum I_c^*(\lambda)\rho_c(\lambda)\tau(\lambda)$ in the following equation.

$$J_I \cdot \left\{\sum I_c^*(\lambda)\rho_c(\lambda)\tau(\lambda)\right\} / J_{I^*} \cong J_{obj} \, . \tag{8}$$

Eq.(8) is just the ratio model using the ratio between two illuminant coordinates which is the most primitive model in the von Kries type models. Eq.(8) derives the solution of the chromatic adaptation for the illuminant spectrum $I_c^*(\lambda)$ and the object spectral reflectance $\rho_c(\lambda)$ corresponding to the centroid coordinate.

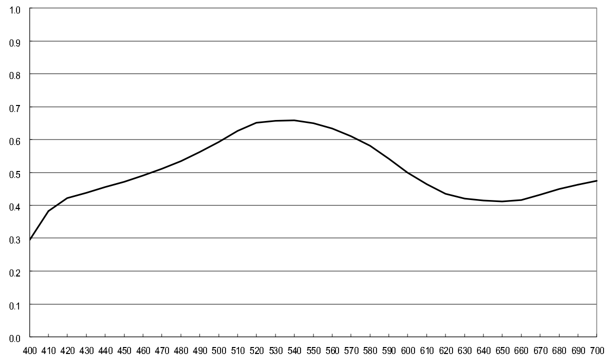
Numerical illustrations and considerations

Figures 1(a)(b)(c) and (d) are examples of $\rho_c(\lambda)$ which satisfy Eq.(8) with less than 7% errors. In this case, the equations predict the choromatic adaptation from under *A* illuminant to *D*65 illuminant. Figure 1 is included as concrete examples providing the theoretical backgrounds. The examples in Figure 1 were derived using a spline curve model[5] for spectral transmittances and the simulated annealing[3] for the optimization minimizing the error of Eq.(8).

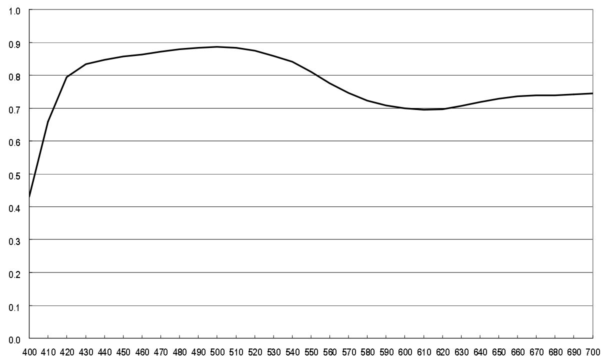
In the existing psychophysical experiments, there has been no theoretical background for the selection of spectral reflectances in the experiments. This is a problem in the development history of chromatic adaptation models. Generally, psychophysical experiments for construction of a model are performed with restricted number of spectral reflectances and typical illuminants which will not corresponds to the centroid. Deviations from the centroids will cause various variations from the simplest model.

Considering the model structure of the von Kries model, also psychophysical experiments will be improved from theoretical point of view.

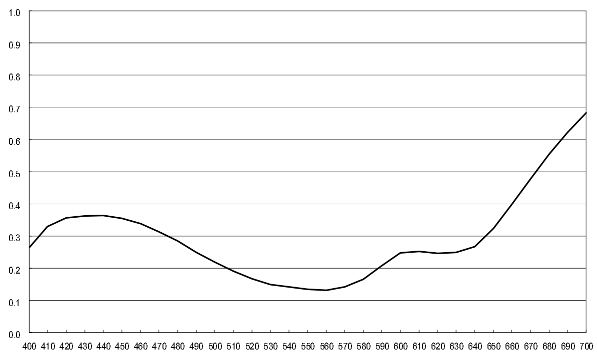
After the discussions above, differences that remain between the human visual system and chromatic adaptation models based on the von Kries model should be investigated.



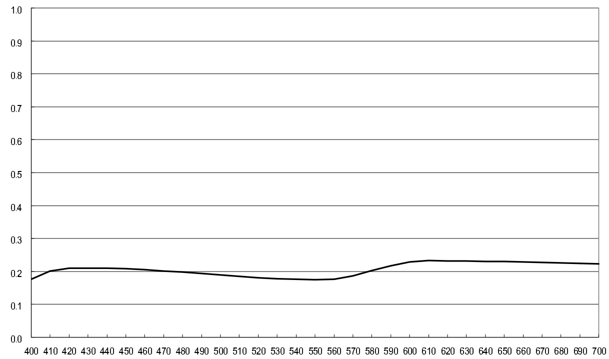
(a)



(b)



(c)



(d)

Figure 1

Conclusions

Chromatic adaptation is among the most important phenomena of the human visual system. However, there remain some important issues related to the von Kries model which have not been clarified up until now. An important issue is what is the conditions in the wavelength range for good von Kries model fitting?

In this paper, our theorems derived for subtractive color mixture problems have been applied to the important issue related to the von Kries model. As the result of this paper, it has been shown that Eq.(8) which is the simplest model in the von Kries type models derives the solution of the chromatic adaptation for the illuminant spectrum and the object spectral reflectance corresponding to the centroid coordinate.

Considering the model structure of the von Kries model, also psychophysical experiments will be improved from theoretical point of view. Hereafter, the fundamental considerations related to the von Kries model will gather efficient data which contribute to reveal the essence of chromatic adaptation.

References

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Biography

Nobuhito Matsushiro received his Ph.D degree (Information engineering) from University of Electro. Communications, Tokyo, Japan, in 1996, and Ph.D degree (Color science) from Chiba University, Chiba, Japan, in 2006. He works for Oki Electric. Co. Ltd., Printing Company.