Euclidean Color-Difference Formula in Chroma Compressed OSA-UCS Space

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Abstract

The very high complexity of the CIEDE2000 formula [1,2] has shown the inadequateness of the CIELAB system to represent a small-medium color-difference formula. This induced us to carry out an analysis on the OSA-UCS system [3,4,5], where the empirical small-medium color differences are represented by a very high regular ellipsoidal equation [6,7]. Particularly, with regard to the whole set of BFD ellipses [8] at constant luminance factor, the ellipses have the long axes lying on lines radiating from the achromatic point and both axes have lengths, that are hue independent and linearly chroma dependent. This regularity induces us to perform a logarithmic compression on chroma producing a new space, where the BFD ellipses appear very close to circles with equal radius and the color-difference formula at constant lightness is represented by the equation of a circle. The goodness of this formula, measured by the performance factor PF/3 [9] over the BFD set, is very close, almost equal, to that obtained with the ellipsoidal formula in OSA-UCS space. This means that the quality of the two formulas is nearly identical and that the small-medium color differences at constant lightness are well represented in the chroma log-compressed OSA-UCS space by the equation of a circle.

Introduction

The definition of the best color-difference formula for small-medium color differences is a very debated problem in color science and is open still today [1]. CIEDE2000 formula [2] has a very high complexity, which shows the inadequateness of the CIELAB system to represent a small-medium color-difference formula. The present analysis is made in the OSA-UCS space [3, 4], spanned by the lightness L_{OSA} and (J, G), corresponding to the empirical j and g [5]. The empirical small-medium color differences are represented in this space (Fig. 1) by a very high regular ellipsoidal equation [6, 7]

$$(\Delta E_{GP})^2 = 10^2\left[\left(\frac{\Delta L_{OSA}}{S_L}\right)^2 + \left(\frac{\Delta C_{OSA}}{S_C}\right)^2 + \left(\frac{\Delta H_{OSA}}{S_H}\right)^2\right] \quad (1)$$

where C_{OSA} is the chroma

$$C_{OSA} = \sqrt{G^2 + J^2}\,, \quad (2)$$

ΔH_{OSA}, for small color differences, is defined as

$$(\Delta H_{OSA})^2 = (\Delta E_0)^2 - (\Delta L_{OSA})^2 - (\Delta C_{OSA})^2 \quad (3)$$

with

$$(\Delta E_0)^2 = (\Delta L_{OSA})^2 + (\Delta J)^2 + (\Delta G)^2\,, \quad (4)$$

in analogy with the CIELAB system, and

$$\begin{aligned} S_L &= 1 \\ S_C &= a_C + b_C\, C_{OSA} \\ S_H &= a_H + b_H C_{OSA} \end{aligned} \quad (5)$$

are weights, where C_{OSA} is the chroma of the color sample considered as standard and a_C, b_C, a_H and b_H are suitable constants.

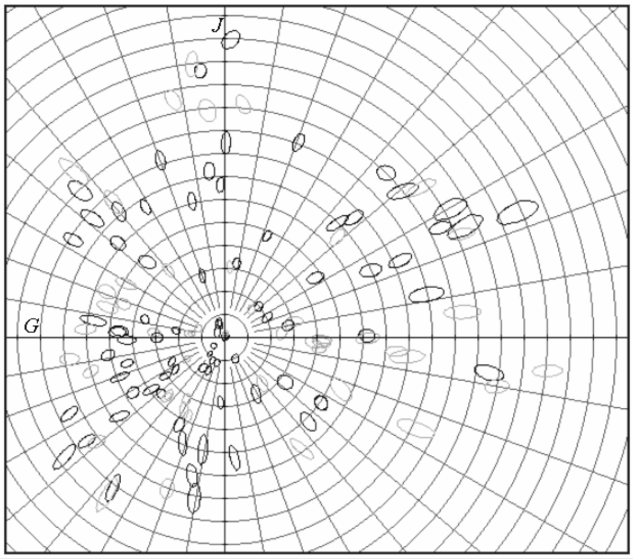

***Figure 1**. (J, G) plane of the OSA-UCS space with the BFD ellipses [8]: BFD-P (black line) and BFD-A (gray line).*

With regard to the whole set of BFD ellipses [8], represented on a constant lightness plane (Fig. 1), evident results are that

1) the ellipses have the long axes lying on lines radiating from the achromatic point,
2) the shape of the ellipses is hue independent,
3) both axes have lengths, that are hue independent and linearly chroma dependent, as represented by Equations (5);
4) consequence of these remarks is that the ellipse centered in the (J, G) coordinate origin is a circle with a radius R approximately equal to 1.

This regularity induces us to perform a logarithmic compression on C_{OSA} producing a new compressed chroma and then a new space, in which the small-medium color-difference formula is well represented by the equation of a sphere. In the next section it is given the derivation of the new chroma log-compressed space and the related color-difference formula. In the third section the compression-parameter optimization is carried on and discussed.

Chroma Log-Compressed OSA-UCS Space

The contribution of the chroma difference in formula (1) is

$$\frac{\Delta C_{OSA}}{a_C + b_C C_{OSA}} , \tag{6}$$

then a new chroma C_E (foot index E means Euclidean), in a chroma log-compressed space

$$C_E \equiv \left(\frac{1}{b_C}\right)\ln\left[1+\frac{b_C}{a_C}C_{OSA}\right] \tag{7}$$

can be defined such that

$$\Delta C_E = \frac{\Delta C_{OSA}}{a_C + b_C C_{OSA}} . \tag{8}$$

Because the chroma log-compression is supposed leaving the hue unmodified, the new log-compressed coordinates are

$$\begin{aligned} G_E &= C_E \cos(h) \\ J_E &= C_E \sin(h) \end{aligned} , \tag{9}$$

where h is the hue angle

$$h = \arctan(-J/G) . \tag{10}$$

This transformation maps the ellipses of a constant luminance factor plane in the OSA-UCS space, with the longer semi-axis situated on the lines radiating from the origin of the coordinate (J, G) and equal to (a_C+b_C C_{OSA}), into circular lines, that are close to circles with radius R in the corresponding chroma log-compressed space. This transformation is hue-angle invariant and depends only on the length of the longer semi-axis (a_C + b_C C_{OSA}). Therefore the shorter semi-axis, that empirically is $\Delta H_{OSA} \approx (a_H + b_H C_{OSA})$, must be dependent on the longer one. This last property is empirically verified, as follows. A circle in the log-compressed space centered at a distance C_E from the origin and with a radius $R = \Delta C_E$, is transformed by inverse equations (9), (10) and (7) into an ellipse-like line, centered in C_{OSA} and with equal hue angle, with the shorter semi-axis ΔH_{OSA}

$$\Delta H_{OSA} = \Delta C_E \frac{C_{OSA}}{C_E} = R\frac{b_C C_{OSA}}{\ln\left(1+\frac{b_C}{a_C}C_{OSA}\right)} . \tag{11}$$

This equation is different from the linear equation $\Delta H_{OSA} \approx (a_H + b_H C_{OSA})$, but both are compatible with the empirical data, as shown in Fig. 2, and the constants a_H, b_H and R have no role in the transformation. This result is very important because states the empirical correctness of the chroma log-compression.

The color-difference formula in the chroma-log-compressed OSA-UCS space is Euclidean

$$\Delta E_E = \sqrt{(\Delta L_{OSA})^2 + (\Delta G_E)^2 + (\Delta J_E)^2} . \tag{12}$$

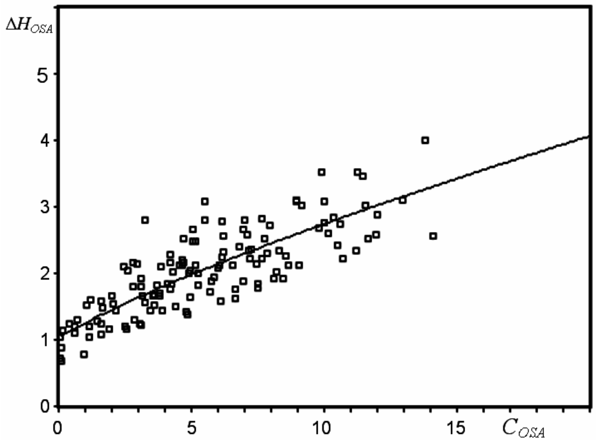

***Figure 2**. Empirical short (□) semi-axes of the whole BFD [8] set of ellipses as functions of the chroma C_{OSA} of the center of the ellipse and the line of equation (11). The values of the related parameters a_C, b_C and R are given in Table 3.*

BFD Ellipses in Chroma Log-Compressed OSA-UCS Space

The parameters a_H, b_H and R are not required for the chroma log-compression, anyway are obtainable by fittings of the empirical evaluation of the short axes of the ellipses (Table 1), or by the *PF*/3 index and RMS value [7] optimization with regard to formula (1) in OSA-UCS space. The optimization of the alone *PF*/3 index for evaluating the parameters is not sufficient, because this index is independent of scale factors, therefore the optimization is obtained by combining this index with the usual Root Mean Square (RMS).

Only the parameters a_C and b_C are needed to perform the chroma log-compression and are obtainable

i. by linear fittings of the empirical evaluation of the long semi-axes of the ellipses (Table 1) [7];
ii. by *PF*/3 index and RMS value optimization on empirical data in OSA-UCS space (Table 2 and 3) [7];
iii. by *PF*/3 index and RMS value optimization on empirical data in chroma log-compressed OSA-UCS space, by using transformation (7), (9) and (10) (Table 4).

The empirical data used in this work for evaluating the transformation parameters and the goodness of the color-difference formulae are constituted by three set ellipses: the BFD-P data set, regarding the Perceptibility, the BFD-A data set, regarding the Acceptability, and the whole BFD dataset, obtained by putting together the BFD-P and BFD-A data sets.

The ellipses of these three datasets, previously considered in OSA-UCS space [6, 7], are represented in the chroma log-compressed OSA-UCS space, where appear very close to circles with equal radius (Fig. 3).

The goodness of the formula (12) is evaluated on parameters obtained by the different techniques i, ii and iii, by the performance factor *PF*/3 over the three BFD data sets and results very close, almost equal, to that obtained by the ΔE_{GP} formula in OSA-UCS space (Tables 1, 2, 3 and 4). This means that the quality of the two formulas is nearly identical and that the small-medium color differences at constant luminance factor are well represented in the chroma log-compressed OSA-UCS space by the equation of a circle.

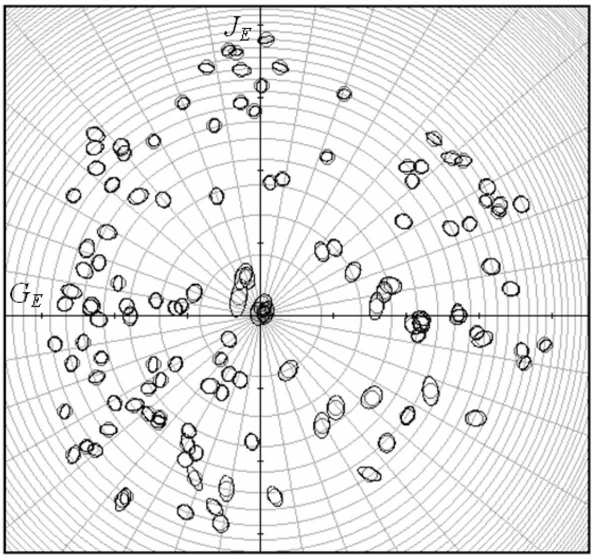

***Figure 3**. (J_E, G_E) plane of the chroma log-compressed space, obtained from the OSA-UCS space, with BFD ellipses (black line), corresponding equal radius circles (dark-grey lines), and net of the cylindrical coordinates of the OSA-UCS space at constant lightness.*

Table 1: Parameters evaluated by fittings on the three empirical sets of BFD ellipses [8], and performance factor *PF*/3 of the color-difference formulas (1) and (12) [7].

	BFD	BFD-P	BFD-A
a_C	1.7513	1.6832	1.4758
b_C	0.0431	0.0428	0.0466
R	0.7483	0.7637	0.8205
PF/3 for ΔE_{GP}	24.5	24.6	22.7
PF/3 for ΔE_E	27.1	27.0	24.1

Table 2: Parameters evaluated by optimization of the performance factor *PF*/3 and of the RMS value in the color-difference formula (1) on the three empirical sets of BFD ellipses [8], and performance factor *PF*/3 of the color-difference formulas (1) and (12) [7].

	BFD	BFD-P	BFD-A
a_C	1.1205	1.1533	0.9366
b_C	0.0513	0.0467	0.0630
R	0.9173	0.9354	0.9153
PF/3 for ΔE_{GP}	21.4	22.7	21.7
PF/3 for ΔE_E	23.1	23.8	21.7

Table 3: Parameters evaluated by optimization of the performance factor *PF*/3 and of the RMS value in the color-difference formula (1) on the COM datasets [2], and performance factor *PF*/3 of the color-difference formulas (1) and (12) [7].

	BFD	BFD-P	BFD-A
a_C	1.2350	1.2350	1.2350
b_C	0.0580	0.0580	0.0580
R	0.8228	0.8191	0.8282
PF/3 for ΔE_{GP}	22.5	22.7	21.9
PF/3 for ΔE_E	23.4	24.0	22.1

Table 4: Parameters evaluated by optimization of the performance factor *PF*/3 multiplied by the RMS value in the color-difference formula (12) on the three empirical sets of BFD ellipses [8], and performance factor *PF*/3 of the color-difference formula (12).

	BFD	BFD-P	BFD-A
a_C	0.9427	0.9151	0.9904
b_C	0.0576	0.0542	0.0624
R	0.9511	0.9917	0.8955
PF/3 for ΔE_E	23.1	23.7	21.6

Conclusions

The Euclidean color-difference formula ΔE_E (12) defined in the chroma log-compressed OSA-UCS space, evaluated on the three sets of BFD ellipses (Table 4) and compared with the previous ΔE_{GP} (1) (Tables 2 and 3), shows a *PF*/3 index equal or greater of only one unit in a percentage scale.

The Euclidean simplicity of the color-difference formula in the chroma log-compressed OSA-UCS space combined with the goodness of the result induces us to believe that any trial to obtain a better agreement between color-difference formula and empirical data available today is an over fitting and almost meaningless work.

Comparisons of the formula ΔE_E (12) with the whole COM dataset [2], using the CIEDE2000 formula [2] and DIN 99d [10] color-difference formulas, are in progress.

Acknowlegments

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