# Spectral Sensitivity Estimation of Digital Cameras

***Carsten Büttner*[1]*, Klaus Bobey*[2]*, and Bernd Schlichting*[1]**
[1] ***Kappa opto-electronics GmbH, Gleichen, Germany***
[2] ***University of Applied Sciences and Arts (HAWK), Göttingen, Germany***

## Abstract

*The knowledge of the spectral sensitivity of a colour camera is an essential requirement to calibrate cameras for a variety of illuminants and to parameterise colour constancy algorithms. The presented method estimates the spectral sensitivity by measuring an optimal set of known spectral stimuli. These are spectrally broadband colour samples generated by illumination of Roscolux transmissive filters of which there is a large number available. However, as a smaller set reduces measurement time in a practical setup, we show an approach to select a reduced set of optimal filters by evaluating Lagrange multipliers. We demonstrate practical results using this optimal set in an experimental setup to determine the spectral system sensitivity of a digital colour camera. The advantage of this new setup compared to existing approaches using test charts is its optical geometry using transmissive filters that allow precise measurements. A result of superior quality is achieved by additional optimisations of the measurement conditions. The quality of the measurements is judged by the prediction error using the estimated sensitivity curves for an independent set of colour samples.*

## Introduction

In order to calculate colour correction transformations to calibrate colour cameras for a variety of illuminants or to test and parameterise colour constancy algorithms, it is useful to take a model-based approach. This allows the usage of spectral power distributions of standard as well as measured illuminants. It also allows the usage of spectral reflectance curves of standard test charts (e. g. Macbeth ColorChecker) as well as measured application-specific objects (e. g. skin or tissue samples). This calibration procedure requires the knowledge of the spectral sensitivity of the camera. This information is often not available or is given only in a limited wavelength range by the sensor manufacturer.

Mainly, there exist two methods for the spectral camera characterisation. The classical and direct way is to use monochromatic light and take the camera response as a measure of the spectral sensitivity of each channel at this particular wavelength. The disadvantage of this method is that it requires expensive equipment and high effort in the optical adjustment. Furthermore, monochromatic light is not the typical stimulus for the camera in most applications. Taking this into account, we follow the other way and use spectrally broadband colour samples to estimate the spectral sensitivity of each channel by its response to the set of samples. Our aim is to develop a precise and quick measuring setup and procedure which are easy to use.

The part of the camera which shall be characterised consists of a CCD image sensor and an analogue front-end including the analogue-to-digital converter at the end of the analogue signal processing chain. The spectral estimation method is based directly on the digitized sensor raw data. Therefore, the correction of a non-linearity caused by signal processing in the camera (see e. g. [1],[2],[3]) is not necessary in our case.

The following is a brief description of the symbols used in this paper and some fundamental equations. The characterisation requires a model of the camera. In this paper, the spectral sensitivity of the k-th channel of a colour camera is called $s_k$ (RGB sensors have $K = 3$ channels). The camera response $r_k$ of each channel can be expressed by the linear model

$$r_k = t_{integ} \cdot \int s_k(\lambda)S(\lambda)\beta(\lambda)d(\lambda) \qquad (1)$$

as the wavelength integral of the product of the spectral transmittance of a colour filter $\beta(\lambda)$ and the spectral energy distribution of the light source $S(\lambda)$ weighted by $s_k(\lambda)$. The integration time $t_{integ}$ is only used as a unit-less factor. For the present, this model is sufficient for integrating image sensors.

Using discrete wavelength samples for the numerical calculations the integral becomes a sum and the spectral functions become vectors or matrices. In our calculations, we use the wavelength range from 380 nm to 750 nm with 10 nm intervals. This results in $N = 38$ wavelength samples. Equation (1) becomes in algebraic notation

$$r_k = t_{integ} \cdot \mathbf{s}_k^T \cdot \mathbf{S} \cdot \beta \qquad (2)$$

where $\mathbf{s}_k$ is a column vector containing discrete samples of $s_k(\lambda)$, $\mathbf{S}$ is a diagonal matrix containing $S(\lambda)$, and $\beta$ is a column vector containing $\beta(\lambda)$. A row vector $\mathbf{r}_k$ containing the camera channel responses to all $M$ colour samples can be calculated in the same way by replacing $\beta$ with the $M \times N$-matrix $\mathbf{B} = \begin{bmatrix} \beta_1 & \cdots & \beta_M \end{bmatrix}$. The camera response of all $K$ channels to a colour sample is a column vector $\mathbf{r}_j$ calculated by using the complete $N \times K$ camera sensitivity matrix $\mathbf{s}$. We call the product of $\mathbf{S}$ and $\mathbf{B}$ the colour sample matrix $\mathbf{C} = \mathbf{S} \cdot \mathbf{B}$. This results in

$$\mathbf{R} = t_{integ} \cdot \mathbf{s}^T \cdot \mathbf{C} \qquad (3)$$

defining the $K \times M$ camera response matrix $\mathbf{R}$. The straightforward approach to determine the spectral sensor sensitivity would be to calculate the product of the camera response matrix and the pseudo-inverse of the colour sample matrix. An alternative approach is to use optimisation algorithms like quadratic programming and to define some constraints. The advantage is its flexibility in easily changing the constraints or adding new ones. For a practical setup using this method, the set of colour filters should be as small as possible to reduce measurement time. To determine such a small set of optimal colour samples, we suggest an approach for the filter selection based on the Lagrange multipliers, which are an additional output of the used quadratic programming algorithm.

## Methodology

There are several methods needed to find the optimal colour samples for the experimental setup, to estimate the camera sensitivity from the measurement data and to evaluate the quality of the estimation. The section starts with the sensitivity estimation method on which everything else is based.

### *Sensitivity estimation*

Several indirect methods for the spectral characterisation have already been described in the literature. Some are based on matrix inversion (e. g. [2],[4],[5]), for example, by using singular value decomposition and deletion of the smallest singular values to reduce the noise influence on the result. Another way is to use linear or quadratic programming with the advantage of easily adding constraints using a-priori knowledge (e. g. [2],[6],[7],[9]). Although there are many suggested constraints, e.g. absolute or relative error, smoothness, uni-modality or positivity, we limited their number to two essential ones, the absolute error and the smoothness, and a minor one, the minimisation of the curve's end points, in order not to limit the solution too much. The absolute error is given by

$$d_j^2 = \left(r_{kj} - \mathbf{s}_k^T \cdot \mathbf{C}_j\right)^2 \ \forall j \tag{4}$$

and the smoothness of the sensitivity functions is defined by the discrete second order derivative at each wavelength sample point corresponding to

$$D_i^2 = \left( \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}^T \cdot \begin{bmatrix} s_{k,i-1} \\ s_{k,i} \\ s_{k,i+1} \end{bmatrix} \right)^2 \ \forall 2 \le i \le N-1 \tag{5}$$

The third constraint minimises the sensitivity curve's end points, in this case the wavelength samples at 380 nm and at 750 nm. Additional factors allow the weighting of these criteria. These factors have to be determined empirically. The quadratic error valuation in the objective function considers all $d_j$ and $D_i$ in a well-balanced way. This seems to be an important advantage compared to the approach using linear programming ([2]). A separate evaluation of mean and maximum error is not necessary in quadratic optimisation.

The usage of basis functions ([7]) or uni-modality ([6]) would restrict the solution to much and thus would lead to too big differences. Because we do not only characterise the image sensor spectrally but the whole image capture system consisting of the colour image sensor, an infrared cut-off filter and an analogue front-end, negative parts in the spectral sensitivity can occur due to, for example, channel cross-talk or analogue filter circuits. Therefore, we do not use the positivity constraint. The author of [6] suggests the minimisation of the relative error of each colour sample given by

$$d_j^2 = \left(\mathbf{s}_k^T \cdot \frac{\mathbf{C}_j}{r_{k,j}} - 1\right)^2 \ \forall j \tag{6}$$

but the described advantages concerning the reduction of the chromaticity error could not be confirmed by us. On the contrary, this approach leads to a higher uncertainty in the estimation result. This is caused by the stronger influence of the worse conditioned colour samples generating lower signal to noise ratios (SNR).

### *Error measures*

To evaluate the quality of the resulting estimated spectral sensitivity curves, it is necessary to use error measures. The aim is to use the characterisation results to predict the camera response to an arbitrary set of colour samples to determine colour space mappings regarding chromatic adaptation to a variety of illuminants, or to test and parameterise colour constancy algorithms. Thus, we propose two error measures to evaluate the quality. The first one is the Euclidean distance of the real and the predicted camera response vectors in relation to the Euclidean length of the illumination response vector. This is formulated by

$$e_{RGB,j} = \frac{\sqrt{\left(\mathbf{r}_j - \mathbf{s}^T \cdot \mathbf{C}_j\right)^T \cdot \left(\mathbf{r}_j - \mathbf{s}^T \cdot \mathbf{C}_j\right)}}{\sqrt{\mathbf{r}_{ref}^T \cdot \mathbf{r}_{ref}}} \tag{7}$$

where index $ref$ refers to the camera response generated by the illumination without a filter (reference). The second error measure is the corresponding geometrical distance in the rg-chromaticity space given by

$$e_{rg,j} = \sqrt{\left(\frac{r_{R,j}}{\sum \mathbf{r}_j} - \frac{\mathbf{s}_R^T \cdot \mathbf{C}_j}{\sum \mathbf{s}^T \cdot \mathbf{C}_j}\right)^2 + \left(\frac{r_{G,j}}{\sum \mathbf{r}_j} - \frac{\mathbf{s}_G^T \cdot \mathbf{C}_j}{\sum \mathbf{s}^T \cdot \mathbf{C}_j}\right)^2} \tag{8}$$

where indices $R$ and $G$ indicate the red and green sensor channel. Both errors are defined in the sensors RGB color space and evaluate the quality of the complete camera sensitivity estimation whereas the characterisation procedure estimates each channel separately. These errors are meaningful here because they apply to the input values for parameterising chromatic adaptation and colour constancy algorithms. This quality evaluation has to be done using a set of colour samples which is completely different from the optimal set of measuring samples.

### *Optimal colour samples*

In the approach described here, a result of superior quality is achieved by optimising the measurement conditions (see experimental results) and by determining an optimal set of colour samples. For a practical setup, others (e. g. [2],[11]) have suggested the use of Roscolux filters made by rosco, of which there is a large number (more than two hundred) available. However, as a smaller set of filters reduces measurement time, we suggest using the Lagrange multipliers, which are an addtional output of the quadratic programming algorithm, to choose the most apt filters for the setup.

The quadratic programming algorithm which is used here is part of the MATLAB Optimization Toolbox. Such an algorithm delivers a set of Lagrange multipliers, one for each constraint, in addition to the solution vector. Like in multi-dimensional function analysis under constraints, the Lagrange multipliers are a measure of influence to the found solution. We suggest using the filters whose corresponding Lagrange multipliers have the highest absolute mean values.

In mathematical optimisation problems, Lagrange multipliers are a method for dealing with constraints. The optimisation algorithm delivers a vector of Lagrange multipliers, one for each constraint. The constraints are given by the $d_j$ and $D_i$. We used a set of the mentioned Roscolux filters leading to 197 absolute error constraints plus $N-2 = 36$ smoothness constraints. The Lagrange multipliers, each corresponding to one of the 197 colour samples, might be used as quality factors for these samples in the estimation procedure. A simulation environment has to be built for this method of filter selection. Therefore, an optimal set consisting of 60 filters is determined in a simulation using a camera noise model allowing for dark and photon noise and photo response non-uniformity (see next section).

## Simulation results

Before doing the experiments, a simulation environment has to be built to test the estimation method by modelling the real camera and to find optimal colour samples for the practical setup.

### Camera model

In the experiments the camera Kappa DXc100 is used with Jenoptik lenses of type Lametar 2.8/25. The camera is build around a CCD image sensor KAI-1020CM made by Kodak. An optical filter of type SCHOTT BG40 with 1 mm thickness is mounted in front of the sensor to cut off the NIR spectral part. The resulting spectral camera model is given by

$$\mathbf{s}_{sim} = \mathbf{f}_{Lam} \cdot \mathbf{f}_{BG40} \cdot \mathbf{s}_{KAI} \tag{9}$$

where $\mathbf{f}_{Lam}$ and $\mathbf{f}_{BG40}$ are diagonal matrices containing the measured transmission of the lenses and the transmission of the SCHOTT filter (extracted from the SCHOTT data base). The spectral sensitivity $\mathbf{s}_{KAI}$ of the colour image sensor is taken from the Kodak data sheet. The systems spectral sensitivity $\mathbf{s}_{sim}$ used in the simulation will be compared with the experimental results. For this first test of the setup, the KAI-1020CM is well-suited because Kodak delivers precise information about its typical spectral sensitivity. This is not the case with many other sensors and most complete cameras.

Ideal camera responses are calculated according to the model using $\mathbf{s}_{sim}$ with equation (3). To adapt the model as good as possible to the real world, the calculation of camera responses is based on a wider spectral range from 380 nm to 1000 nm with 10 nm intervals. The estimation method will be used in the smaller range up to 750 nm as mentioned in the introduction.

In addition to this spectral model, we use a noise model allowing for dark noise, photon noise and photo response non-uniformity (PRNU). The specific standard deviations $\sigma$ of the different noise sources are shown in table 1. In this case, $S$ is the sensor signal value in units of electrons ($e^-$). The noise is added by extending equation (3) to

**Table 1: Noise sources in camera model**

| | |
|---|---|
| Dark noise | $\sigma_{dark} = 40\, e^-$ |
| Photon noise | $\sigma_{phot} = \sqrt{S}$ |
| PRNU | $\sigma_{PRNU} = 0.02 \cdot S$ |

$$\mathbf{R} = t_{integ} \cdot \mathbf{s}^T \cdot \mathbf{C} + \xi \tag{10}$$

with the additive noise term $\xi$. The added noise term considers the mentioned noise sources, a maximum sensor output signal of 80 per cent of the sensors full well (FW) capacity ($S_{FW} \approx 40{,}000\, e^-$) and the formation of mean values over an array of $25 \times 25$ pixels to approximate the real measurement conditions (for further details on noise modelling see e. g. [10]).

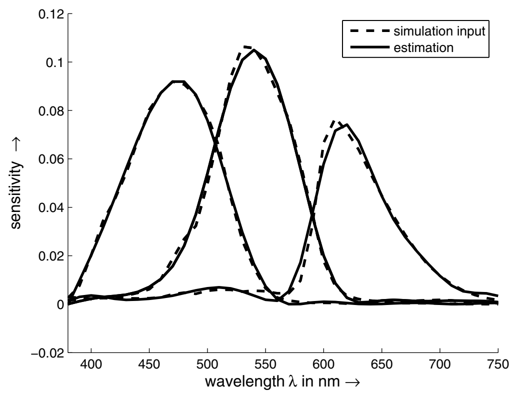


*Figure 1. Spectral system sensitivity: simulation input and estimated*

Figure 1 shows the spectral system sensitivity $\mathbf{s}_{sim}$ (dashed) as the simulation input in comparison to the estimation (solid) which is based on the simulated camera responses according to the camera model described above. The estimated sensitivity curves shown in the figure are a typical result of a simulation pass. This result varies from one pass to another because of the random noise which is generated in each pass. The algorithm itself delivers exactly one result for each distinct set of input data values.

### Optimal colour samples

The simulation environment can be used to test the estimation method like it is described in the previous section and the output of the optimisation algorithm can be evaluated to determine an optimal filter set. In the methodology section, we describe how the Lagrange multipliers may be used for this evaluation. We suggest using the colour samples, or corresponding filters, whose constraints deliver the highest corresponding Lagrange multipliers.

This evaluation depends on the particular noise in each simulation pass. Therefore, the choice is made on a statistical evaluation of a high number of passes $P$ (e. g. $P = 1000$). We suggest choosing the colour samples whose Lagrange multipliers have the highest absolute mean value over all passes. Because we estimate the spectral sensitivity of each camera channel separately we have to combine this information of all three camera channels. This leads to the combined quality factor

$$L_j = \sqrt{\sum_{k=1}^{3} \left( \frac{1}{P} \cdot \sum_{p=1}^{P} L_{k,j,p} \right)^2} \tag{11}$$

for each colour sample $j$. The $L_{k,j,p}$ are the Lagrange multipliers $L$ for each channel $k$, each colour sample $j$ and each simulation pass $p$. This procedure delivers a reproducible set of optimal filters generating the colour sample matrix.

However, it has to be kept in mind that this selection depends on the weighting of the optimisation constraints. In general, the higher the smoothness weighting is the higher are the $L_j$ and the selection changes a little.

## Experimental Results

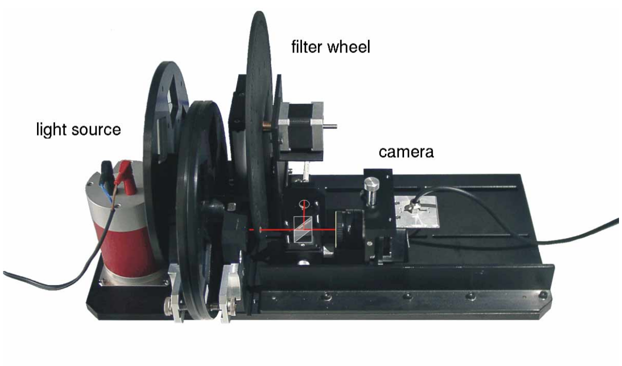


*Figure 2. measurement setup*

The experimental setup consists of a tungsten-halogen light source and some additional optical filters to modify its spectral characteristic so that it is nearly constant over the visible spectrum. The optimised set consists of sixty colour filters which are mounted on an automatically positionable filter wheel. In order to enable the spectral measurement to be performed simultaneously with acquiring the camera response, a beam splitter is positioned in the light path. The spectral measurement is done with a compact calibrated fiber-coupled spectrometer (Ocean-Optics

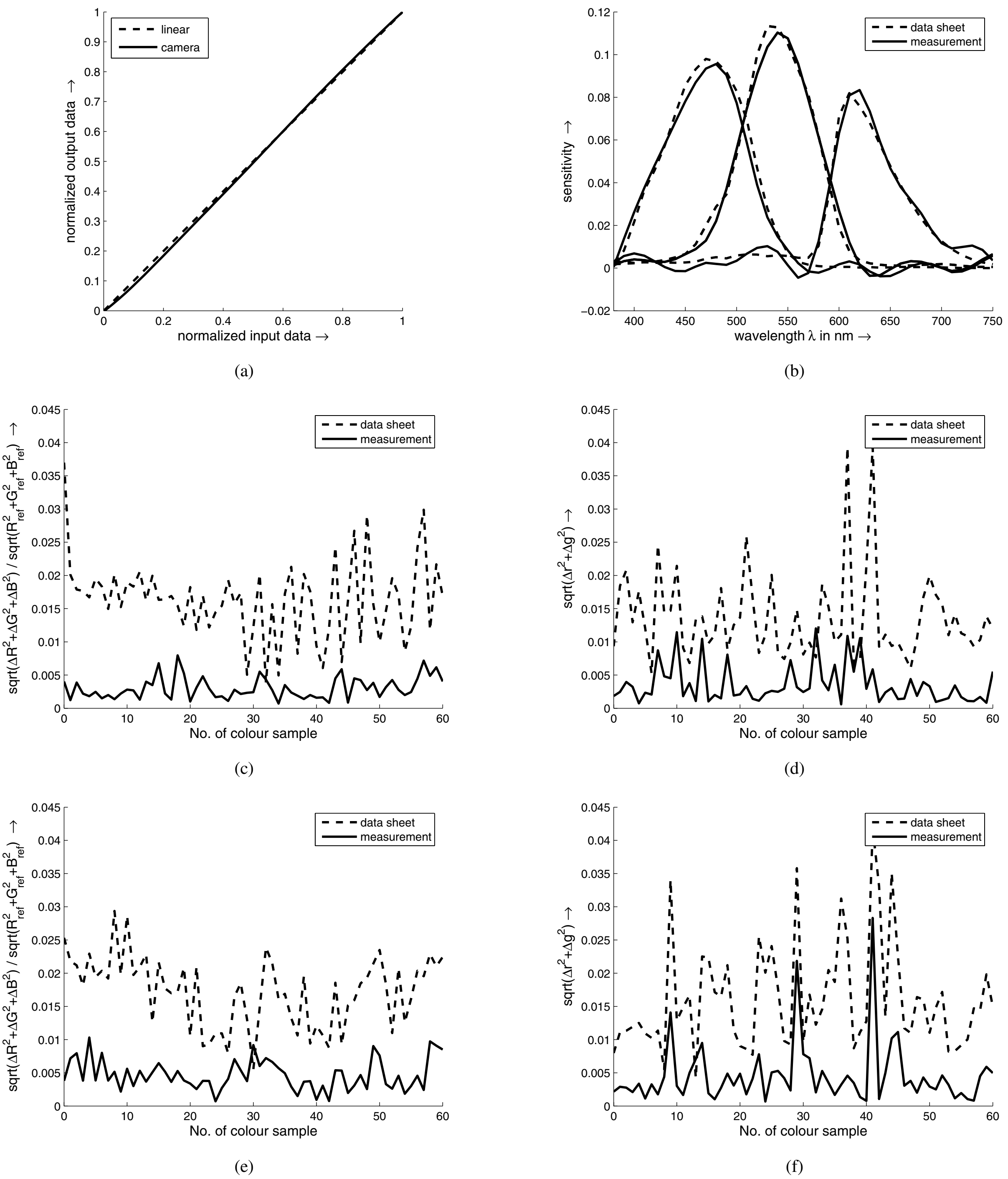


***Figure 3.*** *Experimental results for Kappa DXc100: (a) non-linearity, (b) estimated spectral sensitivity in comparison to data sheets, optimal set: (c) normalised RGB vector difference, (d) distance in rg-chromaticity space, verification set: (e) normalised RGB vector difference, (f) distance in rg-chromaticity space*

USB2000). All these components are spectrally characterised to ensure correct measurements. There are exclusively transmissive optical components used and the setup is organised such that all measurements with the spectrometer as well as the camera are done under a small aperture angle. The camera response is determined from a small area of interest covering $50 \times 50$ pixel in the center part of the image sensor ($25 \times 25$ RGB Bayer macro pixel). This setup geometry has the advantage of avoiding shading effects such that no spatial corrections are needed. This is a main advantage in comparison to all setups using test charts with spatially distributed colour samples (see e. g. [4],[9]) and, therefore, allows superior quality in the sensitivity estimation. The setup is shown in figure 2.

The real measurements show that there is a slight non-linearity in the camera response (see fig. 3 (a)). As this would lead to errors in the sensitivity estimation the model in equation (2) has to be extended by a non-linearity function $\mathscr{F}$ to

$$r_k = \mathscr{F}(\nu_k) = \mathscr{F}\left(t_{integ} \cdot \mathbf{s}_k^T \cdot \mathbf{S} \cdot \beta\right) \quad (12)$$

We determine $\mathscr{F}$ by varying the integration time at constant illumination ([11]) and approximate it by a 7th order polynom. The fitting of the non-linearity has to be very precise and it is, in our case, not possible to use just a power-law form ([1]). Therefore, this quick measurement procedure is done separately. The linear camera responses $v$ are the input to the described spectral sensitivity estimation method.

A further improvement in the measurement conditions is the adjustment of the integration time in such a way that the maximum channel response for each colour sample equals approximately eighty per cent of the sensor's full well capacity. The adjustment of the integration time optimises the SNR of the camera responses. After correcting the slight non-linearity in the camera response, the values are normalised to the integration time at reference illumination and used for the estimation procedure.

We tested the setup and method on two cameras, the Kappa DXc100 equipped with a Kodak KAI-1020CM sensor and the Kappa DX40 with a Sony ICX285AQ sensor. Both are interline transfer progressive scan CCD sensors with an RGB Bayer colour filter array. The estimated spectral system sensitivity of a Kappa DXc100 camera is shown in figure 3 (b) (in comparison to the data sheet values allowing for the used infrared cut-off filter and the Jenoptik lenses). There are essential differences in the sensitivity functions of both cameras, above all in the red channel, but the method works in both cases in the same manner. Therefore, only the results for the DXc100 are shown in this article.

Both error measures mentioned deliver an average error of approximately three per mill for both cameras when tested with the estimated sensitivity and the optimal set of colour samples (see fig. 3 (c) and (d) for the DXc100 results). In comparison, the calculation using the data sheet values (see simulation) delivers an average prediction error of more than one per cent.

A more representative quality check is delivered by an independent set of colour samples which are not used for the estimation procedure. This set is arbitrarily chosen from the rest of the 197 Roscolux filters. The comparison of the real camera responses to the calculated ones using the estimated sensitivity curves shows in both error measures an average error of less than five per mill for the measured Kappa DXc100 and the independent verification set (see fig. 3 (e) and (f)). The average prediction errors using data sheet values are, in this case, also higher than one per cent. Hence, we have proved that our estimation method delivers even better results than the typical data sheet values which qualifies our method. And, as most manufacturers do not publish the sensor's spectral sensitivity in this detail and quality this method may prove a valuable tool to measure it.

## Discussion

This study investigated a new method and setup for the spectral characterisation of colour cameras. The used estimation method was explained and a new approach for the selection of optimal colour samples for a practical setup was described. This reduced set of specifically chosen colour samples leads to shortened measurement times enabling the method for a practical setup. The setup is designed in a transmissive optical geometry using only a small aperture angle. This allows precise measurements without the need for spatial corrections. The measurement results are additionally optimised by integration time adjustment to maximize the SNR.

The experimental results show that the proposed method delivers superior results in the spectral sensitivity estimation. These results are only possible by correcting the slight non-linearity of the camera response using a 7th order polynom. The polynom fitting is based on camera responses acquired in an integration time series at constant illumination. The quality of the estimation results was judged by two error measures defined in the sensor's RGB colour space. Here, these are prefered to $\Delta E$ evaluation because the estimated spectral sensitivity shall be judged and not the transformation to an independent colour space. The error measures compare the real camera responses with the predicted ones using the camera model with the estimated spectral camera sensitivity. A verification set of colour samples which were not part of the estimation procedure was used for the quality judgement. The prediction of camera responses based on the estimated camera sensitivity shows an average error of less than five per mill. This result qualifies the method and setup to measure spectral camera sensitivities for the usage in model-based camera calibration procedures.

## References


[1] V. Cheung, S. Westland, M. Thomson, Accurate Estimation of the Nonlinearity of Input/Output Response for Color Cameras, Color Res. Appl., 29, pp. 406–412 (2004).

[2] F. König, Die Charakterisierung von Farbsensoren, PhD thesis, RWTH Aachen (2001).

[3] P. L. Vora, J. E. Farrell, J. D. Tietz, D. H. Brainard, Digital color cameras - 1 - Response models, Tech. report HPL-97-53 (1997).

[4] J. M. DiCarlo, G. E. Montgomery, S. W. Trovinger, Emissive chart for imager calibration, Proc. CIC: Color Sc. Eng. Sys., Tech., Appl., pp. 295–301 (2004)

[5] G. Sharma, H. J. Trussell, Characterization of Scanner Sensitivity, Proc. CIC: Transf. & Transp. of Color, pp. 103–107 (1993).

[6] K. Barnard, B. Funt, Camera Characterization for Color Research, Color Res. Appl., 27, pp. 153–164 (2002).

[7] G. D. Finlayson, S. Hordley, P. M. Hubel, Recovering Device Sensitivities with Quadratic Programming, 6th CIC: Color Sc., Sys. Appl., pp. 90–95 (1998).

[8] P. L. Vora, J. E. Farrell, J. D. Tietz, D. H. Brainard, Digital color cameras - 2 - Spectral response, Tech. report HPL-97-54 (1997).

[9] D. Paulus, V. Hong, C. Idler, J. Hornegger, L. Csink, Sensitivity Curve Approximation using linear Algebra, Proc. 2nd CGIV, pp. 207–212 (2004).

[10] G. C. Holst, CCD Arrays, Cameras and Displays, 2nd edition, JCD Publishing, 1998.

[11] U. Krüger, Ausgewählte Aspekte der ortsaufgelösten Erfassung stationärer Licht- und Farbverteilungen, PhD thesis, TU Ilmenau (2004).


## Author Biography


*Carsten Büttner received his diploma degree in Measurement Technology in 2003 and his Master of Science in Optical Engineering/Photonics in 2005 from the University of Applied Sciences and Arts (HAWK), Germany. Since January 2006, he has been head of the signal processing group at Kappa opto-electronics GmbH. Currently, he is working on his PhD thesis which is concerned with spectral characterisation methods and calibration of digital colour cameras.*