A Robust Filtering Method using OWA filters: Application to Color Images

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Abstract

This paper addresses the problem of robust filtering in color images. We propose to improve the bilateral filter by using Ordered Weighted Averaging (OWA) filters. Bilateral filter decreases amount of noise preserving edges when filtering. But the results could be strongly affected by the presence of outliers. In this paper, we define the filtering in the framework of fuzzy logic. If filtering is considered as a weighted averaging, then each filter is associated with a fuzzy set. The membership values of the fuzzy set are the weights of average. In this context, the bilateral filter is a conjunction of two fuzzy sets (i.e. aggregation with AND operator): one in the spatial domain and the other one in a photometric domain. Applied to color images, we propose to extend the conjunction to three fuzzy sets: one in the spatial domain, one in the brightness domain and one in the chromatic domain. Taking into account the robustness of rank filters, we propose to define an OWA filter in order to obtain robust adaptive filters in brightness and chromaticity. The application to color images states the filtering ability for removing noise in presence of outliers.

Introduction

The restoration of images from degraded ones is a classical low level problem in image processing. The development of new acquisition devices for multicomponent images increases the risks of system errors and the presence of noise. In this context, the goal of filtering methods is to enhance the quality of these images by suppressing noise that can be composed of outliers. Color images give an example of such complex multicomponent images. They are obtained by three spectral acquisition channels leading to three components: red, green and blue. At this low level of image processing, this paper proposes a new filtering scheme applied to color images. This approach should be adaptive to noise and robust against outliers.

In the field of image processing, many filtering methods are proposed. The classical convolution filters like the mean filter or Gaussian filter reduce the noise by smoothing. But these filters do not preserve the details and edges in images. Moreover these low pass filters are unadapted in presence of outliers. The rank filters [1] are more robust. For instance, the median filter suppresses some outliers when filtering. But it does not preserve small details and shapes in images. In this paper, we propose a filtering approach taking into account the robustness of rank filters when restoring images. Indeed some methods try to preserve details while smoothing but fail in presence of outliers. For instance, the anisotropic diffusion method [2] allows to decrease the amount of noise while preserving the edges in images. Such results are obtained by smoothing along the edges and avoiding to diffuse across these ones. Bilateral filtering [3] is another classical method of anisotropic approach based on the conjunction of spatial and photometric filtering. Unfortunately, these anisotropic methods are strongly affected by outliers. Moreover, these methods require to determine two parameter values in order to separate the noise from the edges. The classical anisotropic diffusion uses both a contrast parameter and a resolution parameter [4]. The bilateral filter [3] depends on both the spatial geometric closeness and the photometric similarity between neighbour pixels. Other techniques like the mean shift [5] have also two parameters controlling the resolution in the spatial and photometric domains. The tuning of these two parameters is not always obvious and remains a drawback. Therefore we propose a new adaptive method for anisotropic filtering which is robust in presence of outliers and does not require too many parameters setup.

The present work addresses an improvement of the bilateral filter by using fuzzy logic operators to gain robustness without any assumption on any noise feature nor outlier presence. Bilateral filtering is based on a weighted averaging of the local neighbourhood samples [6]. The weights are the products of two values, the first one is computed in the spatial domain using Euclidean distance between the center sample and its neighbors. The other one is computed in the photometric domain using Euclidean distance between the photometric values (or vectors) of the center sample and its neighbors. Note that the photometric domain is a RGB color space in this paper. Now let us briefly describe the way we use to improve the bilateral filter.

We define the bilateral filter in the framework of fuzzy logic. Firstly, we consider that the bilateral filter is the conjunction of two fuzzy filters: a spatial filter and a photometric filter. Then we propose to use classical conjunction operators with t-norms [7] to define the collaboration between several filters. Secondly, we consider that the filtered values are the results of a fuzzy aggregation procedure [7]. Then we propose to improve the robustness using Ordered Weighted Averaging (OWA) operators [8]. The aggregation with OWA operator is a weighted averaging procedure. But the weights depend on the rank of similarities between data instead of the similarities themselves. Then OWA aggregation derives advantage from the robustness of rank approach. The OWA operators were first described by Yager [8], so we call our robust filters: OWA filters. Finally, we apply this filtering scheme to color images by separating the colors into two photometric features: brightness and chromaticity. Thus we obtain a trilateral filter by conjunction of three filters: a classical Gaussian filter in the spatial domain, an OWA filter in the brightness domain and an OWA filter in the chromatic domain. The application to color images demonstrates the filtering ability for removing noise in presence of outliers. We propose to compare our filter scheme with three classical filtering methods: the Gaussian filter (i.e. only spatial filtering), the median filter (i.e. only photometric filtering), and the bilateral filter (i.e. both spatial and photometric filtering).

In the following we describe the principle of our filtering scheme. Firstly we go over the bilateral filter by using the conjunction operator. Secondly we describe spatial filters in the framework of fuzzy logic. Then color filtering using OWA fil-
ters is proposed separating brightness and chromaticity. The next section is devoted to experiments and results. Then we conclude this paper on this new approach for robust adaptive filtering of color images.

**Principle**

**Bilateral filtering with conjunction operators**

This section is devoted to describe bilateral filtering in the framework of fuzzy logic. Let us briefly recall the bilateral filtering method. The filtered value \( I'(x) \) of a pixel \( x \) is considered as a weighted average of values \( I(y) \) where \( y \) are the pixels belonging to a neighbourhood of \( x \). This neighbourhood \( N(x) \) is generally a \( n \times n \) spatial window centered on \( x \) where \( n \) is the width in pixel unit. The filtered values are defined by:

\[
I'(x) = \frac{\sum_{y \in N(x)} w(x,y)I(y)}{\sum_{y \in N(x)} w(x,y)}
\]

where \( w(x,y) \) are the weights applied to \( y \) in \( N(x) \). In the case of multicomponent images, the one-dimensional values of \( I(y) \) and \( I'(x) \) are replaced by vectors. In the bilateral filtering scheme, we have \( w(x,y) = w_s(x,y) \times w_p(x,y) \) where \( w_s(x,y) \) is the spatial weight and \( w_p(x,y) \) is the photometric weight. Tomasi and Manduchi [3] proposed to use:

\[
w_s(x,y) = \exp\left(-\frac{d^2_s(x,y)}{2\sigma^2_s}\right)
\]

where \( d_s(x,y) \) is Euclidean distance in spatial domain between the pixels \( x \) and \( y \), and \( \sigma_s \) is geometric spread factor in the spatial domain. Similarly, using \( d_p(x,y) \) as an Euclidean distance in a photometric domain between the pixels \( x \) and \( y \), and \( \sigma_p \) as the photometric spread factor, they define:

\[
w_p(x,y) = \exp\left(-\frac{d^2_p(x,y)}{2\sigma^2_p}\right)
\]

Note that the bilateral filter uses two parameters (a spatial spread factor \( \sigma_s \) and a photometric spread factor \( \sigma_p \)) with empirically selected values.

In this paper, we note that the coefficients \( w_s(x,y) \) and \( w_p(x,y) \) lie between zero and one. So we consider this coefficients as some membership values of two fuzzy subsets inside the image field. The supports of these fuzzy subsets are \( N(x) \) (i.e. the membership values are null outside \( N(x) \)). The product of two membership values corresponds to a conjunction operator (i.e. the AND operator). Thus the final weight of the bilateral filter is the result of a conjunction between two fuzzy subsets where the conjunction is obtained by the product of the two membership values. Such a conjunction could be obtained using any other \( \tau \)-norms as conjunction operators [8] (the product of two membership values is a particular case of conjunction corresponding to the classical algebraic \( \tau \)-norm). But the most classical conjunction operator is the minimum which replace the product in this paper for computing the weights of filters. Lukasiewicz \( \tau \)-norm is another conjunction operator defined by:

\[
w(x,y) = \max\left(w_s(x,y) + w_p(x,y) - 1, 0\right)
\]

Many other \( \tau \)-norm can be used. This approach by using fuzzy aggregation operator let us transform the bilateral filter as a multi-criteria conjunction operator. Thus we can generalize the bilateral filter by aggregating various weights and by suppressing any limitation to spatial and photometric weights. In this paper, we propose to filter color images by defining three weights: the first one \( (w_s(x,y)) \) uses the spatial coordinates of pixels, the second one \( (w_{\text{brightness}}(x,y)) \) uses the brightness of pixels and the last one \( (w_{\text{chroma}}(x,y)) \) uses the chromaticity. Thus our filter becomes the conjunction of three fuzzy sets. The final weight \( w(x,y) \) is computed by:

\[
w(x,y) = \min\left(w_s(x,y), w_{\text{brightness}}(x,y), w_{\text{chroma}}(x,y)\right)
\]

In the next section, we describe the computation of these three weights.

**Spatial filtering with fuzzy filters**

In the spatial domain, well-known filter is the Gaussian filter. Tomasi and Manduchi [3] used this filter when they defined the bilateral filter. We also use this filter. Such a filter could be considered as a weighted averaging procedure where the weights are given by a Gaussian distribution. In this paper, we consider that the weights are membership values of a fuzzy set in the image field. Then the Gaussian filter is a fuzzy filter based on a specific membership function. Note that we could use any classical convolution filter in the place of the Gaussian filter as soon as a fuzzy set represents this filter in the image field (in the spatial domain).

**Color filtering with OWA filters**

The photometric domain of color images is the RGB space. Tomasi and Manduchi [3] do not take into account any color perception. Then the weights \( w_p(x,y) \) are directly computed using an Euclidean distance in the RGB space. In this paper, we improve their approach by separating the brightness and the chromaticity which are the two main features of colors [9]. According to our fuzzy approach of filtering, we have to define two fuzzy sets: one for brightness filtering and the other one for chromaticity filtering. Thus the weight \( w_p(x,y) \) is obtained by the aggregation of two weights \( w_{\text{brightness}}(x,y) \) and \( w_{\text{chroma}}(x,y) \).

**OWA filter in brightness domain**

We define the brightness \( H_1 \) by:

\[
H_1 = \frac{R + G + B}{3}
\]

The weight \( w_{\text{brightness}}(x,y) \) of each pixel \( y \) lying in a spatial window centered on \( x \) depends on the brightness similarity between the pixels \( x \) and \( y \). These weights are membership values of fuzzy neighbourhood of \( x \) which is defined according to the brightness similarity between \( x \) and \( y \) (without taking into account the spatial coordinates of \( y \)). In this paper, we propose a new approach to define filters which remain robust even in presence of outliers.

Indeed, in a photometric domain, the bilateral filter uses Euclidean distance to define similarities between pixels. Unfortunately, this approach using distances is not robust because the distances could be strongly affected by outliers. We propose another way to define such similarities. Taking into account the robustness of rank filters, we propose that the ranks of distances take the place of the distances themselves. According to their ranks, we can give a weight to each pixel. The weights are assigned to ranks and not to the distances. If the rank is not changed, then a large distance or a small distance from \( x \) to \( y \) does not modify the weight assigned to \( y \). Then these weights depending on ranks can deal with outliers: the outliers which
of outliers. Moreover, as quoted before, setting the coefficient of
than the classical ones since they are more efficient in presence
new scheme for building adaptive filters which are more robust
ones are set to 0. A normalization procedure is applied in order
to have the sum of all weights equal to one. Thus we obtain an
scheme for building adaptive filters which are more robust
than the classical ones since they are more efficient in presence
of outliers. Moreover, as quoted before, setting the coefficient of
x to zero may enhance the efficiency of the filter face to outliers.
Note that the median filter (and most of rank filters) can also
exclude the value of x when computing filtered value (or vector).
Note also that this robust filter is even adaptive because we have
no tuning parameter depending on the distance itself.

We apply this approach with OWA filter in the brightness
domain. The distances \( d_{\text{brightness}}(x, y) \) between the brightness values of the center sample \( x \) and its neighbors \( y \) are computed by:

\[
d_{\text{brightness}}(x, y) = |H1(x) - H1(y)|
\]

These distances are sorted by value and the rank are set from
0 to \( N - 1 \) where \( N \) is the number of pixel samples. The weights \( w_{\text{brightness}}(x, y) \) are assigned to each pixel \( y \) in a 3 × 3 window cen-
tered on \( x \) as described previously.

**OWA filter in chromaticity domain**

The same OWA filter is applied in chromaticity domain. We
define the chromaticity space using two components \( H2 \) and \( H3 \)
where \( H2 \) is associated with the opposition Red-Green and \( H3 \) is associated with the opposition Blue-Yellow. Thus we define:

\[
\begin{align*}
H2 & = R - G \\
H3 & = B - \frac{R + G}{2}
\end{align*}
\]

Therefore the linear transformation from RGB to \( H1H2H3 \) sep-
arates color space into brightness space (\( H1 \)) and chromaticity space (\( H2H3 \)). In this chromaticity space, we use the ranks of Euclidean distances between the peaks \( (H2(y), H3(y)) \) and
\( (H2(x), H3(x)) \) to build the OWA filter. The assignment of the
chromaticity weight values \( w_{\text{chroma}}(x, x) = 0 \) follows the same
rules as the brightness weights. Pixels of rank six or less are
weighted by one whereas the others are weighted by zero. Each
weight is then divided by the sum of all weights.

**Experiments and Results**

In this section, we compare our OWA filter with the Gauss-
ian filter, the median filter and the bilateral filter. The spatial
neighborhood is the same for all the four filters and is equal to a
3 × 3 window. The photometric spread parameter \( \sigma_p \) of the bil-
ateral filter is set to almost twice the value of the standard deviation
\( \sigma \) of an hypothetic Gaussian noise deduced from the difference
between the original image and the degraded image. The Gauss-
ian and median filters are applied separately on the three color
bands (red, green and blue).

The filtered images are compared to the original ones using
the peak signal to noise ratio (PSNR) expressed in dB :

\[
\text{PSNR} = 20 \log_{10} \left( \frac{255}{\sqrt{\text{MSE}}} \right)
\]

where MSE stands for the Mean Square Error between the de-
graded image and the original one.

The nature of the restoration problem depends on the kind of
noise corrupting the images. Unfortunately the type is generally
unknown. But robust filters allow us to deal with any kind of
noise. Of course, if the kind of noise is known, then we would
rather use specific filters than this robust filter. The next tables
show the average PSNR obtained by the filters on a set of 25
images. The kind of noise applied to the images is quoted on the
first column.

**Average PSNR obtained on a set of 25 images**

<table>
<thead>
<tr>
<th>PSNR</th>
<th>Gaussian</th>
<th>Median</th>
<th>Bilateral filter</th>
<th>OWA filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian noise</td>
<td>( \sigma = 5 )</td>
<td>26.4</td>
<td>31.1</td>
<td>36.6</td>
</tr>
<tr>
<td></td>
<td>( \sigma = 10 )</td>
<td>26.0</td>
<td>29.6</td>
<td>31.6</td>
</tr>
<tr>
<td>Impulse noise</td>
<td>5%</td>
<td>24.2</td>
<td>31.4</td>
<td>20.9</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>22.5</td>
<td>30.5</td>
<td>17.9</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>20.0</td>
<td>27.8</td>
<td>15.1</td>
</tr>
</tbody>
</table>

The bilateral filter outperforms the other filters when Gauss-
ian noise is present whereas the median filter obtains almost the
best results in presence of impulse noise. It confirms the fact that
bilateral filters are well suited for Gaussian noise whereas med-
ian filters are well adapted to impulse noise. But we notice that
the OWA filter is always coming second. This means that the
OWA filter is robust face to different kind of noise.

To outline the properties of the OWA filter, we present some
results obtained with four common images in image processing
(peppers, house, Lena and mandrill, see table 3). Original im-
ages are corrupted by an additive Gaussian noise combined with
a “salt and pepper” noise:

\[
f'(x, y) = f \left( I(x, y) + \eta_g(x, y) \right)
\]

where \( \eta_g(x, y) \) corresponds to a zero mean Gaussian distribution
with a variance \( \sigma^2 \) and \( f(x) \) simulates a “salt and pepper” noise
with a probability \( p \) by:

\[
f(x) = \begin{cases} 
  x & \text{with probability } 1 - p \\
  \eta_g(x) & \text{with probability } p 
\end{cases}
\]

where \( \eta_g(x) \) is an uniform distribution in the interval \([0, 255]\).

The amount of noise is chosen to be different depending on
the color band. Such experiment makes more difficult the choice
of parameters such as the photometric spread parameter in the
bilateral filter. This may reveal the adaptive ability of the filters.
The parameters of noises applied to each color band is resumed
in table 2.

The results for the four images are quoted in table 3.
As expected, the bilateral filter obtains the worse results in presence of outliers. Indeed, when the central pixel is an outlier (a “salt or pepper” pixel), then the distances with its neighbours in the photometric domain tend to be high and the resultant weights are almost equal to zero except for the central pixel. The degraded image is therefore not affected by the bilateral filter. The result image looks very similar to the degraded ones (see figure 3). Of course, one can improve the results by increasing the spread factor in the photometric space ($\sigma_p$) but it won’t outperform the Gaussian filter results since the bilateral filter will tend in fact to a Gaussian filter (the weights in the photometric domain will all tend to one).

By setting the weight of the central pixel to zero in the OWA filter, the “salt and pepper” pixels are almost removed. However, when the percentage of outliers is greater than 10%, the performance of the OWA filter is degraded compared to the median filter, some outliers are not removed. Thus we consider that OWA filter is less robust than median filter when a very large amount of outliers are feared.

The use of the ranks to determine the weights in a chromatic space should yet prevent the outliers to affect the neighbourhood. One can notice, in the case of the median filter, that the outliers may interfere on the results in the neighbourhood depending on the context. For instance, on the figure 1 which represents a zoom on a region of the peppers image, some wrong colors appear on the limit between the brown and the red regions. These colors have almost the same hue than the outliers which lie in the neighbourhood.

Moreover, the contrast and details tend to be better preserved using OWA filters than median filters (figure 2). The whole image has a less smoothed aspect. This may explain that the OWA filters sometimes slightly outperform the median filter, despite the fact that some sets of outliers are not suppressed.

**Conclusions**

This paper proposes a new version of bilateral filter by using OWA filter. This approach of filtering using fuzzy logic allows to design robust and adaptive filters. More sophisticated filtering strategies are proposed in literature designing filter by fuzzy techniques [10]. Unfortunately these techniques are developed for higher levels of image processing using complex decision models which require a priori knowledges on the images. Thus we reject these methods because our goal is to propose low level adaptive methods without any assumption on the images.

The ability of fuzzy aggregation methods let us develop specific filters for color images. Unlike Tomasi and Manduchi [3], we take into account some elements of color perception by separating RGB vector into brightness and chromaticity vectors. Thus the similarities between colors are not evaluated by Euclidean distance in RGB space. The results show the improvement of color filter with our method when outliers are feared.

The low level filter is developed for obtaining a fast processing method. A future work will consist in developing this kind of fuzzy adaptive filters for image sequences taking into account the time component for improving the quality of color images.

**References**


Bright reflect in the eye is more preserved using OWA filter.

Zoom on Lena image corrupted image

Median filter OWA filter

The gutter contrast is slightly enhanced with the OWA filter compared to the median filter.

Zoom on house image corrupted image

Median filter OWA filter

Figure 2. Zoom on the right eye of Lena image and on the roof gutter of the house image.

Author Biography

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Figure 3. Comparing filters on images with Gaussian and “salt and pepper” noise.