Color Correction by Calculating a Metamer Boundary Descriptor

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Abstract

Color correction describes the transformation process between device RGB values and CIEXYZ resp. CIELab values. In every metamer color reproduction system this is the first color transformation after image acquisition. In general, this mapping is not unique since the spectral sensitivities of most of the devices do not satisfy the Luther condition and because the acquisition and viewing light sources have a different power spectrum.
Therefore, there exists a set of colors with different reflectance spectra which result in the same device RGB response (device metamerism), but have different color appearances for an observer under the viewing light source. We present a novel method to determine this metamere subspace (which depends on the device response) in the viewing CIELab space by calculating a Metamer Boundary Descriptor (MBD) matrix.
This MBD describes the metamer subspace approximately by storing boundary points of the set in every entry. By calculating the center of gravity of this MBD we get a good color choice in the sense of a small mean error. To calculate the entries of the MBD we use *a priori* knowledge about the physics of natural reflectance spectra and a linear programming technique. This method improves the performance of target- and regression-based methods especially in the area of saturated colors. Simulation experiments including a comparison with existing methods are given in the text.

Introduction

In every metamer color reproduction system color correction, i. e. the mapping from device RGB values to CIEXYZ or CIELab colors respectively, is the first color transformation after image acquisition.
There exist two different classes of methods for color correction in linear acquisition systems: target based methods and model based methods. Target based methods use a set of sample CIEXYZ / CIELab colors with corresponding acquired RGB values to determine an approximation of the color correction transformation. The best known method is the linear mapping between the RGB values and CIEXYZ colors. Assuming that the RGB values depend linearly on the CIEXYZ colors this transformation has to be optimal. This method determines a transformation 3×3 matrix A, using linear regression of the target colors.

$$A \cdot (R,G,B)^T = (X,Y,Z)^T \rightarrow (L^*,a^*,b^*)^T \qquad (1)$$

As a result of different acquisition and viewing light sources and non-compliance of the Luther condition by the spectral sensitivities of the image acquisition system the mapping is generally not linear and the method results in high ΔE errors.
An improvement of this method is the use of polynomial mapping of order $n > 1$

$$\begin{bmatrix} P_X(R,G,B) \\ P_Y(R,G,B) \\ P_Z(R,G,B) \end{bmatrix} = (X,Y,Z)^T \rightarrow (L^*,a^*,b^*)^T \qquad (2)$$

with

$$P_x(R,G,B) = \sum_{\substack{i,j,k=0,\\ i+j+k<=n}} a^x_{i,j,k} R^i G^j B^k \qquad (3)$$

for $x = X, Y, Z$. The coefficients can be calculated using multidimensional polynomial regression (**MPR**) of the target colors.
In the previous methods the regression was calculated in the intensity linear CIEXYZ color space. This minimizes the root mean square error between the mapped colors and the corresponding target colors, but our color difference formulas work in the CIELab color space. Therefore, an improvement in respect of small ΔE values should be achieved, if the regression can be performed directly in the CIELab color space.
Hardeberg [2] transformed the R, G, B values at first with a nonlinear function $g(x) = x^{1/3}$ to achieve a nearly linear relationship between the $g(R), g(G), g(B)$ values and the CIELab colors. This leads to a better performance of the subsequent multidimensional polynomial regression and smaller ΔE errors. The reason for choosing $g(x) = x^{1/3}$ is the transformation formula between CIEXYZ and CIELab. We call the method "multidimensional polynomial regression into CIELab based on Hardeberg" (**MPRLabH**).
König [3] transformed the R, G, B values as in equation

(1), but performed a multidimensional polynomial regression to the real target CIELab colors afterwards. We call this method "multidimensional polynomial regression into CIELab based on König" (**MPRLabK**).
If we consider noisy systems the regression based methods result in high error rates. König [3] proposed a new matrix based method as in equation (1), by calculating the coefficients using linear programming. Goal of the calculation are matrix entries, which have nearly the same size but still cause small errors. The resulting matrix does not amplify the noise as much as regression based matrices. We call the method "robust matrix" method (**RM**).
The other class of color correction methods are the model based methods. These methods use the mathematical model of the image acquisition system (see equation (4)) to reconstruct the CIELab colors from the RGB values. As mentioned before the acquisition and viewing light can differ and spectral sensitivities of the image acquisition system might not satisfy the Luther condition (i. e. the linear dependency to the sensitivities of the standard observer) (see [4]). Therefore, the reconstruction is generally not unique. There exists a set of colors with different reflectance spectra which result in the same device RGB response (device metamerism), but have different color appearances for an observer under the viewing light source.
The method of reconstructing a reflexion spectrum from a given RGB response and calculating the appropriate CIE XYZ color is not recommended, because the set of possible spectra, which lead to the given sensor response, is generally too extensive to achieve satisfying results.
Finlayson and Morovic [1] attempted to characterize the metamer subspace of possible color spectra which result in the given sensor response in the viewing CIEXYZ space. They used a linear programming technique to find the smallest cube in which the metamer subspace is located and chose the center of this cube for color correction (Linear Programming Centre of Cube - **LPCC**). They also proposed another technique to determine the desired color by sampling the mentioned cube (as a region of interest) of feasible points and take the centroid of the feasible set. A disadvantage of this method is that the characterization is performed in the intensity linear CIEXYZ color space and not in the CIELab space, where the error metrics are defined. Furthermore, the description of the metamer subspace using an enveloping cube is a very raw characterization.
The following text describes a method for characterizing the metamer subspace in the nearly perceptual uniform CIELab color space by calculating a metamer boundary descriptor matrix. The center of gravity of this matrix is an approximation of the color with the smallest ΔE distance to all other colors in the metamer subspace. This color is the optimal color choice for color correction in the sense of small mean error.

The Metamer Subspace

Supposing a linear acquisition system with the following relationship between the acquisition illuminant L_a, the sensitivities $\mathrm{s}_\mathcal{X}, \mathcal{X} = \mathrm{R}, \mathrm{G}, \mathrm{B}$, the reflection spectrum r, and the sensor response $\mathcal{X} = \mathrm{R}, \mathrm{G}, \mathrm{B}$, with additive noise ϵ and the visible wavelength interval $\Lambda = [400\mathrm{nm}, 700\mathrm{nm}]$ is

$$\mathcal{X} = \int_\Lambda \mathrm{s}_\mathcal{X}(\lambda)\mathrm{L}_a(\lambda)\mathrm{r}(\lambda)d\lambda + \epsilon \tag{4}$$

In our work the system sensitivities $\mathrm{s}_\mathcal{X}$, the acquisition light L_a and the sensor response $\mathcal{X} = \mathrm{R}, \mathrm{G}, \mathrm{B}$ are well-known. The corresponding discrete formulation by sampling the spectra in N equal wavelength intervals is

$$c = \Omega_a r + \epsilon \tag{5}$$

with $c = (R, G, B)^T$, $r = (\mathrm{r}(\lambda_1), \ldots, \mathrm{r}(\lambda_N))^T$ and the acquisition lighting matrix

$$\Omega_a = \begin{bmatrix} \mathrm{s}_R(\lambda_1)\mathrm{L}_a(\lambda_1) & \cdots & \mathrm{s}_R(\lambda_N)\mathrm{L}_a(\lambda_N) \\ \mathrm{s}_G(\lambda_1)\mathrm{L}_a(\lambda_1) & \cdots & \mathrm{s}_G(\lambda_N)\mathrm{L}_a(\lambda_N) \\ \mathrm{s}_B(\lambda_1)\mathrm{L}_a(\lambda_1) & \cdots & \mathrm{s}_B(\lambda_N)\mathrm{L}_a(\lambda_N) \end{bmatrix}$$

The accruement of an observer's tristimulus value $v_r \in$ CIEXYZ by the same reflection spectrum r under the viewing illuminant L_v is analogous. We only present the discrete form here

$$v_r = \Omega_v r \tag{6}$$

with the observer lighting matrix depending on the CIE color matching functions $\bar{x}, \bar{y}, \bar{z}$

$$\Omega_v = \begin{bmatrix} \bar{x}(\lambda_1)\mathrm{L}_v(\lambda_1) & \cdots & \bar{x}(\lambda_N)\mathrm{L}_v(\lambda_N) \\ \bar{y}(\lambda_1)\mathrm{L}_v(\lambda_1) & \cdots & \bar{y}(\lambda_N)\mathrm{L}_v(\lambda_N) \\ \bar{z}(\lambda_1)\mathrm{L}_v(\lambda_1) & \cdots & \bar{z}(\lambda_N)\mathrm{L}_v(\lambda_N) \end{bmatrix}$$

A general solution of (5) for the reflection spectrum r is

$$r = \Omega_a^\ominus(c - \epsilon) + \mathrm{Kern}(\Omega_a) \tag{7}$$

where $\Omega_a^\ominus$ is the pseudo-inverse matrix of Ω_a and $\mathrm{Kern}(\Omega_a) := \{w \mid \Omega_a w = 0\}$. If we collect a basis $w_1, \ldots, w_K$ of $\mathrm{Kern}(\Omega_a)$ into a matrix $W := (w_1, \ldots, w_K)$ we can rewrite (7) as follows

$$r = \Omega_a^\ominus(c - \epsilon) + Wu, \ \forall u \in \mathbb{R}^\mathbb{K} \tag{8}$$

This algebraic solution has to be restricted by physical conditions like the positivity, boundness and smoothness of r. So not all $u \in \mathbb{R}^\mathbb{K}$ are physically useful. We denote by $U \subset \mathbb{R}^\mathbb{K}$ the subset that fulfils these constraints in (8). If we insert (8) in (6) we get the metamer set M_{XYZ} of possible tristimuli in the CIEXYZ color space

$$M_{XYZ}^c = \{\Omega_v\Omega_a^\ominus(c - \epsilon) + \Omega_v W u \mid u \in U\} \tag{9}$$

Our error metrics (color difference formulas) are defined in the CIELab color space, so we have to map the elements of M_{XYZ} from the intensity linear CIEXYZ color space into the nearly perceptual uniform CIELab space. We denote this transformation with $\mathcal{L}$: CIEXYZ $\mapsto$ CIELab and define our metamer subspace by

$$M^c_{Lab} = \mathcal{L}(M^c_{XYZ}) \tag{10}$$

In the next section we will present a method which characterizes this subspace by a metamer boundary descriptor.

The Metamer Boundary Descriptor

The Metamer Boundary Descriptor (MBD) is a $n \times m$ matrix which stores a boundary point of the metamer subspace M^c_{Lab} in every entry. Each row contains m contour points of the set for a fixed L^* value. The boundary points will be calculated in the CIEXYZ space using the following linear programming (LP) problem, which sampled the metamer subspace along a straight line $q + \lambda v$ (figure 1)

$$-\lambda = \min \tag{11}$$

with the linear constraints

$$\begin{aligned} r &\geq 0 && (12)\\ r &\leq 1 && (13)\\ Hr &\leq p && (14)\\ -Hr &\leq p && (15)\\ \Omega_a r &= c + \epsilon && (16)\\ \Omega_v r &= v_r && (17)\\ g + \lambda v &= v_r && (18)\\ \lambda &\geq 0 && (19) \end{aligned}$$

Constraint (12) ensures the positivity of reflectance spectra and constraint (13) the boundness for non-fluorescent surfaces. In addition, we use the smoothness constraint in (14) and (15) with a smoothing parameter $p > 0$ and a convolution matrix H to apply the Laplace operator to r

$$H = \begin{bmatrix} 1 & -2 & 1 & & 0 \\ & \ddots & \ddots & \ddots & \\ 0 & & 1 & -2 & 1 \end{bmatrix} \tag{20}$$

Constraint (16) ensures that we only take into account reflectance spectra r, which lead to the given sensor response $c + \epsilon$. Constraint (17) introduces the observer's lighting matrix with the auxiliary variable v_r which is necessary for the next constraint (18). This constraint together with the objective function (11) allows us to sample in the CIEXYZ space along the mentioned line. For each boundary point the parameters g and v defining the sampling line have to be chosen in a way that the MBD entries are uniformly distributed on the boundary of M^c_{Lab}. An explanation of how to achieve this aim can be found in the next section

After solving this LP we get as results a tristimulus value v_r which is the intersection of the line and the boundary of M^c_{XYZ} and the appropriate reflection spectrum r. There only exists one intersection point, because all the constraints form a convex set. This intersection point v_r has to be transformed into CIELab coordinates and stored in the MBD matrix. After calculating the MBD matrix the

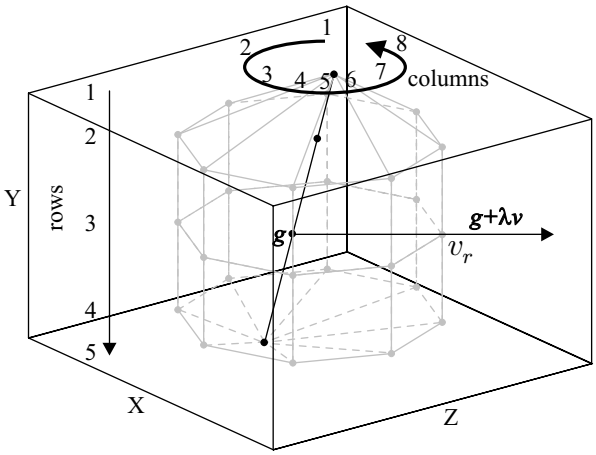

Figure 1: Sampling of the metamer subspace M^c_{XYZ} along the line $q + \lambda v, \lambda \geq 0$. The corresponding MBD matrix has 8 columns and 5 rows

color correction can be realized by choosing the center of gravity of all different points in the matrix. In our experiments this point always lies inside M^c_{Lab}.

Half Line Parameter

As mentioned before the sampling points have to be chosen in a way, that they are uniformly distributed on the boundary of M^c_{Lab}. Since the metamer subspace is sampled in the CIEXYZ color space we need some calculations to achieve this aim.

Anchor Point g: All half lines, which are used to determine a MBD row, have the same anchor point g (figure 1). Therefore, we only have to calculate n anchor points $g_i, i = 1, \ldots, n$ for a $n \times m$ MBD matrix.
We have to select the anchor points in a way that they uniformly cover the luminance spread of M^c_{Lab}. To determine this luminance spread we have to solve the following two LP problems

$$\begin{aligned} -v^Y_r r &= \min && (21)\\ v^Y_r r &= \min, \text{ resp.} && (22) \end{aligned}$$

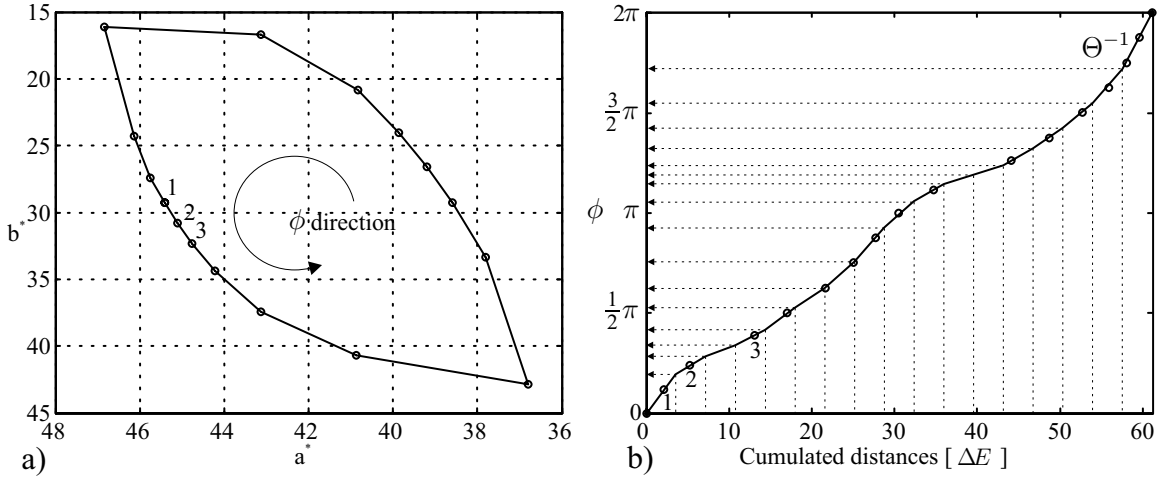

Figure 2: a) Contour plot of the metamer subspace by a special luminance level. Equidistant angles result in a non-uniform distribution of the points on the contour. b) The inverse of Θ shows the angles which result in a uniform distribution of the contour points.

with the linear constraints

$$\begin{aligned} r &\geq 0 && (23)\\ r &\leq 1 && (24)\\ Hr &\leq \rho && (25)\\ -Hr &\leq \rho && (26)\\ \Omega_a r &= c+\epsilon && (27)\\ \Omega_v r &= v_r && (28) \end{aligned}$$

The LP problem containing the objective function (21) returns a point $v_{r,\max} \in M^c_{XYZ}$ with maximal Y value. The LP problem containing the objective function (22) gives a point $v_{r,\min} \in M^c_{XYZ}$ with minimal Y value. We transform both points into the CIELab space to get the luminance spread of M^c_{Lab}

$$[L_{\min}, L_{\max}] := [\mathcal{L}_L(v_{r,\min}), \mathcal{L}_L(v_{r,\max})] \quad (29)$$

and sample this interval on n equidistant positions

$$L_i := L_{\min} + \frac{i-1}{n-1}(L_{\max} - L_{\min}), \;\; i = 1, \ldots, n. \quad (30)$$

Then we can define the desired anchor points in the CIEXYZ space in the following manner, where Y_i corresponds to $L_i, \;\; i = 1, \ldots, n$

$$g_i := v_{r,Y_{min}} + \frac{Y_i - Y_1}{Y_n - Y_1}\left(v_{r,Y_{max}} - v_{r,Y_{min}}\right). \quad (31)$$

All anchor points are located on the line section between $v_{r,\min}$ and $v_{r,\max}$ and therefore in the convex set M^c_{XYZ}.

Direction Vector v: The direction vectors have to be calculated for each MBD matrix entry. Due to the structure of the MBD their Y-components are zero.
For each luminance level, which is defined by an anchor point, we would like to choose the direction vectors in a way, that they uniformly cover the contour of M^c_{XYZ}. Therefore, we use a multigrid strategy: At first, we take a decomposition of $[0, 2\pi]$

$$\phi_k := \frac{2\pi k}{K}, \;\; k = 0, \ldots, K \quad (32)$$

and the appropriate direction vectors

$$v_k := \begin{bmatrix} \cos(\phi_k) \\ 0 \\ \sin(\phi_k) \end{bmatrix}, \;\; k = 0, \ldots, K \quad (33)$$

to calculate $K+1$ boundary points of M^c_{Lab}, using the LP problem (11)-(19). The resulting boundary points $\mathcal{L}(v_{r,k})$, $k = 0, \ldots, K$ are in general not uniformly distributed over the contour of M^c_{Lab} at the regarding luminance level (see figure 2a).
Assigning each angle $\phi_k, \;\; k = 0, \ldots, K$, a cumulative distance

$$\begin{aligned} \phi_0 &\longrightarrow \xi_0 := 0 && (34)\\ \phi_k &\longrightarrow \xi_k := \sum_{j=1}^{k} \Delta E(\mathcal{L}(v_{r,j-1}), \mathcal{L}(v_{r,j})) && (35) \end{aligned}$$

we can define a function $\Theta : [0, 2\pi] \mapsto \mathbb{R}^+$ by linear interpolation of these pairs. This function is strictly monotonic increasing and therefore invertible. Then we can define new angles by inverting Θ (see figure 2b)

$$\tilde{\phi}_i := \Theta^{-1}(\frac{i}{m}\xi_K), \;\; i = 0, \ldots, m \quad (36)$$

The corresponding direction vectors, which are defined by equation (33), result in boundary points, which are more uniformly distributed over the contour of M^c_{Lab} at the regarding luminance level. Iterating this procedure leads to nearly uniformly distributed points. In our experiments we only use one iteration step.

Results

We have compared our method (**MBD**) with the methods, which are described in the introduction, by simulation. For this, we used a Sony sensor [6] with the sensitivities shown in figure 3 and the acquisition illumination F11 (narrow band white fluorescent lamp) under different CIE standard viewing illuminants and with different databases containing reflection spectra (Vrhel database [7]: *Dupont*: 120 Dupont paint chips, *Munsell*: 64 Munsell chips and *Objects*: 170 natural and man-made objects. Additionally we have used a spectrally measured *IT8.7/2* target [5] with 288 color patches). For all target based methods the 288 colors of another *IT8.7/2* target were used as the training set. For the regression based methods we have taken the polynomial order 3, so we have to calculate 20 coefficients for each polynomial. The basis of reflection spectra which is needed by the **LPCC** method consists of 8 spectra which include 99.9% of the energy of the training set.
The MBD matrix which is used by our method has 8 columns and a variable number of rows depending on the luminance spread. We have chosen the maximal distance of two neighbouring luminance levels by $\Delta E = 0.5$, so the number of rows in the MBD matrix is

$$\text{round}(2(L_{max} - L_{min}) + 0.5). \qquad (37)$$

As smoothness parameter we have chosen $p = 0.0035$.
In a second simulation we have added Gaussian noise to the sensor response. The noise amplitude corresponds to 1% of the sensor's maximal response.
The overall results are shown in table 1. Figure 6 shows the results grouped with respect to the analyzed illuminants. Some examples of the metamer subspace structure can be found in figure 4 and 5.

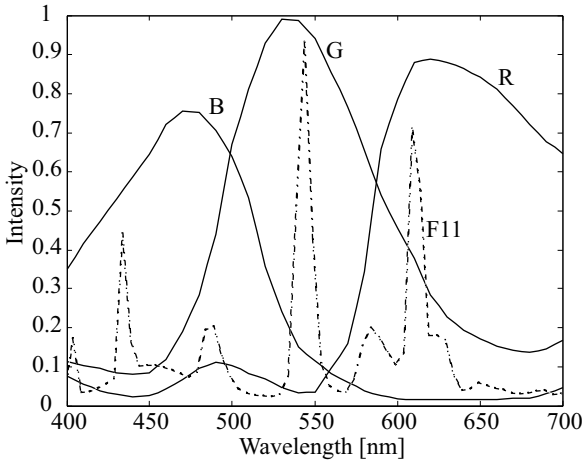

Figure 3: *Sensitivities of the Sony CCD Sensor and the acquisition illumination for simulation experiments (narrow band white fluorescent lamp - F11)*

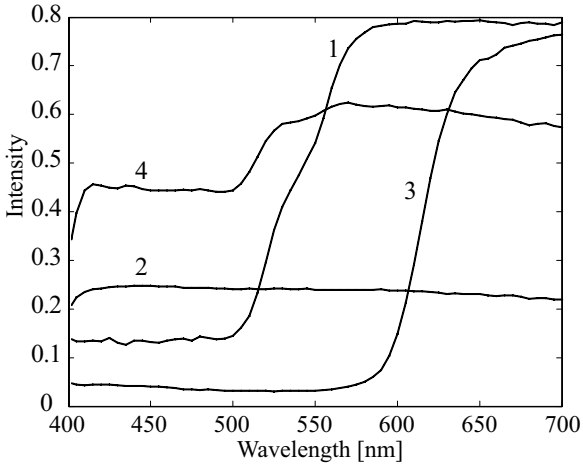

Figure 4: *Examples of some Munsell spectra extracted from the Vrhel database*

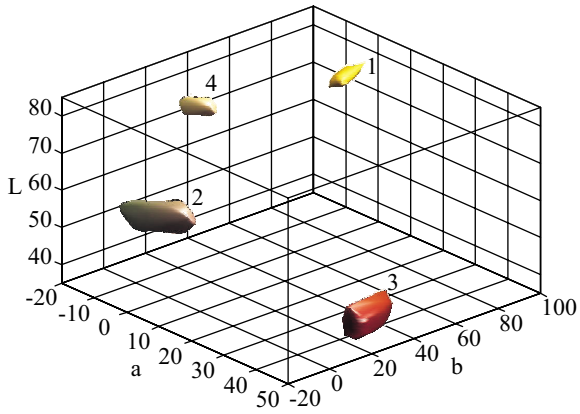

Figure 5: *Metamer subspaces of the spectra in figure 4 under viewing illuminant F2 calculated for the Sony sensor with acquisition illuminant F11*

Conclusion

A new model based method for color correction was presented and compared with other methods by simulation experiments. The new method characterizes the metamer subspace of reflexion spectra, which result in the given sensor response, by calculating a metamer boundary descriptor matrix in the CIELab color space. Using a larger matrix dimension we receive a better representation of the metamer subspace. By calculating the center of gravity of all different matrix entries we achieved a good approximation of the color with the smallest mean ΔE distance to all other possible colors and therefore a good candidate for color correction to obtain small mean errors. The results validate the performance of our method for noisy systems as well.

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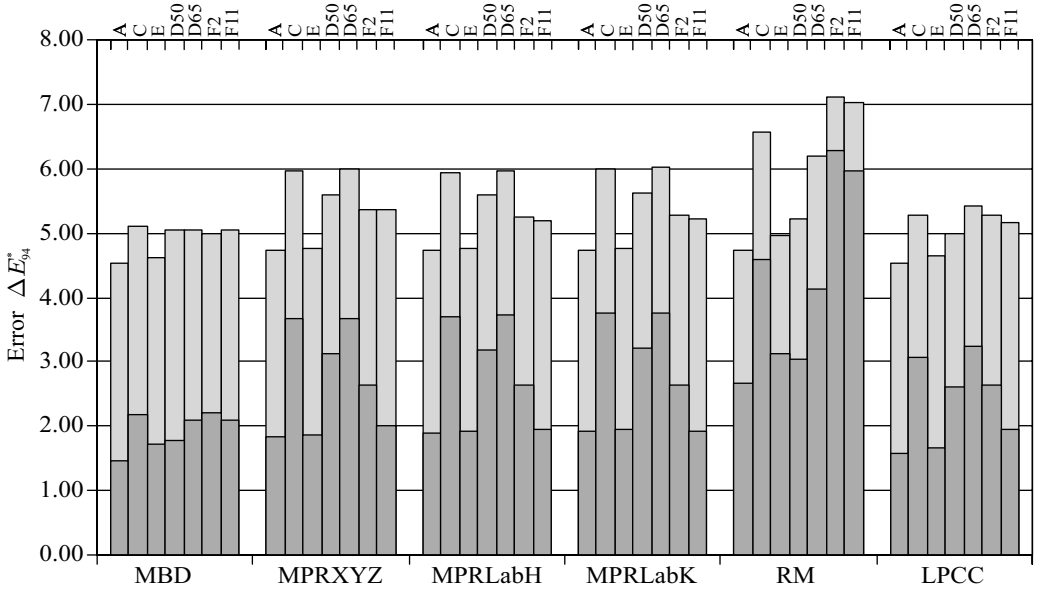

Figure 6: *Results for the Sony ICX434DQN sensor. Values refer to all test spectra and are grouped with respect to the analyzed illuminants. The darker bars correspond to the noise free system, the brighter bars to the 1% noisy system.*

	Noise free			1% Noise		
	E(ΔE^*_{94})	Std(ΔE^*_{94})	Max(ΔE^*_{94})	E(ΔE^*_{94})	Std(ΔE^*_{94})	Max(ΔE^*_{94})
MBD	1.93	1.19	10.18	4.91	0.93	38.07
MPRXYZ	2.69	1.42	10.06	5.39	0.61	40.42
MPRLabH	2.71	1.47	9.89	5.34	1.35	51.20
MPRLabK	2.74	1.53	10.28	5.38	0.90	47.87
RM	4.26	2.60	17.42	5.97	2.65	40.94
LPCC	2.39	1.72	22.34	5.04	2.19	42.88

Table 1: *Results for the Sony ICX434DQN sensor. Values refer to all illuminants and test spectra.*

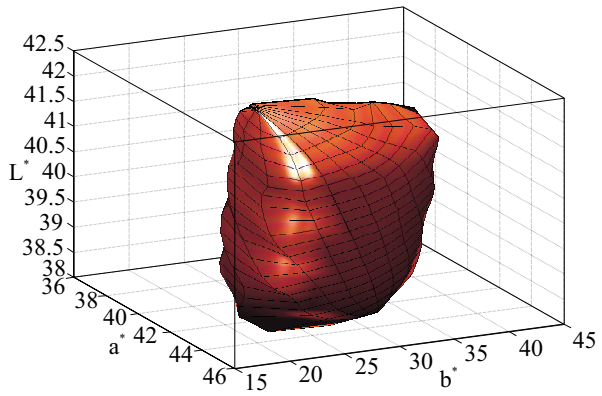

Figure 7: *Detailed view on metamer subspace no. 3 in figure 5*

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Biography

Philipp Urban received his dipl. math at the University of Hamburg, Germany in 1999. Since 1999 he is part of the research group ”Vision Systems” (http://www.tuvision.de) of the Technical University Hamburg-Harburg, Germany, and works for ”Ratio Entwicklungen GmbH”, ICC-member. His research interests are color management, gamut boundary calculation, device calibration and characterization.